



AP[®] Calculus AB 2005 Scoring Guidelines Form B

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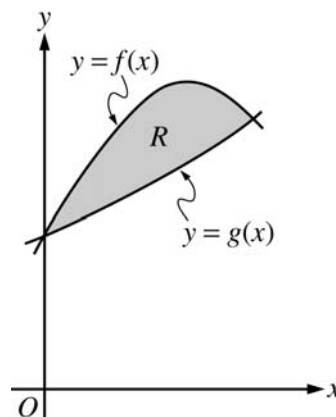
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2005 SCORING GUIDELINES (Form B)

Question 1

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is revolved about the x -axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.

(a)
$$\begin{aligned} \text{Area} &= \int_0^S (f(x) - g(x)) \, dx \\ &= \int_0^S (1 + \sin(2x) - e^{x/2}) \, dx \\ &= 0.429 \end{aligned}$$

(b)
$$\begin{aligned} \text{Volume} &= \pi \int_0^S ((f(x))^2 - (g(x))^2) \, dx \\ &= \pi \int_0^S ((1 + \sin(2x))^2 - (e^{x/2})^2) \, dx \\ &= 4.266 \text{ or } 4.267 \end{aligned}$$

(c)
$$\begin{aligned} \text{Volume} &= \int_0^S \frac{\pi}{2} \left(\frac{f(x) - g(x)}{2} \right)^2 \, dx \\ &= \int_0^S \frac{\pi}{2} \left(\frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 \, dx \\ &= 0.077 \text{ or } 0.078 \end{aligned}$$

1 : correct limits in an integral in (a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ \langle -1 \rangle \text{ each error} \\ \text{Note: } 0/2 \text{ if integral not of form} \\ c \int_a^b (R^2(x) - r^2(x)) \, dx \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 2

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

- (a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?
 (b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?
 (c) At what time t , for $0 \leq t \leq 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
 (d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k .

- (a) No; the amount of water is not increasing at $t = 15$ since $W(15) - R(15) = -121.09 < 0$.

1 : answer with reason

- (b) $1200 + \int_0^{18} (W(t) - R(t)) dt = 1309.788$
 1310 gallons

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

- (c) $W(t) - R(t) = 0$
 $t = 0, 6.4948, 12.9748$

t (hours)	gallons of water
0	1200
6.495	525
12.975	1697
18	1310

3 : $\begin{cases} 1 : \text{interior critical points} \\ 1 : \text{amount of water is least at} \\ \quad t = 6.494 \text{ or } 6.495 \\ 1 : \text{analysis for absolute minimum} \end{cases}$

The values at the endpoints and the critical points show that the absolute minimum occurs when $t = 6.494$ or 6.495 .

- (d) $\int_{18}^k R(t) dt = 1310$

2 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

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Question 3

A particle moves along the x -axis so that its velocity v at time t , for $0 \leq t \leq 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$. The particle is at position $x = 8$ at time $t = 0$.

- (a) Find the acceleration of the particle at time $t = 4$.
 (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?
 (c) Find the position of the particle at time $t = 2$.
 (d) Find the average speed of the particle over the interval $0 \leq t \leq 2$.

(a) $a(4) = v'(4) = \frac{5}{7}$

1 : answer

(b) $v(t) = 0$
 $t^2 - 3t + 3 = 1$
 $t^2 - 3t + 2 = 0$
 $(t-2)(t-1) = 0$
 $t = 1, 2$

3 : $\left\{ \begin{array}{l} 1 : \text{sets } v(t) = 0 \\ 1 : \text{direction change at } t = 1, 2 \\ 1 : \text{interval with reason} \end{array} \right.$

$v(t) > 0$ for $0 < t < 1$
 $v(t) < 0$ for $1 < t < 2$
 $v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
 The particle travels to the left when $1 < t < 2$.

(c) $s(t) = s(0) + \int_0^t \ln(u^2 - 3u + 3) du$
 $s(2) = 8 + \int_0^2 \ln(u^2 - 3u + 3) du$
 $= 8.368$ or 8.369

3 : $\left\{ \begin{array}{l} 1 : \int_0^2 \ln(u^2 - 3u + 3) du \\ 1 : \text{handles initial condition} \\ 1 : \text{answer} \end{array} \right.$

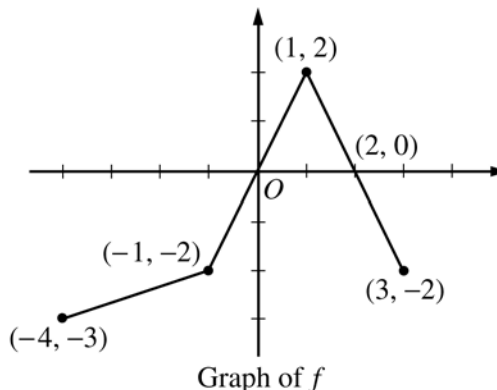
(d) $\frac{1}{2} \int_0^2 |v(t)| dt = 0.370$ or 0.371

2 : $\left\{ \begin{array}{l} 1 : \text{integral} \\ 1 : \text{answer} \end{array} \right.$

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Question 4

The graph of the function f above consists of three line segments.



(a) Let g be the function given by $g(x) = \int_{-4}^x f(t) dt$.

For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x -coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_x^3 f(t) dt$. Find all values of x in the closed interval $-4 \leq x \leq 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

(a) $g(-1) = \int_{-4}^{-1} f(t) dt = -\frac{1}{2}(3)(5) = -\frac{15}{2}$
 $g'(-1) = f(-1) = -2$
 $g''(-1)$ does not exist because f is not differentiable at $x = -1$.

3 : $\begin{cases} 1 : g(-1) \\ 1 : g'(-1) \\ 1 : g''(-1) \end{cases}$

(b) $x = 1$
 $g' = f$ changes from increasing to decreasing at $x = 1$.

2 : $\begin{cases} 1 : x = 1 \text{ (only)} \\ 1 : \text{reason} \end{cases}$

(c) $x = -1, 1, 3$

2 : correct values
 $\langle -1 \rangle$ each missing or extra value

(d) h is decreasing on $[0, 2]$
 $h' = -f < 0$ when $f > 0$

2 : $\begin{cases} 1 : \text{interval} \\ 1 : \text{reason} \end{cases}$

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Question 5

Consider the curve given by $y^2 = 2 + xy$.

(a) Show that $\frac{dy}{dx} = \frac{y}{2y-x}$.

(b) Find all points (x, y) on the curve where the line tangent to the curve has slope $\frac{1}{2}$.

(c) Show that there are no points (x, y) on the curve where the line tangent to the curve is horizontal.

(d) Let x and y be functions of time t that are related by the equation $y^2 = 2 + xy$. At time $t = 5$, the value of y is 3 and $\frac{dy}{dt} = 6$. Find the value of $\frac{dx}{dt}$ at time $t = 5$.

(a) $2yy' = y + xy'$
 $(2y - x)y' = y$
 $y' = \frac{y}{2y - x}$

2 : $\begin{cases} 1 : \text{implicit differentiation} \\ 1 : \text{solves for } y' \end{cases}$

(b) $\frac{y}{2y - x} = \frac{1}{2}$
 $2y = 2y - x$
 $x = 0$
 $y = \pm\sqrt{2}$
 $(0, \sqrt{2}), (0, -\sqrt{2})$

2 : $\begin{cases} 1 : \frac{y}{2y-x} = \frac{1}{2} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{y}{2y - x} = 0$
 $y = 0$
 The curve has no horizontal tangent since
 $0^2 \neq 2 + x \cdot 0$ for any x .

2 : $\begin{cases} 1 : y = 0 \\ 1 : \text{explanation} \end{cases}$

(d) When $y = 3$, $3^2 = 2 + 3x$ so $x = \frac{7}{3}$.

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} = \frac{y}{2y-x} \cdot \frac{dx}{dt}$$

$$\text{At } t = 5, 6 = \frac{3}{6 - \frac{7}{3}} \cdot \frac{dx}{dt} = \frac{9}{11} \cdot \frac{dx}{dt}$$

$$\frac{dx}{dt} \Big|_{t=5} = \frac{22}{3}$$

3 : $\begin{cases} 1 : \text{solves for } x \\ 1 : \text{chain rule} \\ 1 : \text{answer} \end{cases}$

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Question 6

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let

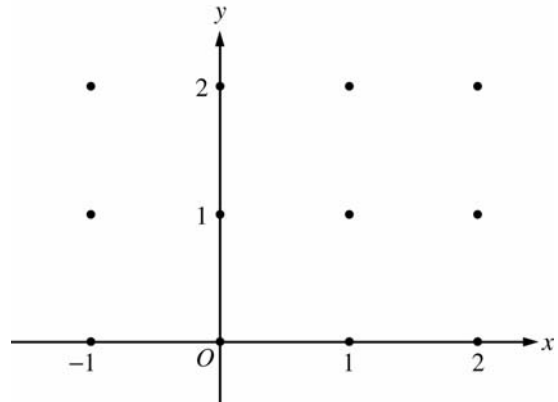
$y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

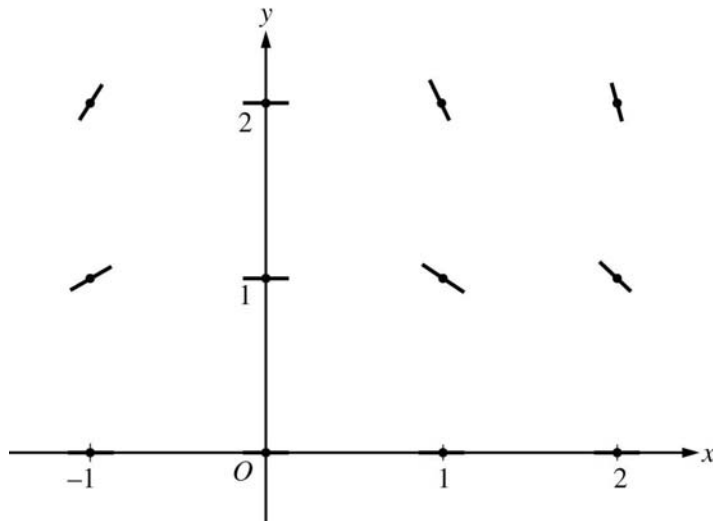
(Note: Use the axes provided in the test booklet.)

- (b) Write an equation for the line tangent to the graph of f at $x = -1$.

- (c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.



(a)



- (b) Slope = $\frac{-(-1)4}{2} = 2$
 $y - 2 = 2(x + 1)$

- (c) $\frac{1}{y^2} dy = -\frac{x}{2} dx$
 $-\frac{1}{y} = -\frac{x^2}{4} + C$
 $-\frac{1}{2} = -\frac{1}{4} + C; C = -\frac{1}{4}$
 $y = \frac{1}{\frac{x^2}{4} + \frac{1}{4}} = \frac{4}{x^2 + 1}$

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \end{cases}$

1 : equation

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables