AP ${ }^{\circledR}$ Statistics 2002 Free-Response Questions Form B

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[^0]Formulas begin on page 3 .
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Formulas
(I) Descriptive Statistics
$\bar{x}=\frac{\sum x_{i}}{n}$
$s_{x}=\sqrt{\frac{1}{n-1} \sum\left(x_{i}-\bar{x}\right)^{2}}$
$s_{p}=\sqrt{\frac{\left(n_{1}-1\right) s_{1}^{2}+\left(n_{2}-1\right) s_{2}^{2}}{\left(n_{1}-1\right)+\left(n_{2}-1\right)}}$
$\hat{y}=b_{0}+b_{1} x$
$b_{1}=\frac{\sum\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sum\left(x_{i}-\bar{x}\right)^{2}}$
$b_{0}=\bar{y}-b_{1} \bar{x}$
$r=\frac{1}{n-1} \sum\left(\frac{x_{i}-\bar{x}}{s_{x}}\right)\left(\frac{y_{i}-\bar{y}}{s_{y}}\right)$
$b_{1}=r \frac{s_{y}}{s_{x}}$
$s_{b_{1}}=\frac{\sqrt{\frac{\sum\left(y_{i}-\hat{y}_{i}\right)^{2}}{n-2}}}{\sqrt{\sum\left(x_{i}-\bar{x}\right)^{2}}}$
(II) Probability
$P(A \cup B)=P(A)+P(B)-P(A \cap B)$
$P(A \mid B)=\frac{P(A \cap B)}{P(B)}$
$E(X)=\mu_{x}=\sum x_{i} p_{i}$
$\operatorname{Var}(X)=\sigma_{x}^{2}=\Sigma\left(x_{i}-\mu_{x}\right)^{2} p_{i}$

If $X$ has a binomial distribution with parameters $n$ and $p$, then:
$P(X=k)=\binom{n}{k} p^{k}(1-p)^{n-k}$
$\mu_{x}=n p$
$\sigma_{x}=\sqrt{n p(1-p)}$
$\mu_{\hat{p}}=p$
$\sigma_{\hat{p}}=\sqrt{\frac{p(1-p)}{n}}$

If $\bar{x}$ is the mean of a random sample of size $n$ from an infinite population with mean $\mu$ and standard deviation $\sigma$, then:
$\mu_{\bar{x}}=\mu$
$\sigma_{\bar{x}}=\frac{\sigma}{\sqrt{n}}$
(III) Inferential Statistics

Standardized test statistic: $\frac{\text { statistic }- \text { parameter }}{\text { standard deviation of statistic }}$
Confidence interval: statistic $\pm$ (critical value) • (standard deviation of statistic)
Single-Sample

| Statistic | Standard Deviation <br> of Statistic |
| :---: | :---: |
| Sample Mean | $\frac{\sigma}{\sqrt{n}}$ |
| Sample Proportion | $\sqrt{\frac{p(1-p)}{n}}$ |

Two-Sample

| Statistic | Standard Deviation <br> of Statistic |
| :---: | :---: |
| Difference of <br> sample means | $\sqrt{\frac{\sigma_{1}^{2}}{n_{1}}+\frac{\sigma_{2}^{2}}{n_{2}}}$ |
| Difference of <br> sample proportions | $\sqrt{\frac{p_{1}\left(1-p_{1}\right)}{n_{1}}+\frac{p_{2}\left(1-p_{2}\right)}{n_{2}}+\frac{1}{n_{2}}}$ |
| Special case when $p_{1}=p_{2}$ |  |
|  | $\sqrt{p(1-p)} \sqrt{\frac{1}{n_{1}}+\frac{1}{n_{2}}}$ |

Chi-square test statistic $=\sum \frac{(\text { observed }- \text { expected })^{2}}{\text { expected }}$

# 2002 AP ${ }^{\circledR}$ STATISTICS FREE-RESPONSE QUESTIONS (Form B) 

## STATISTICS

## SECTION II

Part A
Questions 1-5 Spend about 65 minutes on this part of the exam.

Percent of Section II grade- 75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

1. Animal-waste lagoons and spray fields near aquatic environments may significantly degrade water quality and endanger health. The National Atmospheric Deposition Program has monitored the atmospheric ammonia at swine farms since 1978. The data on the swine population size (in thousands) and atmospheric ammonia (in parts per million) for one decade are given below.

| Year | 1988 | 1989 | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Swine <br> Population | 0.38 | 0.50 | 0.60 | 0.75 | 0.95 | 1.20 | 1.40 | 1.65 | 1.80 | 1.85 |
| Atmospheric <br> Ammonia | 0.13 | 0.21 | 0.29 | 0.22 | 0.19 | 0.26 | 0.36 | 0.37 | 0.33 | 0.38 |

(a) Construct a scatterplot for these data.

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(b) The value for the correlation coefficient for these data is 0.85 . Interpret this value.
(c) Based on the scatterplot in part (a) and the value of the correlation coefficient in part (b), does it appear that the amount of atmospheric ammonia is linearly related to the swine population size?
Explain.
(d) What percent of the variability in atmospheric ammonia can be explained by swine population size?
2. Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

| Number who <br> actually show up | 36 | 37 | 38 | 39 | 40 | 41 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.46 | 0.30 | 0.16 | 0.05 | 0.02 | 0.01 |

Assume that 41 tickets are sold for each flight.
(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?
(b) What is the expected number of no-shows for this flight?
(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?
3. A preliminary study conducted at a medical center in St. Louis has shown that treatment with small, lowintensity magnets reduces the self-reported level of pain in polio patients. During each session, a patient rested on an examining table in the doctor's office while the magnets, embedded in soft pads, were strapped to the body at the site of pain. Sessions continued for several weeks, after which pain reduction was measured.
A new study is being designed to investigate whether magnets also reduce pain in patients suffering from herniated disks in the lower back. One hundred male patients are available for the new study.
(a) Describe an appropriate design for the new study. Your discussion should briefly address treatments used, methods of treatment assignment, and what variables would be measured. Do not describe how the data would be analyzed.
(b) Would you modify the design above if, instead of 100 male patients, there were 50 male and 50 female patients available for the study? If so, how would you modify your design? If not, why not?
4. Each person in a random sample of 1,026 adults in the United States was asked the following question.
"Based on what you know about the Social Security system today, what would you like Congress and the President to do during this next year?"

The response choices and the percentages selecting them are shown below.

| Completely overhaul the system | $19 \%$ |
| :--- | ---: |
| Make some major changes | $39 \%$ |
| Make some minor adjustments | $30 \%$ |
| Leave the system the way it is now | $11 \%$ |
| No opinion | $1 \%$ |

(a) Find a $95 \%$ confidence interval for the proportion of all United States adults who would respond "Make some major changes" to the question. Give an interpretation of the confidence interval and give an interpretation of the confidence level.
(b) An advocate for leaving the system as it is now commented, "Based on this poll, only $39 \%$ of adults in the sample responded that they want some major changes made to the system, while $41 \%$ responded that they want only minor changes or no changes at all. Therefore, we should not change the system." Explain why this statement, while technically correct, is misleading.
5. At a school field day, 50 students and 50 faculty members each completed an obstacle course. Descriptive statistics for the completion times (in minutes) for the two groups are shown below.

|  | Students | Faculty Members |
| :---: | :---: | :---: |
| Mean | 9.90 | 12.09 |
| Median | 9.25 | 11.00 |
| Minimum | 3.75 | 4.50 |
| Maximum | 16.50 | 25.00 |
| Lower quartile | 6.75 | 8.75 |
| Upper quartile | 13.75 | 15.75 |

(a) Use the same scale to draw boxplots for the completion times for students and for faculty members.
(b) Write a few sentences comparing the variability of the two distributions.
(c) You have been asked to report on this event for the school newspaper. Write a few sentences describing student and faculty performances in this competition for the paper.

# 2002 AP ${ }^{\circledR}$ STATISTICS FREE-RESPONSE QUESTIONS (Form B) 

# STATISTICS 

## SECTION II

## Part B

Question 6

## Spend about 25 minutes on this part of the exam.

## Percent of Section II grade-25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.
6. In September 1990, each student in a random sample of 200 biology majors at a large university was asked how many lab classes he or she was enrolled in. The sample results are shown below.

| Number of Lab Classes | Number of Students |
| :---: | :---: |
| 0 | 28 |
| 1 | 62 |
| 2 | 58 |
| 3 | 28 |
| 4 | 16 |
| 5 | 8 |
| (Total) | 200 |

$$
\bar{x}=1.83
$$

$$
s=1.29
$$

To determine whether the distribution has changed over the past 10 years, a similar survey was conducted in September 2000 by selecting a random sample of 200 biology majors. Results from the year 2000 sample are shown below.

| Number of Lab Classes | Number of Students |
| :---: | :---: |
| 0 | 20 |
| 1 | 72 |
| 2 | 60 |
| 3 | 10 |
| 4 | 26 |
| 5 | 12 |
| (Total) | 200 |

$$
\bar{x}=1.93
$$

$$
s=1.37
$$

(a) Do the data provide evidence that the mean number of lab classes taken by biology majors in September 2000 was different from the mean number of lab classes taken in 1990 ? Perform an appropriate statistical test using $\alpha=0.10$ to answer this question.
(b) Does the test in (a) address the question of whether the distribution of number of lab classes was different in 2000 than it was in 1990 ? If so, explain your reasoning. If not, carry out an appropriate statistical test using $\alpha=0.10$ to answer this question.

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(c) Use the results of your analyses in (a) and (b) to write a few sentences that summarize how the distribution of the number of lab classes did or did not differ. Use appropriate graphs to help communicate your message. This summary should be understandable to someone who has not studied statistics.

## END OF EXAMINATION

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Table entry for $z$ is the probability lying below $z$.


Table A Standard normal probabilities

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.4 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0002 |
| -3.3 | . 0005 | . 0005 | . 0005 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0003 |
| -3.2 | . 00007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | . 0010 | . 0009 | . 0009 | . 0009 | . 0008 | . 0008 | . 0000 | . 0008 | . 0007 | . 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
| -2.9 | . 0019 | . 0018 | . 0018 | . 0017 | . 0016 | . 0016 | . 0015 | . 0015 | . 0014 | . 0014 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | . 0019 |
| -2.7 | . 0035 | . 0034 | . 0033 | . 0032 | . 0031 | . 0030 | . 0029 | . 0028 | . 0027 | . 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | . 0036 |
| -2.5 | . 0062 | . 0060 | . 0059 | . 0057 | . 0055 | . 0054 | . 0052 | . 0051 | . 0049 | . 0048 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | . 0066 | . 0064 |
| -2.3 | . 0107 | . 0104 | . 0102 | . 0099 | . 0096 | . 0094 | . 0091 | . 0089 | . 0087 | . 0084 |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | . 0110 |
| -2.1 | . 0179 | . 0174 | . 0170 | . 0166 | . 0162 | . 0158 | . 0154 | . 0150 | . 0146 | . 0143 |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
| -1.9 | . 0287 | . 0281 | . 0274 | . 0268 | . 0262 | . 0256 | . 0250 | . 0244 | . 0239 | . 0233 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | . 0294 |
| -1.7 | . 0446 | . 0436 | . 0427 | . 0418 | . 0409 | . 0401 | . 0392 | . 0384 | . 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | . 0495 | . 0485 | . 0475 | . 0465 | . 0455 |
| -1.5 | . 0668 | . 0655 | . 0643 | . 0630 | . 0618 | . 0606 | . 0594 | . 0582 | . 0571 | . 0559 |
| -1.4 | . 0808 | . 0793 | . 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
| -1.3 | . 0968 | . 0951 | . 0934 | . 0918 | . 0901 | . 0885 | . 0869 | . 0853 | . 0838 | . 0823 |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | . 1075 | . 1056 | . 1038 | . 1020 | . 1003 | . 0985 |
| -1.1 | . 1357 | . 1335 | . 1314 | . 1292 | . 1271 | . 1251 | . 1230 | . 1210 | . 1190 | . 1170 |
| -1.0 | . 1587 | . 1562 | . 1539 | . 1515 | . 1492 | . 1469 | . 1446 | . 1423 | . 1401 | . 1379 |
| -0.9 | . 1841 | . 1814 | . 1788 | . 1762 | . 1736 | . 1711 | . 1685 | . 1660 | . 1635 | . 1611 |
| -0.8 | . 2119 | . 2090 | . 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | . 1894 | . 1867 |
| -0.7 | . 2420 | . 2389 | . 2358 | . 2327 | . 2296 | . 2266 | . 2236 | . 2206 | . 2177 | . 2148 |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| -0.5 | . 3085 | . 3050 | . 3015 | . 2981 | . 2946 | . 2912 | . 2877 | . 2843 | . 2810 | . 2776 |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -0.3 | . 3821 | . 3783 | . 3745 | . 3707 | . 3669 | . 3632 | . 3594 | . 3557 | . 3520 | . 3483 |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | . 4602 | . 4562 | . 4522 | . 4483 | . 4443 | . 4404 | . 4364 | . 4325 | . 4286 | . 4247 |
| -0.0 | . 5000 | . 4960 | . 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

2002 AP ${ }^{\circledR}$ STATISTICS FREE-RESPONSE QUESTIONS (Form B)

Table entry for $z$ is the probability lying below $z$.


Table A (Continued)

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

## 2002 AP ${ }^{\circledR}$ STATISTICS FREE-RESPONSE QUESTIONS (Form B)

Table entry for $p$ and $C$ is the point $t^{*}$ with probability $p$ lying above it and probability $C$ lying between $-t^{*}$ and $t^{*}$.


Table B $\boldsymbol{t}$ distribution critical values

|  |  |  |  |  |  |  |  |  |  |
| ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |

Confidence level $C$

2002 AP ${ }^{\circledR}$ STATISTICS FREE-RESPONSE QUESTIONS (Form B)

Table entry for $p$ is the point ( $\chi^{2}$ ) with probability $p$ lying above it.

$\left(\chi^{2}\right)$

Table C $\quad \chi^{2}$ critical values

| df | Tail probability $p$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . 25 | . 20 | . 15 | . 10 | . 05 | . 025 | . 02 | . 01 | . 005 | . 0025 | . 001 | . 0005 |
| 1 | 1.32 | 1.64 | 2.07 | 2.71 | 3.84 | 5.02 | 5.41 | 6.63 | 7.88 | 9.14 | 10.83 | 12.12 |
| 2 | 2.77 | 3.22 | 3.79 | 4.61 | 5.99 | 7.38 | 7.82 | 9.21 | 10.60 | 11.98 | 13.82 | 15.20 |
| 3 | 4.11 | 4.64 | 5.32 | 6.25 | 7.81 | 9.35 | 9.84 | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4 | 5.39 | 5.99 | 6.74 | 7.78 | 9.49 | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5 | 6.63 | 7.29 | 8.12 | 9.24 | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6 | 7.84 | 8.56 | 9.45 | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.2 ! | 22.4 | 24.10 |
| 7 | 9.04 | 9.80 | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.0 | 24.3 | 26.02 |
| 8 | 10.22 | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.1 | 27.87 |
| 9 | 11.39 | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.8 | 29.67 |
| 10 | 12.55 | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.5 | 31.42 |
| 11 | 13.70 | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12 | 14.85 | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13 | 15.98 | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14 | 17.12 | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15 | 18.25 | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16 | 19.37 | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.44 | 39.2 | 41.31 |
| 17 | 20.49 | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.9: | 40.7 | 42.88 |
| 18 | 21.60 | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.3 | 44.43 |
| 19 | 22.72 | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.8 ¢ | 43.8 | 45.97 |
| 20 | 23.83 | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | $42.3<$ | 45.3 | 47.50 |
| 21 | 24.93 | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22 | 26.04 | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23 | 27.14 | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24 | 28.24 | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25 | 29.34 | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26 | 30.43 | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.8: | 54.0 | 56.41 |
| 27 | 31.53 | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.4 | 57.86 |
| 28 | 32.62 | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.5 | 56.8 | 59.30 |
| 29 | 33.71 | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.3 | 60.73 |
| 30 | 34.80 | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.3: | 59.7 | 62.16 |
| 40 | 45.62 | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50 | 56.33 | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60 | 66.98 | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80 | 88.13 | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1 | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |


[^0]:    These materials were produced by Educational Testing Service ${ }^{\circledR}\left(\right.$ ETS $\left.^{\circledR}\right)$, which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

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