# $\int^{\bullet}$ CollegeBoard Advanced Placement Program 

AP ${ }^{\oplus}$ Statistics 2007 Scoring Guidelines<br>Form B

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# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 1

## Intent of Question

The three primary goals of this question are to assess a student's ability to: (1) construct a stemplot from a given data set; (2) describe the important features of the plot; and (3) discuss how a single measure of centrality fails to convey important features of the plot.

## Solution

## Part (a):

0| 89
1| 26878993640
2|
3| 3856
4| $143 \quad$ Legend: $1 \mid 2$ represents 12 questions answered correctly
$O R$, with ordered leaves (not required)
0| 89
1| 02346678899
2|
3| 3568
4|134 Legend: $1 \mid 2$ represents 12 questions answered correctly
$O R$, with repeated stems (leaves may be ordered or not)

0H 89
1L 0234
1H 6678899
2 L
2H
3L 3
3H 568 Legend: 3H 6 represents 36 questions answered correctly
4L 134
4L 1 represents 41 questions answered correctly

## Part (b):

The most striking feature of the plot is that the scores cluster into two groups, one concentrated in the mid-teens and the other in the high 30s (or one with relatively low scores on the exam and one with relatively high scores). There are no scores in the 20 s.

## Part (c):

A measure of center might fall between the two groups (as does the mean of 22.95 here) where there is no data and would not provide an accurate picture of student performance on the exam. It would not indicate that students tended to score either very well or very poorly on the exam.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 1 (continued)

## Scoring

This question is scored in four sections: section 1 is part (a), and sections 2 to 4 consist of elements of parts (b) and (c).

Section 1 is scored as either essentially correct (E) or incorrect (I).
Section 1 is essentially correct (E) if in part (a) the student gives a correctly constructed stemplot. Any other type of plot is incorrect (I).

NOTE: One or two misplaced or omitted leaves can still be considered essentially correct as long as the important features of the display are not altered.

Parts (b) and (c) are scored together in three sections, each of which is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 2 is essentially correct (E) if in either part (b) or (c) the student clearly notices:

1. that there are two groups;
2. that there is a gap in the middle of the distribution;
3. the relative or specific positions of the two groups, OR
the location of the gap, OR
a general measure of location (such as mean, median, or the fact that most scores fall between 10 and 19). $($ Median $=18$, mean $=22.95)$

Section 2 is partially correct $(\mathrm{P})$ if the student notes two out of the three.
Section 3 is essentially correct (E) if in part (b) or part (c) the solution is given in the context of the problem and is communicated well.

Section 3 is partially correct $(\mathrm{P})$ if the student mentions the context (for instance, using the word "scores"), but communication of the context is weak.

Section 3 is incorrect (I) if the context is not mentioned at all.
Section 4 is essentially correct (E) if in part (c) a valid reason is given for why a measure of center is not sufficient for data of this type (with the two groups and a gap). If, for instance, the reasoning would apply equally well to other shapes, it is not sufficient.

Section 4 can be at most partially correct $(\mathrm{P})$ if a student does not recognize the groups or gap. It is partially correct if the student compares the mean and median and cites outliers or skewness as the reason why a measure of center is not sufficient, or if a general reason is given for why a measure of center is not sufficient. (For instance, the student may say that center alone without some measure of spread is never sufficient.)

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 1 (continued)

4 Complete Response
All four sections essentially correct

## 3 Substantial Response

Three sections essentially correct and no sections partially correct
OR
Two sections essentially correct and two sections partially correct

## 2 Developing Response

Two sections essentially correct and no sections partially correct
OR
One section essentially correct and two sections partially correct
Note: A score cannot exceed 2 if (1) the student fails to notice either of the two distinct groups of scores or the gap between the groups, and (2) the response to part (c) mentions neither the two groups nor the gap.

1 Minimal Response
One section essentially correct and no sections partially correct
OR
No sections essentially correct and two sections partially correct
If a response is between two scores (for example, $2^{1 / 2}$ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 2

## Intent of Question

The three primary goals of this question are to assess a student's ability to: (1) calculate a probability from a display of population frequencies; (2) calculate a binomial probability; and (3) describe a sampling distribution of a sample mean for a moderately large sample.

## Solution

## Part (a):

$$
P(X>3)=0.07+0.04+0.04+0.02=0.17 .
$$

## Part (b):

$Y=$ number of households in violation.
$Y$ has a binomial distribution with $n=10$ and $p=0.17$.

$$
P(Y=2)=\binom{10}{2}(0.17)^{2}(0.83)^{8}=0.2929
$$

## Part (c):

The distribution of $\bar{X}$ will:

1. be approximately normal;
2. have mean $\mu_{\bar{x}}=\mu=1.65$;
3. have standard deviation $\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}=\frac{1.851}{\sqrt{150}}=0.1511$.

## Scoring

This question is scored in four sections. Each section is scored as either essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is part (a), section 2 is part (b), and sections 3 and 4 consist of elements of part (c). This scoring gives part (c) double weight relative to either part (a) or part (b).

Section 1 is essentially correct (E) if $P(X>3)$ is correctly computed and work is shown in part (a).
Section 1 is partially correct ( P ) if:
$P(X \geq 3)=0.26$ is computed;
OR
a correct numerical answer is given but no work is shown.

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## Question 2 (continued)

Section 2 is essentially correct (E) if in part (b):

1. the probability from part (a) is correctly used to calculate the probability that exactly 2 households are in violation, either using the binomial pdf or using general probability rules, AND
2. work is shown.

Section 2 is partially correct $(\mathrm{P})$ if in part (b):
the student computes $P(Y \geq 2)=0.5270$ or $P(Y \leq 2)=0.7659$ instead of $P(Y=2)$;
OR
the correct probability is given but no work is shown;
OR
the binomial coefficient is omitted $\left[(0.83)^{8}(0.17)^{2}=0.0065\right]$.
Section 3 is essentially correct (E) if the response to part (c):
recognizes that the distribution of $\bar{X}$ will be approximately normal;
OR
the response says that the distribution of $\bar{X}$ is more symmetric than the population distribution AND mentions that the population distribution is highly skewed.

Section 3 is partially correct ( P ) if the response to part (c) reports a normal distribution for $\bar{X}$ without indicating that the normal distribution is an approximation.

Section 4 is essentially correct (E) if the response to part (c) provides the appropriate mean $\mu_{\bar{x}}=\mu=1.65$ and standard deviation $\sigma_{\bar{X}}=\frac{\sigma}{\sqrt{n}}=\frac{1.851}{\sqrt{150}}=0.1511$ for $\bar{X}$.

Section 4 is partially correct ( P ) if the response to part (c):
provides either the correct mean or the correct standard deviation for $\bar{X}$, but not both;
OR
provides correct numerical values for both the mean and standard deviation but sample notation
( $\bar{X}$ and $s$ ) is used instead of population notation ( $\mu$ and $\sigma_{\bar{X}}=\sigma / \sqrt{n}$ );
OR
says only that the $\bar{X}$ distribution is centered in the same place as the population and has a smaller standard deviation than the population (and does not give the values of 1.65 and 0.1511 ).

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 2 (continued)

4 Complete Response
All four sections essentially correct
3 Substantial Response
Three sections essentially correct and no section partially correct
OR
Two sections essentially correct and two sections partially correct
2 Developing Response
Two sections essentially correct and no sections partially correct
OR
One section essentially correct and two sections partially correct
OR
No sections essentially correct and four sections partially correct
1 Minimal Response
One section essentially correct and no sections partially correct
OR
No sections essentially correct and two sections partially correct
If a response is between two scores (for example, $2^{1 / 2}$ points) use a holistic approach to determine whether to score up or down depending on the strength of the response and communication. If the word "approximately" is missing in part (c), round down.

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## Question 3

## Intent of Question

The primary goals of this question are to: (1) assess a student's ability to use blocking in designing an experiment, and (2) describe a mechanism for randomly assigning treatments to experimental units in the context of the selected blocking.

## Solution

## Part (a):

Acceptable blocking schemes:
\(\left.\begin{array}{llll}Blocks: \& \begin{array}{l}1 and 12 <br>

6 and 7\end{array} \& 2 and 3 \& 8 and 9\end{array}\right]\)\begin{tabular}{l}
10 and 5 <br>
<br>
<br>
Blocks:

 

1 and 12 \& 2 and 5 \& 3 and 4 <br>
6 and 7 \& 8 and 11 \& 9 and 10
\end{tabular}

Blocks: 1 and $12 \quad 2,3,4$, and $5 \quad 6$ and $7 \quad 8,9,10$, and 11
We want to create blocks of homogeneous "units." Exposure (side of house) would have an effect on heat gain through a window, so the best blocking scheme would take side of house into account when creating blocks.

The blocking schemes above create blocks that are similar with respect to exposure (side of house). Since there are two treatments (types of windows), the optimal blocking scheme would create blocks consisting of two window boxes each.

## Part (b):

For each block we could select one of the window boxes and then flip a coin to determine which type of window would be installed in that window box. For example, if the coin lands face up, install type A; otherwise install type B. Continue this process until half of the windows in the block are assigned to one type, then install the remaining window type in the other boxes.

## Scoring

Each part is scored as either essentially correct (E), partially correct (P), or incorrect (I).
Part (a) is essentially correct (E) if:

1. one of the sets of blocks given in the solution is identified;
2. the justification for the blocking scheme demonstrates an understanding that windows of both types should be used in equal numbers on each side of the house because of differing exposure to sun, light, heat, etc.

Part (a) is partially correct (P) if it includes one of the two elements above.

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## Question 3 (continued)

Part (a) can be partially correct $(\mathrm{P})$ if the student confuses treatments with blocks, for instance by assigning the two "blocks" to be even- and odd-numbered window boxes. The justification must be that the exposure is similar for the boxes on each side of the house, and thus each side must receive equal numbers of type A and type B windows.

Part (b) is essentially correct (E) if it:

1. assigns window types at random to the window boxes in each block in a way that is consistent with the blocks the student identifies in part (a) and that ensures an equal number of each type of window within each block;
2. describes a mechanism for random assignment, such as a coin toss, roll of a die, use of random number table, etc.

Part (b) is partially correct $(\mathrm{P})$ if it includes only one of the two elements above.

## NOTES:

- If students confuse treatments with blocks in part (a), they receive credit for the first element above only if they are logically consistent. It is consistent if they use a scheme that randomly assigns half of each type of window to go on each wall.
- By itself, "at random" is not sufficient as an answer and should be scored as incorrect.
- By itself, "at random within each block" is not sufficient for an essentially correct answer but can be scored as partially correct.


## 4 Complete Response

Both parts essentially correct

## 3 Substantial Response

One part essentially correct and the other part partially correct

## 2 Developing Response

One part essentially correct and the other part incorrect
OR
Both parts partially correct

## 1 Minimal Response

One part partially correct

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 4

## Intent of Question

The goals of this question are to assess a student's ability to: (1) plot a least squares regression line; (2) examine a residual; and (3) discuss the effect of an additional observation on an estimated correlation coefficient and on the least squares estimate of the slope of a line.

## Solution

Parts (a) and (b):


When $x=67, \hat{y}=35.1+0.427(67)=63.709$
and the residual $=y-\hat{y}=61-63.709=-2.709$.

## Part (c):

See the new point indicated in the plot above. The slope would remain about the same since the new point is consistent with the linear pattern in the original plot (i.e., close to the line).

The correlation coefficient would increase. We know that $b=r \frac{S_{y}}{S_{x}}$. The added point will increase $S_{x}$ more than it will increase $S_{y}$ so $\frac{S_{y}}{S_{x}}$ will be less than 1 . If the slope is to stay the same, $r$ must increase. OR
This point fits the pattern well and has an $x$ value that is far from $\bar{X}$.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 4 (continued)

## Scoring

This problem is scored in 4 sections. Section 1 consists of the graphical parts of (a) and (b) together. Section 2 consists of the numerical parts of (b). Section 3 consists of the first part of (c). Section 4 consists of the second part of (c).

Each section is scored as either essentially correct (E), partially correct (P), or incorrect (I).
Section 1 (graphical parts of $a$ and $b$ ) is essentially correct (E) if:

1. the regression line is drawn correctly on the scatterplot;
2. the point $(67,61)$ is circled and the vertical segment corresponding to the residual is drawn on the scatterplot.

Section 1 is partially correct $(\mathrm{P})$ if the response includes one of the above two elements.
Section 2 (numerical part of $b$ ) is essentially correct ( E ) if the residual is correctly computed as -2.709 ; OR
the response states that the residual was approximated using the graph, a reasonable value for the residual is given, and the sign of the residual is correct.

Section 2 is partially correct $(\mathrm{P})$ if the magnitude of the residual is correct but the sign is wrong.
Section 3 (first part of (c)) is essentially correct (E) if it:

1. states that the slope will remain about the same (or change slightly);
2. provides an explanation based on the new point fitting the pattern in the original plot.

Section 3 is partially correct $(\mathrm{P})$ if it states that the slope will be about the same, but the explanation is missing or incorrect.

NOTE: If the line is drawn incorrectly in part (a), and the answer to this part is consistent with the line drawn, section 3 is essentially correct (E).

Section 4 (second part of (c)) is essentially correct (E) if it:

1. states that the value of the correlation coefficient will increase;
2. provides an explanation based on the relative changes in $s_{x}$ and $s_{y}$ OR
based on the fact that the new point fits the pattern AND is far out in the $x$ direction, OR because the linear pattern is stronger.

Section 4 is partially correct $(\mathrm{P})$ if it states that the value of the correlation coefficient will increase, but the explanation is missing or incorrect.

NOTE: If the response just says that the correlation coefficient will increase because the point is close to the line, section 4 is partially correct.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 4 (continued)

## 4 Complete Response

All four sections essentially correct

## 3 Substantial Response

Three sections essentially correct and no sections partially correct
OR
Two sections essentially correct and two sections partially correct

2 Developing Response
Two sections essentially correct and no sections partially correct
OR
One section essentially correct and two sections partially correct
OR
Four parts partially correct

1 Minimal Response
One section essentially correct and no sections partially correct
OR
No sections essentially correct and two sections partially correct
If a response is between two scores (for example, $2^{1 ⁄ 2}$ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 5

## Intent of Question

The primary intent of this question is to assess a student's ability to make an inference about the difference in two population means using a two-sample $t$-test, including the identification of the null and alternative hypotheses and good communication of test results and conclusions.

## Solution

Part 1: States a correct pair of hypotheses

$$
\mathrm{H}_{\mathrm{o}}: \mu_{\mathrm{S}}-\mu_{N}=0 \text { versus } \mathrm{H}_{\mathrm{a}}: \mu_{S}-\mu_{N}<0
$$

OR

$$
\mathrm{H}_{\mathrm{o}}: \mu_{N}-\mu_{\mathrm{S}}=0 \text { versus } \mathrm{H}_{\mathrm{a}}: \mu_{N}-\mu_{\mathrm{S}}>0
$$

OR

$$
\mathrm{H}_{\mathrm{o}}: \mu_{\mathrm{S}}=\mu_{\mathrm{N}} \text { versus } \mathrm{H}_{\mathrm{a}}: \mu_{\mathrm{S}}<\mu_{N}
$$

where $\mu_{s}=$ mean decrease in cholesterol for standard drug
and $\mu_{N}=$ mean decrease in cholesterol for new drug.

Part 2: Identifies a correct test (by name or by formula) and checks appropriate assumptions.
Two-sample $t$-test (or $z$-test)

$$
t=\frac{\bar{X}_{s}-\bar{X}_{N}}{\sqrt{\frac{s_{s}^{2}}{n_{S}}+\frac{s_{N}^{2}}{n_{N}}}}
$$

Assumptions:

1. random assignment of subjects to treatments;
2. normal population distributions or large samples.

Both sample sizes were large (50), and there was random assignment of subjects to treatments.
NOTE: A two-sample $z$-test is acceptable as long as the large sample sizes are noted. A pooled $t$-test is also acceptable, but the student must also state and comment on the plausibility of the equal population variances assumption.

Part 3: Correct mechanics, including the value of the test statistic, df (stated or implied by calculator work), and $p$-value (or rejection region).

$$
t=\frac{\bar{X}_{S}-\bar{X}_{N}}{\sqrt{\frac{s_{S}^{2}}{n_{S}}+\frac{s_{N}^{2}}{n_{N}}}}=\frac{10-18}{\sqrt{\frac{8^{2}}{50}+\frac{12^{2}}{50}}}=-3.92
$$

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) <br> Question 5 (continued) 

$\mathrm{df}=85, p$-value $=0.000088$, or from table $p$-value $<0.001$
OR
$\mathrm{df}=49, p$-value $=0.00014$.
For two-sample $z$-test, $z=-3.92$, , $p$-value $=0.000044$.
For pooled $t$-test, $t=-3.92, \mathrm{df}=98, p$-value $=0.000081$.

Rejection regions:

$$
\begin{aligned}
& \alpha=0.10: t<-1.303(\mathrm{df}=40), t<-1.292(\mathrm{df}=80), t<-1.290(\mathrm{df}=100) \mathrm{OR} z<-1.28 \\
& \alpha=0.05: t<-1.684(\mathrm{df}=40), t<-1.664(\mathrm{df}=80), t<-1.660(\mathrm{df}=100) \text { OR } z<-1.645 \\
& \alpha=0.01: t<-2.423(\mathrm{df}=40), t<-2.374(\mathrm{df}=80), t<-2.364(\mathrm{df}=100) \mathrm{OR} z<-2.33
\end{aligned}
$$

## Part 4: Stating a correct conclusion in the context of the problem, using the result of the statistical test.

Because the $p$-value $<$ selected $\alpha$ (or because the $p$-value is so small), reject $H_{0}$. There is convincing evidence that the mean cholesterol reduction is greater for the new drug.

## Scoring

Each part is scored as either essentially correct (E), partially correct (P), or incorrect (I).

Part 1 is essentially correct ( E ) if the response:

1. includes the correct pair of hypotheses;
2. defines the parameters in the hypotheses in the context of the problem.

Part 1 is partially correct $(\mathrm{P})$ if the hypotheses are stated correctly, but notation is not defined.
Part 2 is essentially correct ( E ) if the response:

1. identifies the correct test by name or formula;
2. checks appropriate assumptions (including equal variance if pooled $t$-test is used).

Part 2 is partially correct $(\mathrm{P})$ if it includes only one of the two elements above.

Part 3 is essentially correct ( E ) if the response includes:

1. correct mechanics, including the value of the test statistic;
2. df and $p$-value or rejection region consistent with the hypotheses in part 1.

Part 3 is partially correct ( P ) if it includes only one of the two elements above.

Part 4 is essentially correct (E) if the response includes:

1. a conclusion in context consistent with the hypotheses in part 1 ;
2. linkage between the results of the test in part 3 and the conclusion, and this is communicated well.

Part 4 is partially correct ( P ) if it includes only one of the two elements above.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 5 (continued)

NOTES:

- If both an $\alpha$ and a $p$-value are given, the linkage is implied. If no $\alpha$ is given, the solution must be explicit about the linkage by giving a correct interpretation of the $p$-value or explaining how the conclusion follows from the $p$-value.
- If the $p$-value in part 3 is incorrect but the conclusion is consistent with the computed $p$-value, part 4 can be considered essentially correct (E).


## 4 Complete Response

All four parts essentially correct

## 3 Substantial Response

Three parts essentially correct and no parts partially correct
OR
Two parts essentially correct and two parts partially correct

## 2 Developing Response

Two parts essentially correct and no parts partially correct
$O R$
One part essentially correct and two parts partially correct
OR
Four parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct
OR
No parts essentially correct and two parts partially correct
If a response is between two scores (for example, $\mathbf{2}^{1 ⁄ 2}$ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 6

## Intent of Question

The primary intent of this question is to assess a student's ability to: (1) make an inference about the difference in two population proportions; (2) examine a regression model for a linear trend in proportions; and (3) construct a confidence interval for a slope. The investigative part of this question requires a student to use a regression model to estimate survival probabilities for two different situations and make an inference about the expected number of surviving species that would be achieved.

## Solution

## Part (a):

Part 1: States a correct pair of hypotheses

$$
\mathrm{H}_{\mathrm{o}}: p_{L}-p_{S}=0 \quad \text { versus } \quad \mathrm{H}_{\mathrm{a}}: p_{L}-p_{S}<0
$$

OR

$$
\mathrm{H}_{\mathrm{o}}: p_{S}-p_{L}=0 \quad \text { versus } \quad \mathrm{H}_{\mathrm{a}}: p_{S}-p_{L}>0
$$

OR

$$
\mathrm{H}_{\mathrm{o}}: p_{L}=p_{S} \quad \text { versus } \quad \mathrm{H}_{\mathrm{a}}: p_{L}<p_{S}
$$

Where
$p_{L}$ is the proportion going extinct on large islands, and
$p_{S}$ is the proportion going extinct on small islands.

Part 2: Identifies a correct test (by name or by formula) and checks appropriate assumptions.
Two-sample test for proportions

$$
Z=\frac{\hat{p}_{L}-\hat{p}_{S}}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_{L}}+\frac{\hat{p}(1-\hat{p})}{n_{S}}}}
$$

Assumptions: independent observations and large sample sizes.
The problem states that whether one species becomes extinct is independent of whether another species becomes extinct, and that the probability of extinction is the same for all species on large islands and for all species on small islands, so it is reasonable to assume that observations are independent.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) 

## Question 6 (continued)

$$
\begin{array}{ll}
\hat{p}_{L}=0.091 & \hat{p}_{S}=0.221 \\
n_{L} \hat{p}_{L}=19 & n_{L}\left(1-\hat{p}_{L}\right)=189 \\
n_{S} \hat{p}_{S}=66 & n_{S}\left(1-\hat{p}_{S}\right)=233
\end{array}
$$

All are greater than 5 (or 10), so the sample sizes are large enough to proceed.
Part 3: Correct mechanics, including the value of the test statistic and $p$-value (or rejection region).

$$
\begin{aligned}
& \quad \hat{p}=\frac{19+66}{208+299}=\frac{85}{507}=0.168 \\
& z=\frac{\hat{p}_{L}-\hat{p}_{S}}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_{L}}+\frac{\hat{p}(1-\hat{p})}{n_{S}}}}=\frac{0.091-0.221}{\sqrt{\frac{(0.168)(0.832)}{208}+\frac{(0.168)(0.832)}{299}}}=\frac{-0.130}{0.034}=-3.82 \\
& p \text {-value }=0.00006
\end{aligned}
$$

(from table $p$-value $\approx 0$; graphing calculator: $z=-3.836233478, p$-value $=0.00006$ )
Part 4: Stating a correct conclusion in the context of the problem, using the result of the statistical test.
Because the $p$-value is less than the stated $\alpha$ (or because the $p$-value is so small, or because the test statistic is in the rejection region), reject $\mathrm{H}_{0}$. There is sufficient evidence that the proportion of species becoming extinct is smaller for large islands than for small islands.

If both an $\alpha$ and a $p$-value are given, the linkage is implied. If no $\alpha$ is given, the solution must be explicit about the linkage by giving a correct interpretation of the $p$-value or explaining how the conclusion follows from the $p$-value.

If the $p$-value in part 3 is incorrect but the conclusion is consistent with the computed $p$-value, part 4 can be considered as correct.

## Part (b):

Compute a 95 percent confidence interval for the slope of the regression line.
Part 1: Identifies appropriate confidence interval by name or by formula.
The confidence interval for the slope of the regression line is $b \pm t s_{b}$.
Part 2: Checks appropriate assumptions.
Assumptions: The residual plot shows no unusual patterns that would suggest violation of the assumptions, so it is reasonable to proceed.

# AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B) <br> Question 6 (continued) 

Part 3: Correct mechanics.

$$
\begin{aligned}
\mathrm{df}=n-2 & =13-2=11 \\
& -0.05323 \pm 2.20(0.00618) \\
& -0.05323 \pm 0.013596 \\
& (-0.0668,-0.0396)
\end{aligned}
$$

## Part 4: Interpretation.

We are 95 percent confident that the mean proportion of species going extinct decreases by somewhere between 0.03 and 0.06 with each increase of 1 unit in $\ln (a r e a)$. The proportion of species going extinct decreases with increasing area.

## Part (c):

From part (b) it appears that the proportion of species going extinct decreases with increasing area. Therefore the proportion of species going extinct is related to the size of the island. Because the island sizes differed within the large island group and within the small island group, the assumption is probably not reasonable.

## Scoring

Each part is scored as either essentially correct (E), partially correct (P), or incorrect (I).
Part (a) is essentially correct (E) if three or four parts of the hypothesis test are correct.
Part (a) is partially correct ( P ) if one or two parts of the hypothesis test are correct.
NOTE: For part 2 of (a), the independent observations assumption does not have to be addressed in the response to get credit for this part, since this is given in the stem of the problem.

Part (b) is essentially correct (E) if three or four parts of the confidence interval are correct.
Part (b) is partially correct ( P ) if one or two parts of the confidence interval are correct.
Part (c) is essentially correct (E) if the response:

1. states the assumptions are not reasonable, $A N D$
2. gives a justification based on the information in part (b);

OR
says that the assumptions are reasonable based on an incorrect conclusion in part (b) that island size is not related to extinction proportion, with an appropriate explanation.

## AP ${ }^{\circledR}$ STATISTICS <br> 2007 SCORING GUIDELINES (Form B)

## Question 6 (continued)

Part (c) is partially correct $(\mathrm{P})$ if:
it says that the assumption is not reasonable, but the explanation is weak or does not appeal to the information in part (b);
OR
it says that the assumption is not reasonable because the negative estimate of the slope given in part (b) is misinterpreted to suggest that survival rates decrease as area increases;
OR
it appeals to part (b) but says that the assumption is reasonable because within each group (large/small), the island sizes don't vary too much;
OR
it says the assumption is reasonable because the negative estimate of the slope given in part (b) is misinterpreted;
OR
the justification appeals to the differing proportions in the original data table only.
Part (c) is incorrect if a choice is made but no justification is given.
Part (d) is essentially correct (E) if the large preserve is chosen and the decision is well supported based on the expectation that a larger number of species will be preserved, in comparison to the expected number preserved on the five small islands.

Part (d) is partially correct ( P ) if:
the large preserve is chosen based on the results from parts (a) and/or (b);
OR
the large preserve is chosen but the justification is weak;
OR
the five small preserves are chosen based on an incorrect computation of the number of species saved for the two scenarios.

Part (d) is incorrect if:
a choice is made (large or five small) but no justification is given;
OR
five small preserves are chosen based only on the fact that there are 80 rather than 70 species at the outset.

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## Question 6 (continued)

4 Complete Response
All four parts essentially correct
3 Substantial Response
Three parts essentially correct and no parts partially correct
OR
Two parts essentially correct and two parts partially correct
2 Developing Response
Two parts essentially correct and no parts partially correct
OR
One part essentially correct and two parts partially correct
Four parts partially correct
1 Minimal Response
One part essentially correct and no parts partially correct
OR
No parts essentially correct and two parts partially correct
If a response is between two scores (for example, $2^{1 / 2}$ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

