# **AP**<sup>°</sup>

# **AP<sup>®</sup> Statistics** 2014 Free-Response Questions

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Formulas begin on page 3. Questions begin on page 6. Tables begin on page 14.

# Formulas

(I) **Descriptive Statistics** 

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_{x} = \sqrt{\frac{1}{n-1} \Sigma (x_{i} - \overline{x})^{2}}$$

$$s_{p} = \sqrt{\frac{(n_{1} - 1)s_{1}^{2} + (n_{2} - 1)s_{2}^{2}}{(n_{1} - 1) + (n_{2} - 1)}}$$

$$s_p = \sqrt{(n_1 - 1) + (n_2 - 1)}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

 $b_0 = \overline{y} - b_1 \overline{x}$ 

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \overline{x}}{s_x} \right) \left( \frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum \left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\sum \left(x_i - \overline{x}\right)^2}}$$

#### (II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

 $E(X) = \mu_X = \sum x_i p_i$ 

$$\operatorname{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If *X* has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

 $\mu_{\chi} = np$ 

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If  $\overline{x}$  is the mean of a random sample of size *n* from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

 $\mu_{\overline{x}} = \mu$ 

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$ 

Confidence interval: statistic  $\pm$  (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

#### Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	Special case when $\sigma_1 = \sigma_2$ $\sigma_1 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
	Special case when $p_1 = p_2$
	$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Chi-square test statistic =	$\sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

#### STATISTICS SECTION II Part A

#### Questions 1-5

#### Spend about 65 minutes on this part of the exam.

#### Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- 1. An administrator at a large university is interested in determining whether the residential status of a student is associated with level of participation in extracurricular activities. Residential status is categorized as on campus for students living in university housing and off campus otherwise. A simple random sample of 100 students in the university was taken, and each student was asked the following two questions.
  - Are you an on campus student or an off campus student?
  - In how many extracurricular activities do you participate?

The responses of the 100 students are summarized in the frequency table shown.

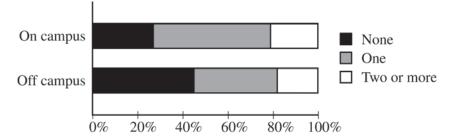
	Resident	Residential Status					
Level of Participation in Extracurricular Activities	On campus	Off campus	Total				
No activities	9	30	39				
One activity	17	25	42				
Two or more activities	7	12	19				
Total	33	67	100				

(a) Calculate the proportion of on campus students in the sample who participate in <u>at least one</u> extracurricular activity and the proportion of off campus students in the sample who participate in <u>at least one</u> extracurricular activity.

On campus proportion:

Off campus proportion:

The responses of the 100 students are summarized in the segmented bar graph shown.



(b) Write a few sentences summarizing what the graph reveals about the association between residential status and level of participation in extracurricular activities among the 100 students in the sample.

- (c) After verifying that the conditions for inference were satisfied, the administrator performed a chi-square test of the following hypotheses.
  - $H_0$ : There is no association between residential status and level of participation in extracurricular activities among the students at the university.
  - H<sub>a</sub> : There is an association between residential status and level of participation in extracurricular activities among the students at the university.

The test resulted in a *p*-value of 0.23. Based on the *p*-value, what conclusion should the administrator make?

- 2. Nine sales representatives, 6 men and 3 women, at a small company wanted to attend a national convention. There were only enough travel funds to send 3 people. The manager selected 3 people to attend and stated that the people were selected at random. The 3 people selected were women. There were concerns that no men were selected to attend the convention.
  - (a) Calculate the probability that randomly selecting 3 people from a group of 6 men and 3 women will result in selecting 3 women.
  - (b) Based on your answer to part (a), is there reason to doubt the manager's claim that the 3 people were selected at random? Explain.
  - (c) An alternative to calculating the exact probability is to conduct a simulation to estimate the probability. A proposed simulation process is described below.

Each trial in the simulation consists of rolling three fair, six-sided dice, one die for each of the convention attendees. For each die, rolling a 1, 2, 3, or 4 represents selecting a man; rolling a 5 or 6 represents selecting a woman. After 1,000 trials, the number of times the dice indicate selecting 3 women is recorded.

Does the proposed process correctly simulate the random selection of 3 women from a group of 9 people consisting of 6 men and 3 women? Explain why or why not.

- 3. Schools in a certain state receive funding based on the number of students who attend the school. To determine the number of students who attend a school, one school day is selected at random and the number of students in attendance that day is counted and used for funding purposes. The daily number of absences at High School A in the state is approximately normally distributed with mean of 120 students and standard deviation of 10.5 students.
  - (a) If more than 140 students are absent on the day the attendance count is taken for funding purposes, the school will lose some of its state funding in the subsequent year. Approximately what is the probability that High School A will lose some state funding?
  - (b) The principals' association in the state suggests that instead of choosing one day at random, the state should choose 3 days at random. With the suggested plan, High School A would lose some of its state funding in the subsequent year if the mean number of students absent for the 3 days is greater than 140. Would High School A be more likely, less likely, or equally likely to lose funding using the suggested plan compared to the plan described in part (a)? Justify your choice.
  - (c) A typical school week consists of the days Monday, Tuesday, Wednesday, Thursday, and Friday. The principal at High School A believes that the number of absences tends to be greater on Mondays and Fridays, and there is concern that the school will lose state funding if the attendance count occurs on a Monday or Friday. If one school day is chosen at random from each of 3 typical school weeks, what is the probability that none of the 3 days chosen is a Tuesday, Wednesday, or Thursday?

- 4. As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be \$189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was \$189,952!"
  - (a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?
  - (b) The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.
    - Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.
    - <u>Method 2</u>: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.

Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988 ? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

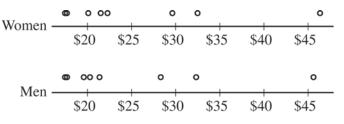
5. A researcher conducted a study to investigate whether local car dealers tend to charge women more than men for the same car model. Using information from the county tax collector's records, the researcher randomly selected one man and one woman from among everyone who had purchased the same model of an identically equipped car from the same dealer. The process was repeated for a total of 8 randomly selected car models.

The purchase prices and the differences (woman – man) are shown in the table below. Summary statistics are also shown.

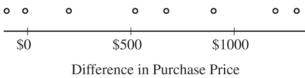
Car model	1	2	3	4	5	6	7	8
Women	\$20,100	\$17,400	\$22,300	\$32,500	\$17,710	\$21,500	\$29,600	\$46,300
Men	\$19,580	\$17,500	\$21,400	\$32,300	\$17,720	\$20,300	\$28,300	\$45,630
Difference	\$520	-\$100	\$900	\$200	-\$10	\$1,200	\$1,300	\$670

	Mean	Standard Deviation
Women	\$25,926.25	\$9,846.61
Men	\$25,341.25	\$9,728.60
Difference	\$585.00	\$530.71

Dotplots of the data and the differences are shown below.



Purchase Price (in thousands of dollars)



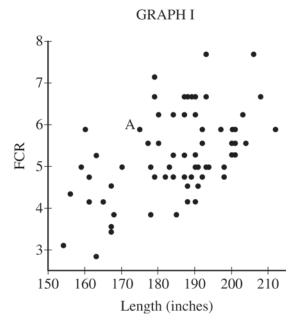
(woman – man, in dollars)

Do the data provide convincing evidence that, on average, women pay more than men in the county for the same car model?

## STATISTICS SECTION II Part B Question 6 Spend about 25 minutes on this part of the exam. Percent of Section II score—25

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Jamal is researching the characteristics of a car that might be useful in predicting the fuel consumption rate (FCR); that is, the number of gallons of gasoline that the car requires to travel 100 miles under conditions of typical city driving. The length of a car is one explanatory variable that can be used to predict FCR. Graph I is a scatterplot showing the lengths of 66 cars plotted with the corresponding FCR. One point on the graph is labeled A.



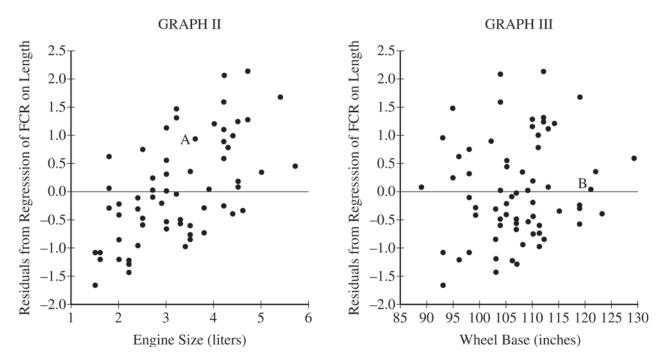
Jamal examined the scatterplot and determined that a linear model would be a reasonable way to express the relationship between FCR and length. A computer output from a linear regression is shown below.

Linear Fit FCR = -1.595789 + 0.0372614 \* Length

Summary of FitRSquare0.250401Root Mean Square Error0.902382Observations66

(a) The point on the graph labeled A represents one car of length 175 inches and an FCR of 5.88. Calculate and interpret the residual for the car relative to the least squares regression line.

Jamal knows that it is possible to predict a response variable using more than one explanatory variable. He wants to see if he can improve the original model of predicting FCR from length by including a second explanatory variable in addition to length. He is considering including engine size, in liters, or wheel base (the length between axles), in inches. Graph II is a scatterplot showing the engine size of the 66 cars plotted with the corresponding residuals from the regression of FCR on length. Graph III is a scatterplot showing the regression of FCR on length.



(b) In graph II, the point labeled A corresponds to the same car whose point was labeled A in graph I. The measurements for the car represented by point A are given below.

FCR	Length (inches)	Engine Size (liters)	Wheel Base (inches)
5.88	175	3.6	93

- (i) Circle the point on graph III that corresponds to the car represented by point A on graphs I and II.
- (ii) There is a point on graph III labeled B. It is very close to the horizontal line at 0. What does that indicate about the FCR of the car represented by point B?

- (c) Write a few sentences to compare the association between the variables in graph II with the association between the variables in graph III.
- (d) Jamal wants to predict FCR using length and one of the other variables, engine size or wheel base. Based on your response to part (c), which variable, engine size or wheel base, should Jamal use in addition to length if he wants to improve the prediction? Explain why you chose that variable.

#### STOP

#### END OF EXAM

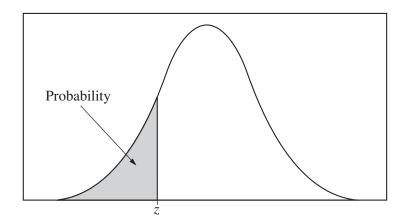


Table entry for z is the probability lying below z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

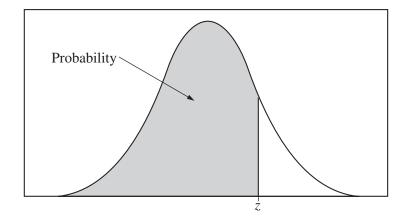


Table entry for z is the probability lying below z.

#### Table A(Continued)

z.00.01.02.03.04.05.06.07.080.0.5000.5040.5080.5120.5160.5199.5239.5279.53190.1.5398.5438.5478.5517.5557.5596.5636.5675.57140.2.5793.5832.5871.5910.5948.5987.6026.6064.61030.3.6179.6217.6255.6293.6331.6368.6446.6443.64800.4.6554.6591.66628.6064.6700.6736.6772.6808.68440.5.6915.6950.6985.7019.7054.7088.7123.7157.71900.6.7257.7291.7324.7357.7738.7423.7454.7486.75170.7.7580.7611.7642.7673.7704.7734.7764.7794.78230.8.7881.7910.7397.7995.8023.8051.8078.81060.9.8159.8166.8212.8238.8264.8289.8315.8340.83651.0.8413.8461.8485.8508.8531.8554.8577.85991.1.8643.8665.8082.9099.9113.9147.91621.2.8849.8869.8888.8907.8925.8944.8962.8980.89971.3.9032.9046.9945<										. (00000	
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$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	.5359	.5319	.5279	.5239	.5199	.5160	.5120	.5080	.5040	.5000	0.0
0.3	.5753	.5714	.5675	.5636	.5596	.5557	.5517	.5478	.5438	.5398	0.1
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	.6141	.6103	.6064	.6026	.5987	.5948	.5910	.5871	.5832	.5793	0.2
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	.6517	.6480	.6443	.6406	.6368	.6331	.6293	.6255	.6217	.6179	0.3
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	.6879	.6844	.6808	.6772	.6736	.6700	.6664	.6628		.6554	0.4
$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	.7224	.7190	.7157	.7123	.7088	.7054	.7019	.6985	.6950	.6915	0.5
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	.7549	.7517		.7454		.7389	.7357	.7324		.7257	0.6
0.9.8159.8186.8212.8238.8264.8289.8315.8340.83651.0.8413.8438.8461.8485.85088531.8554.8577.85991.1.8643.8665.8686.8708.8729.8749.8770.8790.88101.2.8849.8869.8888.8907.8925.8944.8962.8980.89971.3.9032.9049.9066.9082.9099.9115.9131.9147.91621.4.9192.9207.9222.9236.9251.9265.9279.9292.93061.5.9332.9345.9357.9370.9382.9394.9406.9418.94291.6.9452.9463.9474.9484.9495.9505.9515.9525.95351.7.9554.9564.9573.9582.9591.9599.9608.9616.96251.8.9641.9649.9656.9664.9671.9678.9686.9693.96991.9.9713.9719.9726.9732.9738.9744.9750.9756.97612.0.9772.9778.9783.9788.9878.9803.9808.98122.1.9821.9866.9890.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9906.9909.9911.99132	.7852	.7823	.7794	.7764	.7734	.7704	.7673	.7642	.7611	.7580	0.7
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.8133	.8106		.8051		.7995		.7939	.7910	.7881	0.8
1.1.8643.8665.8686.8708.8729.8749.8770.8790.8810 $1.2$ .8849.8869.8888.8907.8925.8944.8962.8980.8997 $1.3$ .9032.9049.9066.9082.9099.9115.9131.9147.9162 $1.4$ .9192.9207.9222.9236.9251.9265.9279.9292.9306 $1.5$ .9332.9345.9357.9370.9382.9394.9406.9418.9429 $1.6$ .9452.9463.9474.9484.9495.9505.9515.9525.9535 $1.7$ .9554.9564.9573.9582.9591.9599.9608.9616.9625 $1.8$ .9641.9649.9656.9664.9671.9678.9686.9693.9699 $1.9$ .9713.9719.9726.9732.9738.9744.9750.9756.9761 $2.0$ .9772.9778.9783.9783.9793.9788.9803.9808.9812 $2.1$ .9821.9826.9830.9834.9838.9842.9846.9850.9854 $2.2$ .9861.9864.9868.9871.9875.9878.9881.9884.9887 $2.3$ .9893.9896.9898.9901.9904.9906.9909.9911.9913 $2.4$ .9918.9920.9922.9927.9929.9931 <td>.8389</td> <td>.8365</td> <td></td> <td>.8315</td> <td>.8289</td> <td>.8264</td> <td>.8238</td> <td>.8212</td> <td>.8186</td> <td>.8159</td> <td>0.9</td>	.8389	.8365		.8315	.8289	.8264	.8238	.8212	.8186	.8159	0.9
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.8621	.8599	.8577	.8554	8531	.8508	.8485	.8461	.8438	.8413	1.0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.8830	.8810		.8770		.8729		.8686	.8665	.8643	1.1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.9015	.8997	.8980	.8962	.8944			.8888	.8869	.8849	1.2
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.9177	.9162		.9131						.9032	1.3
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.9319	.9306	.9292	.9279	.9265	.9251	.9236	.9222	.9207	.9192	1.4
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	.9441	.9429	.9418	.9406	.9394	.9382	.9370	.9357	.9345	.9332	1.5
1.8.9641.9649.9656.9664.9671.9678.9686.9693.96991.9.9713.9719.9726.9732.9738.9744.9750.9756.97612.0.9772.9778.9783.9788.9793.9798.9803.9808.98122.1.9821.9826.9830.9834.9838.9842.9846.9850.98542.2.9861.9864.9868.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9545	.9535	.9525	.9515	.9505	.9495	.9484	.9474	.9463	.9452	1.6
1.9.9713.9719.9726.9732.9738.9744.9750.9756.97612.0.9772.9778.9783.9788.97939798.9803.9808.98122.1.9821.9826.9830.9834.9838.9842.9846.9850.98542.2.9861.9864.9868.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9633	.9625	.9616	.9608	.9599	.9591	.9582	.9573	.9564	.9554	1.7
2.0.9772.9778.9783.9788.97939798.9803.9808.98122.1.9821.9826.9830.9834.9838.9842.9846.9850.98542.2.9861.9864.9868.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9706	.9699	.9693	.9686	.9678	.9671	.9664	.9656	.9649	.9641	1.8
2.1.9821.9826.9830.9834.9838.9842.9846.9850.98542.2.9861.9864.9868.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9767	.9761	.9756	.9750		.9738	.9732	.9726	.9719	.9713	1.9
2.2.9861.9864.9868.9871.9875.9878.9881.9884.98872.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9817	.9812	.9808	.9803	9798	.9793	.9788	.9783		.9772	2.0
2.3.9893.9896.9898.9901.9904.9906.9909.9911.99132.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9970.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9857	.9854	.9850	.9846		.9838	.9834	.9830	.9826	.9821	2.1
2.4.9918.9920.9922.9925.9927.9929.9931.9932.99342.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9970.9971.9972.99732.8.9974.9975.9976.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9890	.9887	.9884	.9881	.9878	.9875	.9871	.9868	.9864	.9861	2.2
2.5.9938.9940.9941.9943.9945.9946.9948.9949.99512.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9970.9971.9972.99732.8.9974.9975.9976.9977.9977.9978.9979.99802.9.9981.9982.9982.9983.9984.9984.9985.9985.9986										.9893	2.3
2.6.9953.9955.9956.9957.9959.9960.9961.9962.99632.7.9965.9966.9967.9968.9969.9970.9971.9972.99732.8.9974.9975.9976.9977.9977.9978.9979.99802.9.9981.9982.9983.9984.9984.9985.9985.9986	.9936	.9934		.9931				.9922	.9920	.9918	2.4
2.7.9965.9966.9967.9968.9969.9970.9971.9972.99732.8.9974.9975.9976.9977.9977.9978.9979.99802.9.9981.9982.9982.9983.9984.9984.9985.9985.9986	.9952	.9951		.9948						.9938	2.5
2.8.9974.9975.9976.9977.9977.9978.9979.99802.9.9981.9982.9982.9983.9984.9984.9985.9985.9986											
2.9         .9981         .9982         .9983         .9984         .9984         .9985         .9986								.9967		.9965	
	.9981	.9980									
	.9986	.9986	.9985	.9985	.9984	.9984	.9983	.9982	.9982	.9981	2.9
		.9990	.9989	.9989	9989	.9988	.9988	.9987	.9987	.9987	3.0
3.1 .9990 .9991 .9991 .9991 .9992 .9992 .9992 .9993	.9993	.9993								.9990	
3.2 .9993 .9993 .9994 .9994 .9994 .9994 .9995 .9995		.9995		.9994		.9994				.9993	3.2
3.3         .9995         .9995         .9996         .9996         .9996         .9996         .9996         .9996         .9996											3.3
3.4 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9997 .9997	.9998	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	3.4

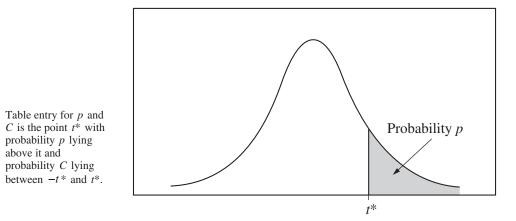


 Table B
 t distribution critical values

	Tail probability <i>p</i>											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
$\infty$	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
					(	Confidence	evel C					

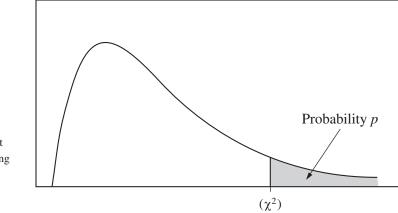


Table entry for p is the point  $(\chi^2)$  with probability p lying above it.

Table C	$\chi^2$	critical values

						Tail prol	pability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2