AP Physics B – Practice Workbook – Book 1

Mechanics, Fluid Mechanics and Thermodynamics

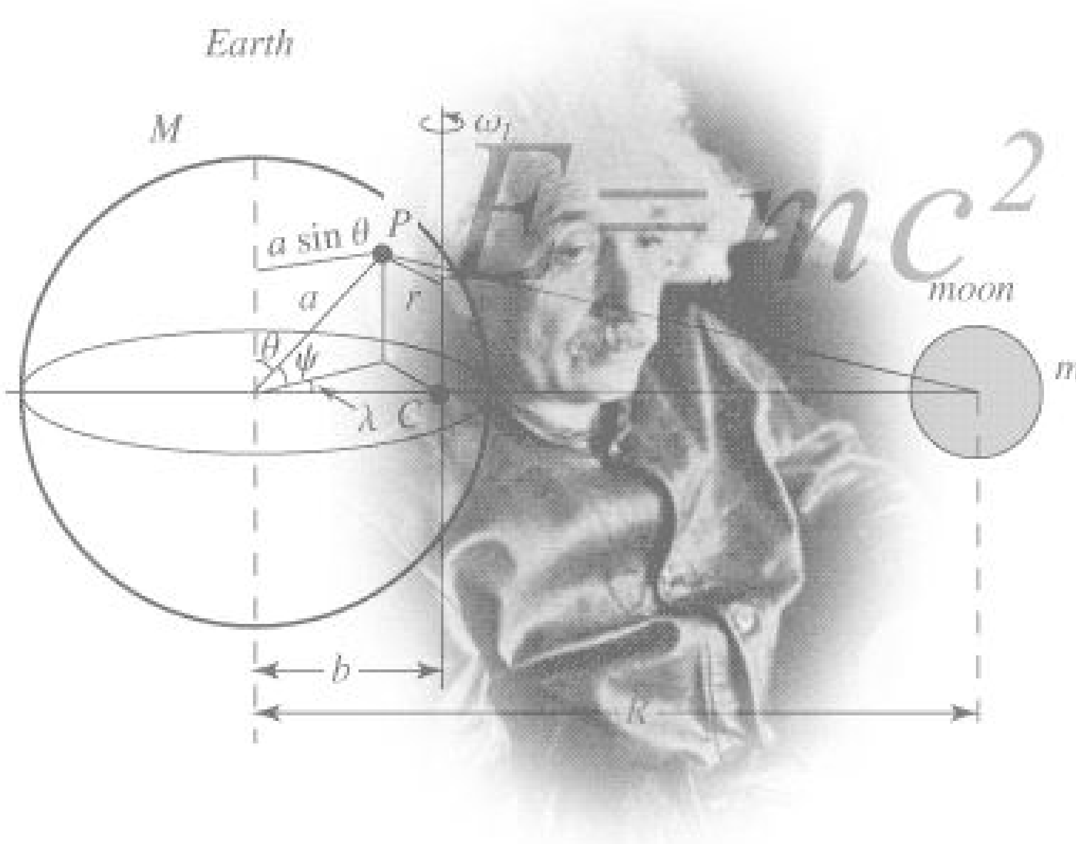


Table of Contents

Chapter 9 Thermodynamics

This book is a compilation of all the problems published by College Board in AP Physics B and AP Physics C that are appropriate for the AP B level as well as problems from AAPT's Physics Bowl and U.S. Physics Team Qualifying Exams organized by topic.

The problems vary in level of difficulty and type and this book represents an invaluable resource for practice and review and should be used… often. Whether you are struggling or confident in a topic, you should be doing these problems as a reinforcement of ideas and concepts on a scale that could never be covered in the class time allotted.

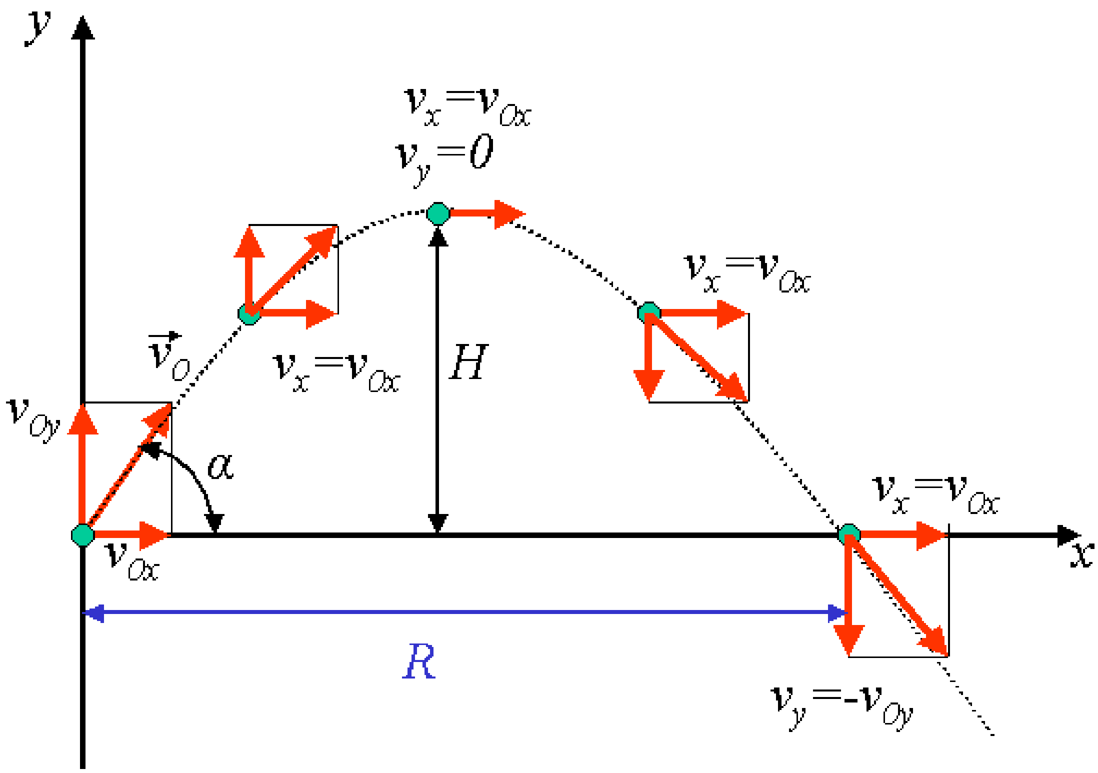
The answers as presented are not the only method to solving many of these problems and physics teachers may present slightly different methods and/or different symbols and variables in each topic, but the underlying physics concepts are the same and we ask you read the solutions with an open mind and use these differences to expand your problem solving skills.

Finally, we *are* fallible and if you find any typographical errors, formatting errors or anything that strikes you as unclear or unreadable, please let us know so we can make the necessary announcements and corrections.

Problems marked with an asterisk (*) are challenging problems that some would consider to be outside the scope of the course, but rely on the concepts taught within the course or they may require information taught in a later part of the course. These are for those students who wish to go beyond the level needed, but are not required for success in the AP B course.

Chapter 1

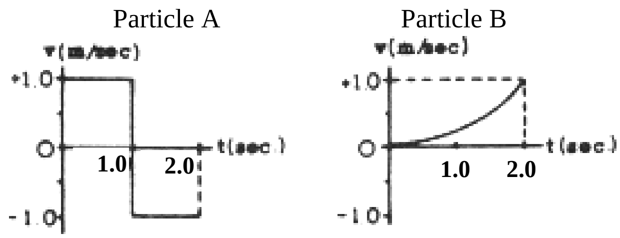
Kinematics



AP Physics Multiple Choice Practice – Kinematics

1. A car travels 30 miles at an average speed of 60 miles per hour and then 30 miles at an average speed of 30 miles per hour. The average speed the car over the 60 miles is
(A) 35 m.p.h. (B) 40 m.p.h. (C) 45 m.p.h. (D) 10 m.p.h. (E) 53 m.p.h.

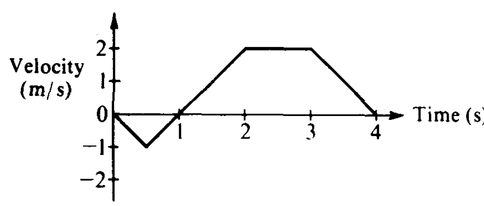
Questions 2 – 4 relate to two particles that start at x = 0 at t = 0 and move in one dimension independently of one another. Graphs, of the velocity of each particle versus time are shown below



2. Which particle is farthest from the origin at t = 2 seconds.
(A) A (B) B (C) they are in the same location at t = 2 seconds (D) They are the same distance from the origin, but in opposite directions (E) It is not possible to determine

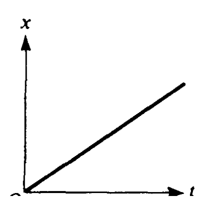
3. Which particle moves with constant non-zero acceleration?
(A) A (B) B (C) both A and B (D) neither A nor B (E) It is not possible to determine

4. Which particle is in its initial position at t = 2 seconds?
(A) A (B) B (C) both A and B (D) neither A nor B (E) It is not possible to determine



5. The graph above shows the velocity versus time for an object moving in a straight line. At what time after t = 0 does the object again pass through its initial position?
(A) Between 0 and 1 s (B) 1 s (C) Between 1 and 2 s (D) 2 s (E) Between 2 and 3 s

6. A body moving in the positive x direction passes the origin at time t = 0. Between t = 0 and t = 1 second, the body has a constant speed of 24 meters per second. At t = 1 second, the body is given a constant acceleration of 6 meters per second squared in the negative x direction. The position x of the body at t = 11 seconds is
(A) + 99m (B) + 36m (C) – 36 m (D) – 75 m (E) – 99 m

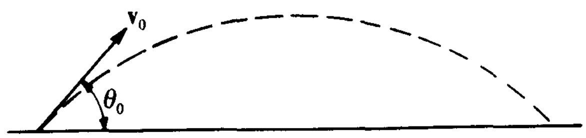


7. The displacement, x, of an object moving along the x-axis is shown above as a function of time, t. The acceleration of this object must be
(A) zero (B) constant but not zero (C) increasing (D) decreasing (E) equal to g

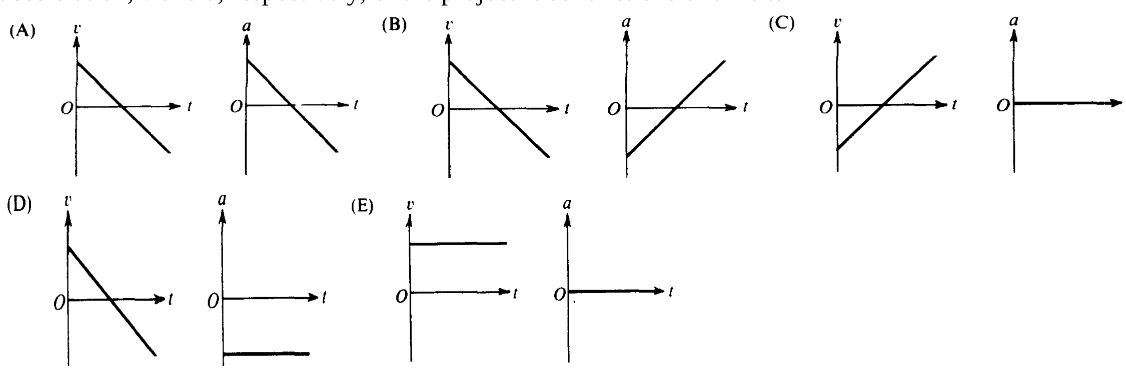
8. A 2-kilogram block rests at the edge of a platform that is 10 meters above level ground. The block is launched horizontally from the edge of the platform with an initial speed of 3 meters per second. Air resistance is negligible. The time it will take for the block to reach the ground is most nearly
(A) 0.3 s (B) 1.0 s (C) 1.4 s (D) 2.0 s (E) 3.0 s

9. A diver initially moving horizontally with speed v dives off the edge of a vertical cliff and lands in the water a distance *d* from the base of the cliff. How far from the base of the cliff would the diver have landed if the diver initially had been moving horizontally with speed 2v?
(A) d (B) $\sqrt{2d}$ (C) $2d$ (D) $4d$ (E) can't be determined without knowing the height of the cliff

10. A truck traveled 400 meters north in 80 seconds, and then it traveled 300 meters east in 70 seconds. The magnitude of the average velocity of the truck was most nearly
(A) 1.2 m/s (B) 3.3 m/s (C) 4.6 m/s (D) 6.6 m/s (E) 9.3 m/s

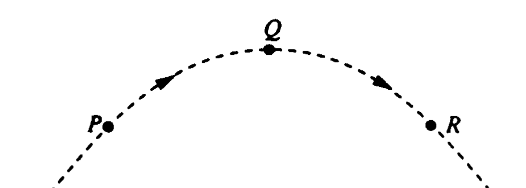


11. A projectile is fired with initial velocity v_o at an angle θ_0 with the horizontal and follows the trajectory shown above. Which of the following pairs of graphs best represents the vertical components of the velocity and acceleration, v and a, respectively, of the projectile as functions of time t?



12. An object is released from rest on a planet that has no atmosphere. The object falls freely for 3.0 meters in the first second. What is the magnitude of the acceleration due to gravity on the planet?
(A) 1.5 m/s^2 (B) 3.0 m/s^2 (C) 6.0 m/s^2 (D) 10.0 m/s^2 (E) 12.0 m/s^2

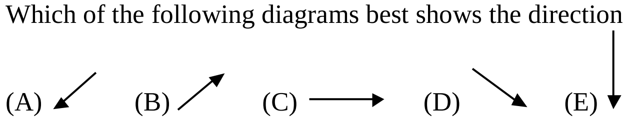
<u>Questions 13 – 14</u>

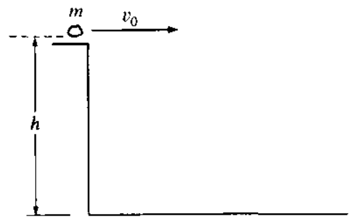


A ball is thrown and follows the parabolic path shown above. Air friction is negligible. Point Q is the highest point on the path. Points P and R are the same height above the ground.

13. How do the speeds of the ball at the three points compare?
(A) $v_P < v_Q < v_R$ (B) $v_R < v_Q < v_P$ (C) $v_Q < v_R < v_P$ (D) $v_Q < v_P = v_R$ (E) $v_P = v_R < v_Q$

14. Which of the following diagrams best shows the direction of the acceleration of the ball at point P?

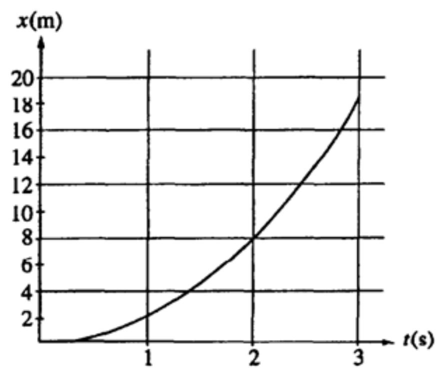




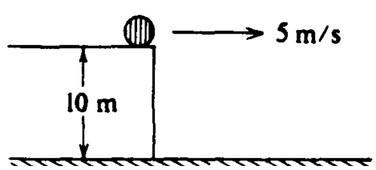
15. A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0. How much time does it take the rock to travel from the edge of the building to the ground?

(A) $\sqrt{hv_o}$ (B) $\frac{h}{v_0}$ (C) $\frac{hv_0}{g}$ (D) $\frac{2h}{g}$ (E) $\sqrt{2h/g}$

16. A ball is thrown straight up in the air. When the ball reaches its highest point, which of the following is true?
(A) It is in equilibrium. (B) It has zero acceleration. (C)It has maximum momentum
(D) It has maximum kinetic energy. (E) None of the above



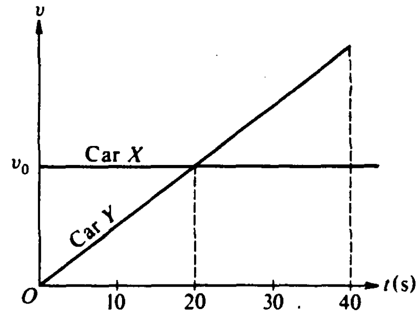
17. The graph above represents position x versus time t for an object being acted on by a constant force. The average speed during the interval between 1 s and 2 s is most nearly
(A) 2 m/s (B) 4 m/s (C) 5 m/s (D) 6 m/s (E) 8 m/s



18. An object slides off a roof 10 meters above the ground with an initial horizontal speed of 5 meters per second as shown above. The time between the object's leaving the roof and hitting the ground is most nearly

(A) $\frac{1}{2}$ s (B) $\frac{1}{\sqrt{2}}$ s (C) $\sqrt{2}$ s (D) 2 s (E) $5\sqrt{2}$ s

Questions 19 – 20

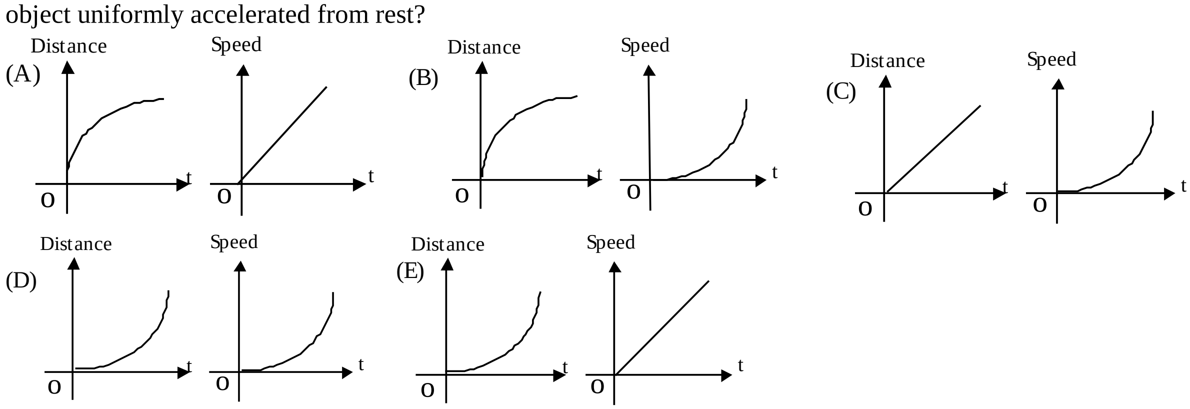


At time t = 0, car X traveling with speed v_0 passes car Y which is just starting to move. Both cars then travel on two parallel lanes of the same straight road. The graphs of speed v versus time t for both cars are shown above.

19. Which of the following is true at time t = 20 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

20. From time t = 0 to time t = 40 seconds, the areas under both curves are equal. Therefore, which of the following is true at time t = 40 seconds?
(A) Car Y is behind car X. (B) Car Y is passing car X. (C) Car Y is in front of car X.
(D) Both cars have the same acceleration. (E) Car X is accelerating faster than car Y.

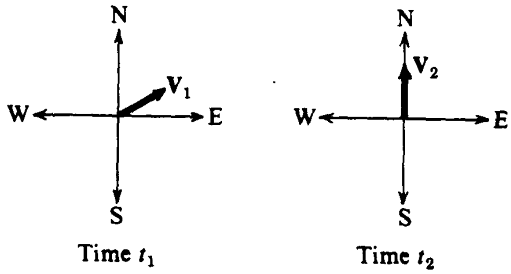
21. Which of the following pairs of graphs shows the distance traveled versus time and the speed versus time for an object uniformly accelerated from rest?



22. An object released from rest at time t = 0 slides down a frictionless incline a distance of 1 meter during the first second. The distance traveled by the object during the time interval from t = 1 second to t = 2 seconds is
(A) 1 m (B) 2 m (C) 3 m (D) 4 m (E) 5 m

23. Two people are in a boat that is capable of a maximum speed of 5 kilometers per hour in still water, and wish to cross a river 1 kilometer wide to a point directly across from their starting point. If the speed of the water in the river is 5 kilometers per hour, how much time is required for the crossing?
(A) 0.05 hr (B) 0.1 hr (C) 1 hr (D) 10 hr
(E) The point directly across from the starting point cannot be reached under these conditions.

24. A projectile is fired from the surface of the Earth with a speed of 200 meters per second at an angle of 30° above the horizontal. If the ground is level, what is the maximum height reached by the projectile?
(A) 5 m (B) 10 m (C) 500 m (D) 1,000 m (E) 2,000 m



25. Vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ shown above have equal magnitudes. The vectors represent the velocities of an object at times t_1, and t_2, respectively. The average acceleration of the object between time t_1 and t_2 was
(A) zero (B) directed north (C) directed west (D) directed north of east (E) directed north of west

26. A rock is dropped from the top of a 45-meter tower, and at the same time a ball is thrown from the top of the tower in a horizontal direction. Air resistance is negligible. The ball and the rock hit the level ground a distance of 30 meters apart. The horizontal velocity of the ball thrown was most nearly
(A) 5 m/s (B) 10 m/s (C) 14.1 m/s (D) 20 m/s (E) 28.3 m/s

27. In the absence of air friction, an object dropped near the surface of the Earth experiences a constant acceleration of about 9.8 m/s^2. This means that the
(A) speed of the object increases 9.8 m/s during each second
(B) speed of the object as it falls is 9.8 m/s
(C) object falls 9.8 meters during each second
(D) object falls 9.8 meters during the first second only
(E) rate of change of the displacement with respect to time for the object equals 9.8 m/s^2

28. A 500-kilogram sports car accelerates uniformly from rest, reaching a speed of 30 meters per second in 6 seconds. During the 6 seconds, the car has traveled a distance of
(A) 15 m (B) 30 m (C) 60 m (D) 90 m (E) 180 m

*29. At a particular instant, a stationary observer on the ground sees a package falling with speed v_1 at an angle to the vertical. To a pilot flying horizontally at constant speed relative to the ground, the package appears to be falling vertically with a speed v_2 at that instant. What is the speed of the pilot relative to the ground?

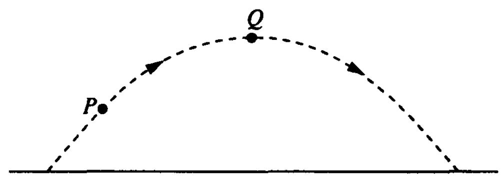
(A) $v_1 + v_2$ (B) $v_1 - v_2$ (C) $v_2 - v_1$ (D) $\sqrt{v_1^2 - v_2^2}$ (E) $\sqrt{v_1^2 + v_2^2}$

30. An object is shot vertically upward into the air with a positive initial velocity. Which of the following correctly describes the velocity and acceleration of the object at its maximum elevation?

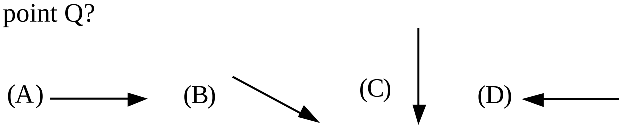
	Velocity	Acceleration
(A)	Positive	Positive
(B)	Zero	Zero
(C)	Negative	Negative
(D)	Zero	Negative
(E)	Positive	Negative

*31. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

(A) h/4 (B) $\frac{h}{2\sqrt{2}}$ (C) h/2 (D) $\frac{h}{\sqrt{2}}$ (E) h



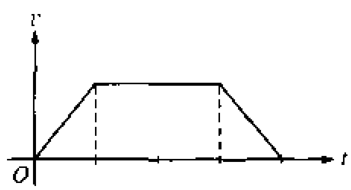
32. A ball is thrown and follows a parabolic path, as shown above. Air friction is negligible. Point Q is the highest point on the path. Which of the following best indicates the direction of the acceleration, if any, of the ball at point Q?



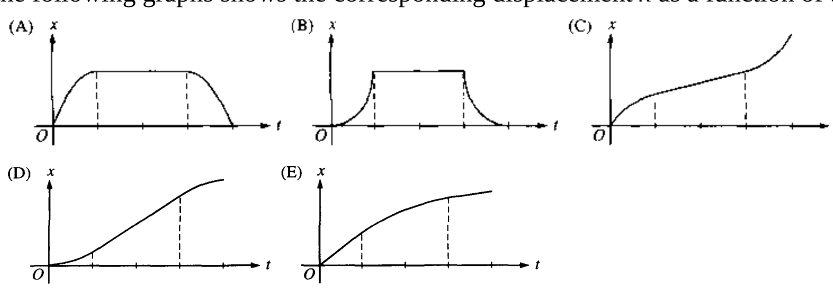
(E) There is no acceleration of the ball at point Q.

33. The velocity of a projectile at launch has a horizontal component v_h and a vertical component v_v. Air resistance is negligible. When the projectile is at the highest point of its trajectory, which of the following shows the vertical and horizontal components of its velocity and the vertical component of its acceleration?

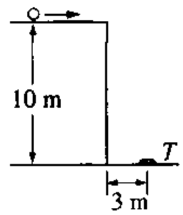
	Vertical Velocity	Horizontal Velocity	Vertical Acceleration
(A)	v_v	v_h	0
(B)	v_v	0	0
(C)	0	v_h	0
(D)	0	0	g
(E)	0	v_h	g



34. The graph above shows the velocity *v* as a function of time *t* for an object moving in a straight line. Which of the following graphs shows the corresponding displacement *x* as a function of time *t* for the same time interval?

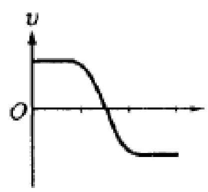


35. An object is dropped from rest from the top of a 400 m cliff on Earth. If air resistance is negligible, what is the distance the object travels during the first 6 s of its fall?
(A) 30 m (B) 60 m (C) 120 m (D) 180 m (E) 360 m

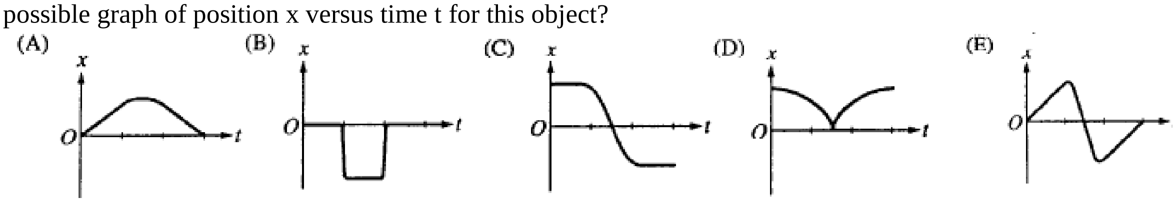


36. A target T lies flat on the ground 3 m from the side of a building that is 10 m tall, as shown above. A student rolls a ball off the horizontal roof of the building in the direction of the target. Air resistance is negligible. The horizontal speed with which the ball must leave the roof if it is to strike the target is most nearly

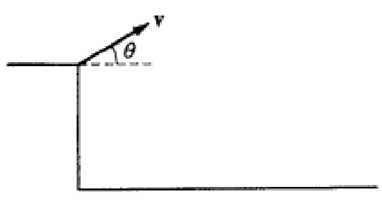
(A) 3/10 m/s (B) $\sqrt{2}$ m/s (C) $\frac{3}{\sqrt{2}}$ m/s (D) 3 m/s (E) $10\sqrt{\frac{5}{3}}$ m/s



37. The graph above shows velocity v versus time t for an object in linear motion. Which of the following is a possible graph of position x versus time t for this object?

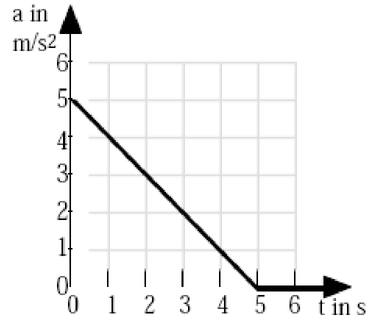


*38. A student is testing the kinematic equations for uniformly accelerated motion by measuring the time it takes for light-weight plastic balls to fall to the floor from a height of 3 m in the lab. The student predicts the time to fall using g as 9.80 m/s^2 but finds the measured time to be 35% greater. Which of the following is the most likely cause of the large percent error?
(A) The acceleration due to gravity is 70% greater than 9.80 m/s^2 at this location.
(B) The acceleration due to gravity is 70% less than 9.80 m/s^2 at this location.
(C) Air resistance increases the downward acceleration.
(D) The acceleration of the plastic balls is not uniform.
(E) The plastic balls are not truly spherical.



Note: Figure not drawn to scale.

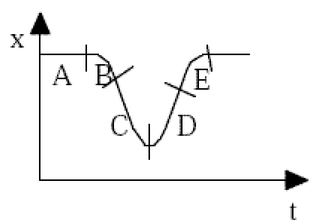
*39. An object is thrown with velocity **v** from the edge of a cliff above level ground. Neglect air resistance. In order for the object to travel a maximum horizontal distance from the cliff before hitting the ground, the throw should be at an angle θ with respect to the horizontal of
(A) greater than 60° above the horizontal (B) greater than 45° but less than 60° above the horizontal
(C) greater than zero but less than 45° above the horizontal (D) zero
(E) greater than zero but less than 45° below the horizontal



*40. Starting from rest at time t = 0, a car moves in a straight line with an acceleration given by the accompanying graph. What is the speed of the car at t = 3 s?
(A) 1.0 m/s (B) 2.0 m/s (C) 6.0 m/s (D) 10.5 m/s (E) 12.5 m/s

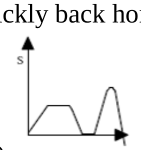
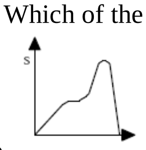
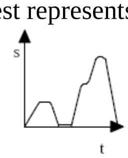
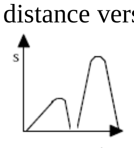
41. A flare is dropped from a plane flying over level ground at a velocity of 70 m/s in the horizontal direction. At the instant the flare is released, the plane begins to accelerate horizontally at 0.75 m/s^2. The flare takes 4.0 s to reach the ground. Assume air resistance is negligible. Relative to a spot directly under the flare at release, the flare lands
(A) directly on the spot. (B) 6.0 m in front of the spot. (C) 274 m in front of the spot.
(D) 280 m in front of the spot. (E) 286 m in front of the spot.

42. As seen by the pilot of the plane (in question #41) and measured relative to a spot directly under the plane when the flare lands, the flare lands
(A) 286 m behind the plane. (B) 6.0 m behind the plane. (C) directly under the plane.
(D) 12 m in front of the plane. (E) 274 m in front of the plane



43. The graph above is a plot of position versus time. For which labeled region is the velocity positive and the acceleration negative?
(A) A (B) B (C) C (D) D (E) E

44. A child left her home and started walking at a constant velocity. After a time she stopped for a while and then continued on with a velocity greater than she originally had. All of a sudden she turned around and walked very quickly back home. Which of the following graphs best represents the distance versus time graph for her walk?

(A)  (B)  (C)  (D)  (E) 

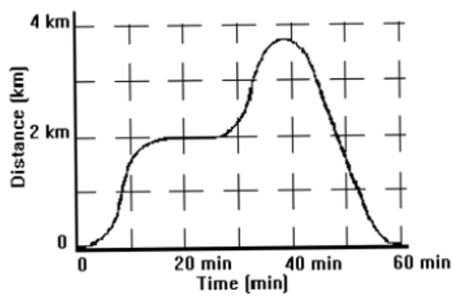
45. In a rescue attempt, a hovering helicopter drops a life preserver to a swimmer being swept downstream by a river current of constant velocity *v*. The helicopter is at a height of 9.8 m. The swimmer is 6.0 m upstream from a point directly under the helicopter when the life preserver is released. It lands 2.0 m in front of the swimmer. How fast is the current flowing? Neglect air resistance.
(A) 13.7 m/s (B) 9.8 m/s (C) 6.3 m/s (D) 2.8 m/s (E) 2.4 m/s

*46. A child tosses a ball directly upward. Its total time in the air is *T*. Its maximum height is *H*. What is its height after it has been in the air a time *T*/4? Neglect air resistance.
(A) H/4 (B) H/3 (C) H/2 (D) 2H/3 (E) 3H/4

47. A whiffle ball is tossed straight up, reaches a highest point, and falls back down. Air resistance is not negligible. Which of the following statements are true?
I. The ball's speed is zero at the highest point.
II. The ball's acceleration is zero at the highest point.
III. The ball takes a longer time to travel up to the highest point than to fall back down.

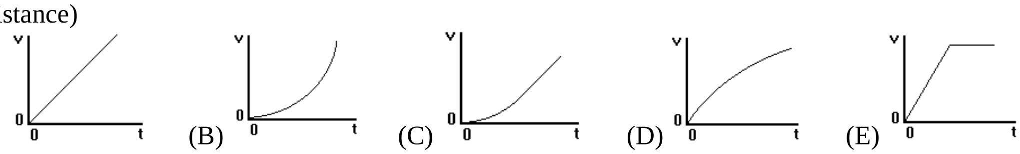
(A) I only (B) II only (C) I & II only (D) I & III only (E) I, II, & III

48. A truck driver travels three-fourths the distance of his run at one velocity (*v*) and then completes his run at one half his original velocity (½*v*). What was the trucker's average speed for the trip?
(A) 0.85*v* (B) 0.80*v* (C) 0.75*v* (D) 0.70*v* (E) 0.65*v*

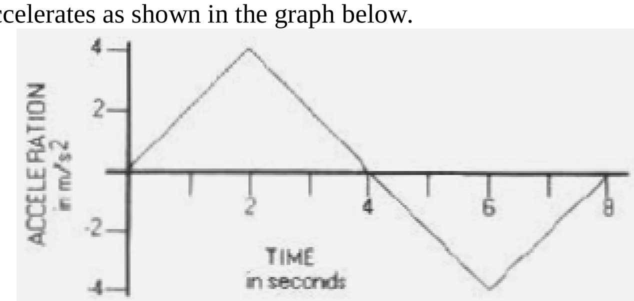


49. Above is a graph of the distance vs. time for car moving along a road. According the graph, at which of the following times would the automobile have been accelerating positively?
(A) 0, 20, 38, & 60 min. (B) 5, 12, 29, & 35 min. (C) 5, 29, & 57 min. (D) 12, 35, & 41 min.
(E) at all times from 0 to 60 min

50. A large beach ball is dropped from the ceiling of a school gymnasium to the floor about 10 meters below. Which of the following graphs would best represent its velocity as a function of time? (do not neglect air resistance)



Questions 51 – 52
A car starts from rest and accelerates as shown in the graph below.

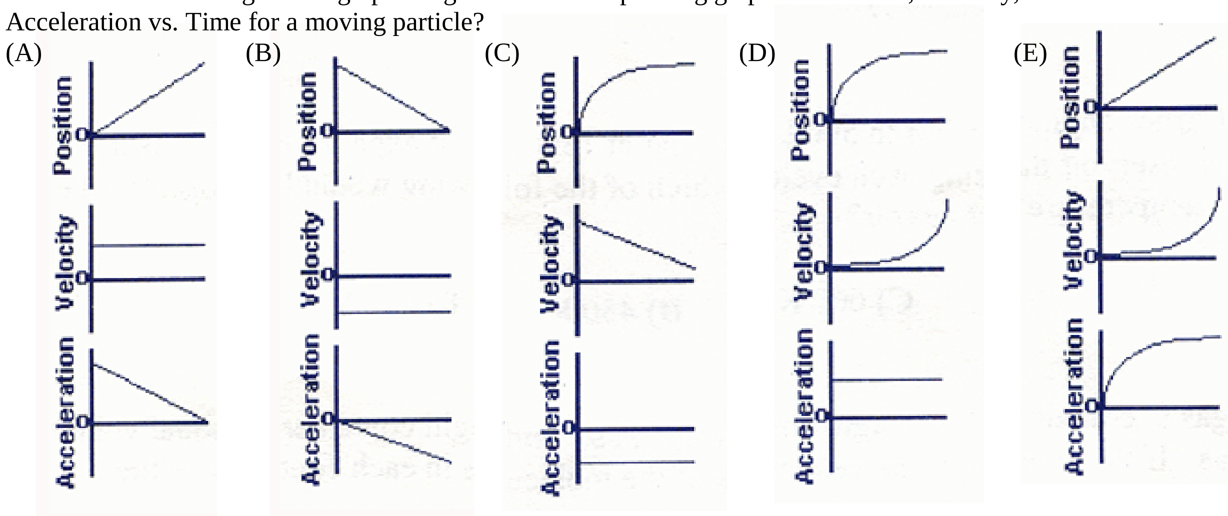


51. At what time would the car be moving with the greatest velocity?
(A) 0 seconds (B) 2 seconds (C) 4 seconds (D) 6 seconds (E) 8 seconds

*52. At what time would the car be farthest from its original starting position?
(A) 0 seconds (B) 2 seconds (C) 4 seconds (D) 6 seconds (E) 8 seconds

53. A ball is dropped 1.0 m to the floor. If the speed of the ball as it rebounds from the floor is 75% of the speed at which it struck the floor, how high will the ball rise?
(A) 0.28 m (B) 0.35 m (C) 0.56 m (D) 0.75 m (E) 0.84 m

54. Which of the following sets of graphs might be the corresponding graphs of Position, Velocity, and Acceleration vs. Time for a moving particle?



*55. An object is thrown with a fixed initial speed v_0 at various angles α relative to the horizon. At some constant height h above the launch point the speed v of the object is measured as a function of the initial angle α. Which of the following best describes the dependence of v on α? (Assume that the height h is achieved, and assume that there is no air resistance.)
(A) v will increase monotonically with α.
(B) v will increase to some critical value v_{max} and then decrease.
(C) v will remain constant, independent of α.
(D) v will decrease to some critical value v_{min} and then increase.
(E) None of the above.

56. A bird is flying in a straight line initially at 10 m/s. It uniformly increases its speed to 15 m/s while covering a distance of 25 m. What is the magnitude of the acceleration of the bird?
(A) 5.0 m/s^2 (B) 2.5 m/s^2 (C) 2.0 m/s^2 (D) 0.5 m/s^2 (E) 0.2 m/s^2

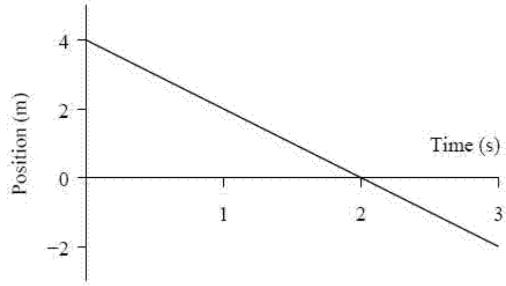
57. A person standing on the edge of a fire escape simultaneously launches two apples, one straight up with a speed of 7 m/s and the other straight down at the same speed. How far apart are the two apples 2 seconds after they were thrown, assuming that neither has hit the ground?
(A) 14 m (B) 20 m (C) 28 m (D) 34 m (E) 56 m

*58. A certain football quarterback can throw a football a maximum range of 80 meters on level ground. What is the highest point reached by the football if thrown this maximum range? Ignore air friction.
(A) 10 m (B) 20 m (C) 30 m (D) 40 m (E) 50 m

59. A bird flying in a straight line, initially at 10 m/s, uniformly increases its speed to 18 m/s while covering a distance of 40 m. What is the magnitude of the acceleration of the bird?
(A) 0.1 m/s^2 (B) 0.2 m/s^2 (C) 2.0 m/s^2 (D) 2.8 m/s^2 (E) 5.6 m/s^2

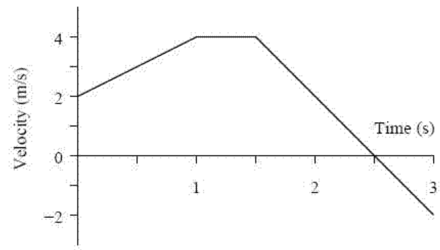
*60. A cockroach is crawling along the walls inside a cubical room that has an edge length of 3 m. If the cockroach starts from the back lower left hand corner of the cube and finishes at the front upper right hand corner, what is the magnitude of the displacement of the cockroach?
(A) $3\sqrt{3}\,m$ (B) $3\sqrt[3]{2}\,m$ (C) $\sqrt{3}\,m$ (D) 3 m (E) 9 m

61. The position vs. time graph for an object moving in a straight line is shown below. What is the instantaneous velocity at $t = 2$ s?



(A) – 2 m/s (B) ½ m/s (C) 0 m/s (D) 2 m/s (E) 4 m/s

62. Shown below is the velocity vs. time graph for a toy car moving along a straight line. What is the maximum displacement from start for the toy car?



(A) 3 m (B) 5 m (C) 6.5 m (D) 7 m (E) 7.5 m

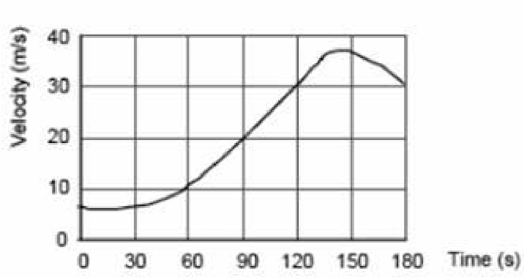
*63. A cannon fires projectiles on a flat range at a fixed speed but with variable angle. The maximum range of the cannon is L. What is the range of the cannon when it fires at an angle $\pi/6$ above the horizontal? Ignore air resistance.

(A) $\frac{\sqrt{3}}{2}L$ (B) $\frac{1}{\sqrt{2}}L$ (C) $\frac{1}{\sqrt{3}}L$ (D) $\frac{1}{2}L$ (E) $\frac{1}{3}L$

*64. A ball is launched upward from the ground at an initial vertical speed of v_0 and begins bouncing vertically. Every time it rebounds, it loses a proportion of the magnitude of its velocity due to the inelastic nature of the collision, such that if the speed just before hitting the ground on a bounce is v, then the speed just after the bounce is rv, where $r < 1$ is a constant. Calculate the total length of time that the ball remains bouncing, assuming that any time associated with the actual contact of the ball with the ground is negligible.

(A) $\frac{2v_0}{g}\frac{1}{1-r}$ (B) $\frac{v_0}{g}\frac{r}{1-r}$ (C) $\frac{2v_0}{g}\frac{1-r}{r}$ (D) $\frac{2v_0}{g}\frac{1}{1-r^2}$ (E) $\frac{2v_0}{g}\frac{1}{1+(1-r)^2}$

65. The graph shows velocity as a function of time for a car. What was the acceleration at time $t = 90$ seconds?



(A) 0.22 m/s^2 (B) 0.33 m/s^2 (C) 1.0 m/s^2 (D) 9.8 m/s^2 (E) 30 m/s^2

66. An object is released from rest and falls a distance h during the first second of time. How far will it fall during the next second of time?

(A) h (B) $2h$ (C) $3h$ (D) $4h$ (E) h^2

67. A stone is thrown straight downward with a speed of 20 m/s from the top of a tall building. If the stone strikes the ground 3.0 s later, about how tall is the building? Assume air resistance is negligible.
(A) 45 m (B) 60 m (C) 90 m (D) 105 m (E) 120 m

68. A coyote can run at a speed of 20 m/s while a prairie dog can manage only 5.5 m/s. If a prairie dog is 45 m in front of a coyote, what is the maximum time it has to reach its hole without being caught?
(A) 2.3 s (B) 3.1 s (C) 5.4 s (D) 5.9 s (E) 8.2 s

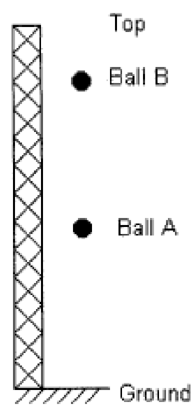
69. A model rocket accelerates from rest upwards at 50 m/s^2 for 2.0 s before its engine burns out. The rocket then coasts upward. What is the maximum height that the rocket reaches? You may assume air resistance is negligible.
(A) 100 m (B) 510 m (C) 610 m (D) 1020 m (E) 1220 m

70. A hunter in a forest walks 800 m west. He then turns south and walks 400 m before turning west again and walking a final 300 m. At the end of the walk, what is the magnitude of the hunter's displacement from the beginning?
(A) 640 m (B) 890 m (C) 1170 m (D) 1390 m (E) 1500 m

71. Robin Hood aims his longbow horizontally at a target's bull's eye 30 m away. If the arrow strikes the target exactly 1.0 m below the bull's eye, how fast did the arrow move as it was shot from the bow? Assume air resistance is negligible.
(A) 6.0 m/s (B) 13 m/s (C) 33 m/s (D) 67 m/s (E) 150 m/s

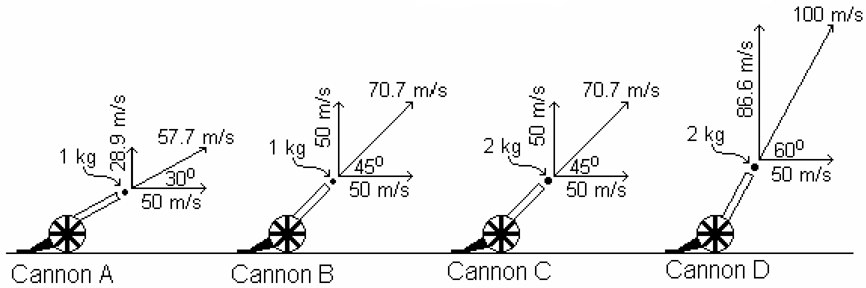
*72. A baseball is thrown vertically into the air with a velocity *v*, and reaches a maximum height *h*. At what height was the baseball moving with one-half its original velocity? Assume air resistance is negligible.
(A) 0.25 *h* (B) 0.33 *h* (C) 0.50 *h* (D) 0.67 *h* (E) 0.75 *h*

73. Two identical bowling balls A and B are each dropped from rest from the top of a tall tower as shown in the diagram below. Ball A is dropped 1.0 s before ball B is dropped but both balls fall for some time before ball A strikes the ground. Air resistance can be considered negligible during the fall. After ball B is dropped but before ball A strikes the ground, which of the following is true?



(A) The distance between the two balls decreases.
(B) The velocity of ball A increases with respect to ball (B)
(C) The velocity of ball A decreases with respect to ball (B)
(D) The distance between the two balls remains constant.
(E) The distance between the two balls increases.

74. The diagram below shows four cannons firing shells with different masses at different angles of elevation. The horizontal component of the shell's velocity is the same in all four cases. In which case will the shell have the greatest range if air resistance is neglected?



(A) cannon A (B) cannon B only (C) cannon C only (D) cannon D
(E) Both cannons B and C have the greatest range

75. Relief supplies are being dropped to flood victims from an airplane flying horizontally at a speed v. If the airplane is at an altitude of h above the ground, what distance d in front of the landing site should the supplies be dropped?
(A) $2v\sqrt{\frac{h}{g}}$ (B) $\frac{2vh}{g}$ (C) $2\sqrt{\frac{vh}{g}}$ (D) $\frac{2vh^2}{g^2}$ (E) $v\sqrt{\frac{2h}{g}}$

*76. An airliner flies at a speed of 500 km/hr with respect to the air. The jet stream blows from west to east with a speed of 100 km/hr. What is the minimum time in which the airliner could fly 3000 km due west and then back to its original starting position?
(A) 10.0 hr (B) 12.0 hr (C) 12.5 hr (D) 13.5 hr (E) 15.0 hr

77. A punter in a football game kicks the ball with an initial speed of 28.3 m/s at an angle of 60° with respect to the ground. The ball is in the air for a total of 5.00 s before hitting the ground. If we assume that air resistance is negligible, what would be the ball's horizontal displacement?
(A) 0 m (B) 14.2 m (C) 24.5 m (D) 70.8 m (E) 122.5 m

78. Starting from rest, object 1 falls freely for 4.0 seconds, and object 2 falls freely for 8.0 seconds. Compared to object 1, object 2 falls:
(A) half as far (B) twice as far (C) three times as far (D) four times as far (E) sixteen times as far

79. A car starts from rest and uniformly accelerates to a final speed of 20.0 m/s in a time of 15.0 s. How far does the car travel during this time?
(A) 150 m (B) 300 m (C) 450 m (D) 600 m (E) 800 m

80. A ball is thrown off a high cliff with no horizontal velocity. It lands 6.0 s later with a velocity of 40 m/s. What was the initial velocity of the ball?
(A) 100 m/s up (B) 20 m/s up (C) 0 (D) 20 m/s down (E) 100 m/s down

81. An arrow is aimed horizontally, directly at the center of a target 20 m away. The arrow hits 0.050 m below the center of the target. Neglecting air resistance, what was the initial speed of the arrow?
(A) 20 m/s (B) 40 m/s (C) 100 m/s (D) 200 m/s (E) 400 m/s

82. A freely falling body is found to be moving downwards at 27 m/s at one instant. If it continues to fall, one second later the object would be moving with a downward velocity closest to:
(A) 270 m/s (B) 37 m/s (C) 27 m/s (D) 17 m/s (E) 10 m/s

83. A rocket near the surface of the earth is accelerating vertically upward at 10 m/s^2. The rocket releases an instrument package. Immediately after release the acceleration of the instrument package is:
(A) 20 m/s^2 up (B) 10 m/s^2 up (C) 0 (D) 10 m/s^2 down (E) 20 m/s^2 down

84. A car starts from rest and accelerates at 0.80 m/s^2 for 10 s. It then continues at constant velocity. Twenty seconds (20 s) after it began to move, the car has a
(A) velocity of 8.0 m/s and has traveled 40 m.
(B) velocity of 8.0 m/s and has traveled 80 m.
(C) velocity of 8.0 m/s and has traveled 120 m.
(D) velocity of 16 m/s and has traveled 160 m.
(E) velocity of 16 m/s and has traveled 320 m.

85. When a falling object reaches terminal velocity, it
(A) is no longer subject to the friction of air (B) moves downward with constant velocity.
(C) has an acceleration of approximately 10 m/s^2 (D) has no downward velocity.
(E) has an upward acceleration

86. A ball which is dropped from the top of a building strikes the ground with a speed of 30 m/s. Assume air resistance can be ignored. The height of the building is approximately:
(A) 15 m (B) 30 m (C) 45 m (D) 75 m (E) 90 m

87. At a certain time, an object in free fall has velocity 4.0 m/s in the upward direction. What is the approximate velocity of the object one second later?
(A) 14 m/s up (B) 10 m/s up (C) 4.0 m/s up (D) 6.0 m/s down (E) 10 m/s down

88. A motorist travels 400 km at 80 km/h and 400 km at 100 km/h. What is the average speed of the motorist on this trip?
(A) 84 km/h (B) 89 km/h (C) 90 km/h (D) 91 km/h (E) 95 km/h

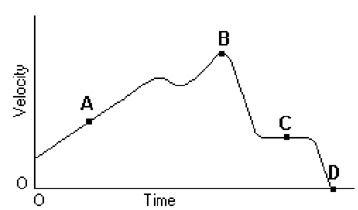
89. How long must a 2.5 m/s^2 acceleration act to change the velocity of a 2.0-kg object by 3.0 m/s?
(A) 0.83 s (B) 1.2 s (C) 1.7 s (D) 2.5 s (E) 7.5 s

90. A freely falling object is found to be moving downward at 18 m/s. If it continues to fall, two seconds later the object would be moving with a speed of
(A) 8.0 m/s (B) 10 m/s (C) 18 m/s (D) 38 m/s (E) 180 m/s

91. The change of distance per unit time without reference to a particular direction is called
(A) inertia (B) speed (C) velocity (D) acceleration (E) position

92. Which of the following terms is NOT related conceptually to the others?
(A) vector (B) resultant (C) component (D) exponent (E) equilibrant

93. In the absence of air resistance, if an object were to fall freely near the surface of the Moon,
(A) its velocity could never exceed 10 m/s.
(B) its acceleration would gradually decrease until the object moves with a terminal velocity.
(C) the acceleration is constant.
(D) it will fall with a constant speed.
(E) the acceleration is zero

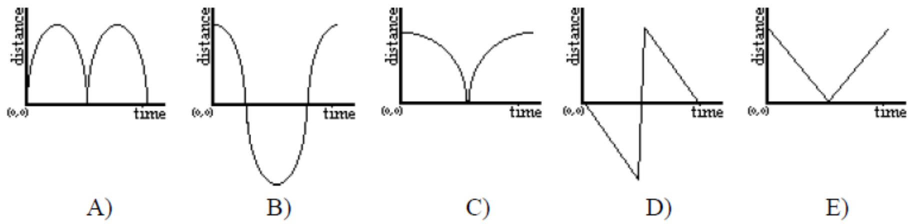


94. Given the graph of the velocity vs. time of a duck flying due south for the winter. At what point did the duck stop its forward motion?
(A) A (B) B (C) C (D) D (E) none of these points

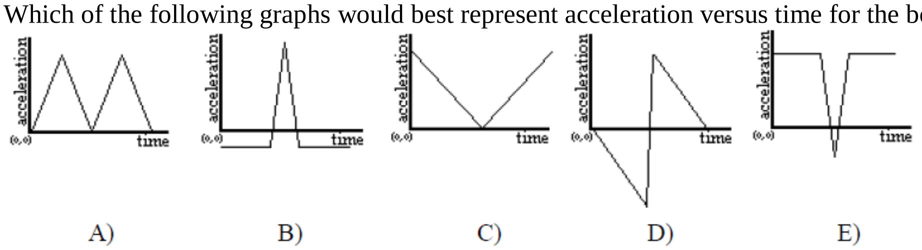
Questions 95 – 96

The following TWO questions refer to the following information. An ideal elastic rubber ball is dropped from a height of about 2 meters, hits the floor and rebounds to its original height.

95. Which of the following graphs would best represent the distance above the floor versus time for the above bouncing ball?

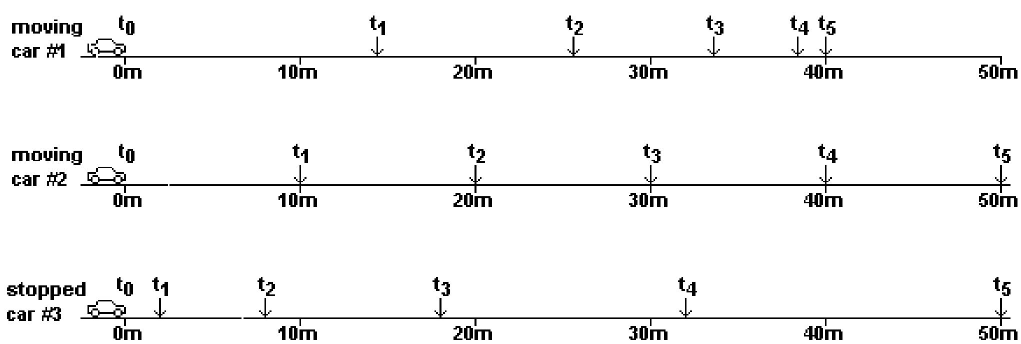


96. Which of the following graphs would best represent acceleration versus time for the bouncing ball?



Questions 97 through 100 refer to the following scenario:

At t_0, two cars moving along a highway are side-by-side as they pass a third car stopped on the side of the road. At this moment the driver of the first car steps on the brakes while the driver of the stopped car begins to accelerate. The diagrams below show the positions of each car for the next 5 seconds.



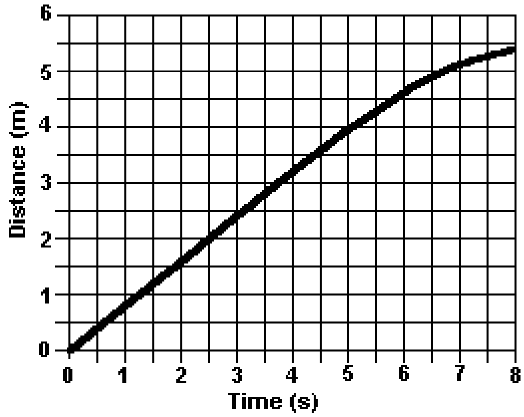
97. During which time interval would cars #2 and #3 be moving at the same average speed?
(A) t_0 to t_1 (B) t_1 to t_2 (C) t_2 to t_3 (D) t_3 to t_4 (E) t_4 to t_5

98. About what position after t_0 would car #1 and car #2 have been side by side?
(A) 0 m (B) 15 m (C) 26 m (D) 37 m (E) 39 m

99. Which of the three cars had the greatest average speed during these 5 seconds?
(A) car #1 (B) car #2 and car #3 had the same average speed (C) car #2
(D) all three cars had the same average speed (E) car #3

100. If car #3 continues to constantly accelerate at the same rate what will be its position at the end of 6 seconds?
(A) 22 m (B) 68 m (C) 72 m (D) 78 m (E) 94 m

Questions 101 – 102



101. The graph represents the relationship between distance and time for an object that is moving along a straight line. What is the instantaneous speed of the object at t = 5.0 seconds?
(A) 0.0 m/s (B) 0.8 m/s (C) 2.5 m/s (D) 4.0 m/s (E) 6.8 m/s

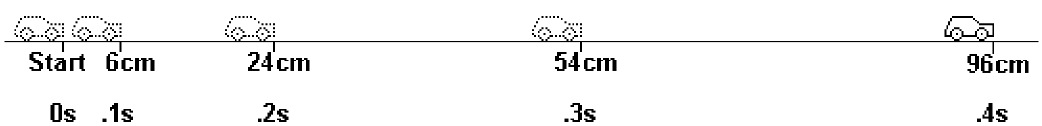
102. Between what times did the object have a non-zero acceleration?
(A) 0 s only (B) 0 s to 8 s (C) 0 s to 5 s (D) the object was not accelerating at any time (E) 5 s to 8 s

*103. An airplane takes off and flies 300 miles at an angle of 30º north of east. It then changes direction and flies 600 miles due west before landing. In what direction is the plane's landing point from its starting point?
(A) 14.2º north of west (B) 66.2º north of west (C) 23.8º north of west (D) 75.9º north of west
(E) 37.4º north of west

104. If a ball is thrown directly upwards with twice the initial speed of another, how much higher will it be at its apex?
(A) 8 times (B) 2 times (C) 4 times (D) 2 times (E) $2\sqrt{2}$ times

Questions 105 – 107

The diagram below represents a toy car starting from rest and uniformly accelerating across the floor. The time and distance traveled from the start are shown in the diagram.



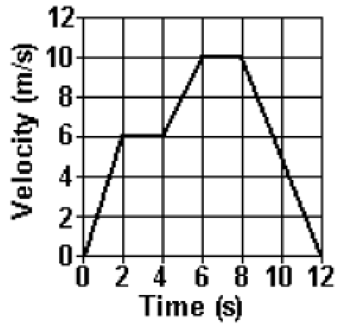
105. What was the average speed of the cart between 0.1 seconds and 0.3 seconds?
(A) 0.6 m/s (B) 4.8 m/s (C) 1.9 m/s (D) 60 m/s (E) 2.4 m/s

106. What was the acceleration of the cart during the first 0.4 seconds?
(A) 6.0 m/s^2 (B) 25 m/s^2 (C) 9.8 m/s^2 (D) 50 m/s^2 (E) 12 m/s^2

107. What was the instantaneous velocity of the cart at 96 centimeters from the start?
(A) 0.6 m/s (B) 4.8 m/s (C) 1.9 m/s (D) 60 m/s (E) 2.4 m/s

Questions 108 – 109

The motion of a circus clown on a unicycle moving in a straight line is shown in the graph below



108. What would be the acceleration of the clown at 5 s?
(A) 1.6 m/s^2 (B) 8.0 m/s^2 (C) 2.0 m/s^2 (D) none of the above (E) 3.4 m/s^2

109. After 12 seconds, how far is the clown from her original starting point?
(A) 0 m (B) 10 m (C) 34 m (D) 47 m (E) 74 m

110. A box falls to the ground from a delivery truck traveling at 30 m/s. After hitting the road, it slides 45 meters to rest. How long does it take the box to come to rest?
(A) 0.67 s (B) 1.5 s (C) 2.0 s (D) 3.0 s (E) 6.0 s

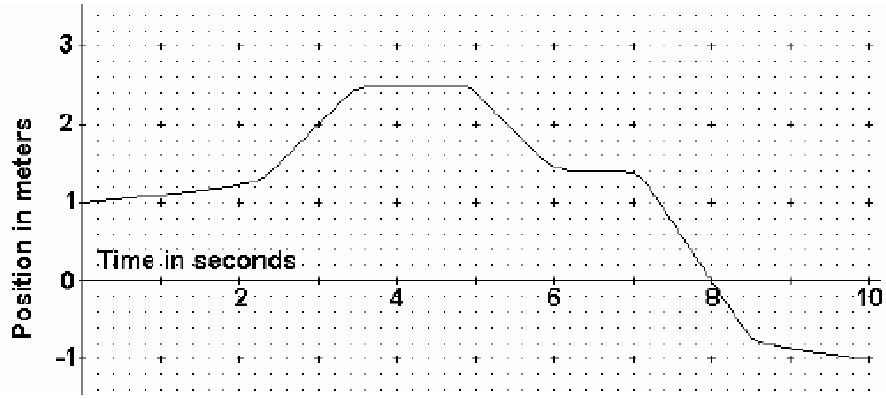
111. When an object falls freely in a vacuum near the surface of the earth
(A) the velocity cannot exceed 10 m/s
(B) the terminal velocity will be greater than when dropped in air
(C) the velocity will increase but the acceleration will be zero
(D) the acceleration will constantly increase
(E) the acceleration will remain constant

112. Two arrows are launched at the same time with the same speed. Arrow A at an angle greater than 45 degrees, and arrow B at an angle less than 45 degrees. Both land at the same spot on the ground. Which arrow arrives first?
(A) arrow A arrives first (B) arrow B arrives first (C) they both arrive together
(D) it depends on the elevation where the arrows are launched
(E) it depends on the elevation where the arrows land

113. A ball is thrown into the air at an angle θ as measured from the horizontal with a velocity v. The horizontal velocity of the ball will be directly proportional to which of the following
(A) the angle θ (B) the sine of the angle θ (C) the cosine of the angle θ (D) the tangent of the angle θ
(E) the value of the gravitational acceleration

Questions 114 – 116

The accompanying graph describes the motion of a marble on a table top for 10 seconds.



114. For which time interval(s) did the marble have a negative velocity?
(A) from t = 8.0 s to t = 10.0 s only (B) from t = 6.9 s to t = 10.0 s only
(C) from t = 4.8 s to t = 10.0 s only (D) from t = 4.8 s to t = 6.2 s and from t = 6.9 s to t = 10.0 s only
(E) from t = 3.2 s to t = 3.6 s, from t = 4.8 s to t = 5 s, and from t = 6.8 s to t = 7.2 s only

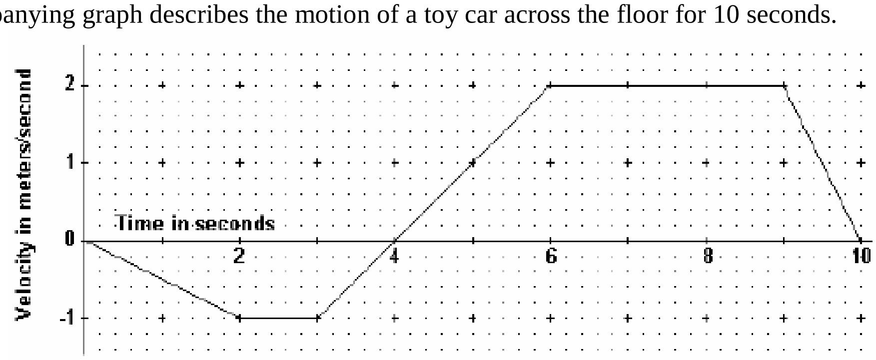
115. For which time interval(s) did the marble have a positive acceleration?
(A) from t = 0.0 s to t = 8.0 s only (B) from t = 0.0 s to t = 3.6 only
(C) from t = 3.8 s to t = 4.8 s and t = 6.2 s to t = 6.8 s only
(D) from t = 2.0 s to t = 2.5 s, from t = 5.8 s to t = 6.2 s, and from t = 8.4 s to t = 8.8 s only
(E) from t = 3.3 s to t = 3.7 s, from t = 4.8 s to t = 5.0 s, and from t = 6.8 s to t = 7.2 s only

116. What is the marbles average acceleration between t = 3.1 s and t = 3.8 s
(A) –2.0 m/s^2 (B) 2.0 m/s^2 (C) 0.0 m/s^2 (D) 3.0 m/s^2 (E) 0.8 m/s^2

117. A car accelerates uniformly from rest for a time of 2.00 s through a distance of 4.00 m. What was the acceleration of the car?
(A) 0.50 m/s^2 (B) 0.71 m/s^2 (C) 1.00 m/s^2 (D) 1.41 m/s^2 (E) 2.00 m/s^2

Questions 118 – 120

The accompanying graph describes the motion of a toy car across the floor for 10 seconds.



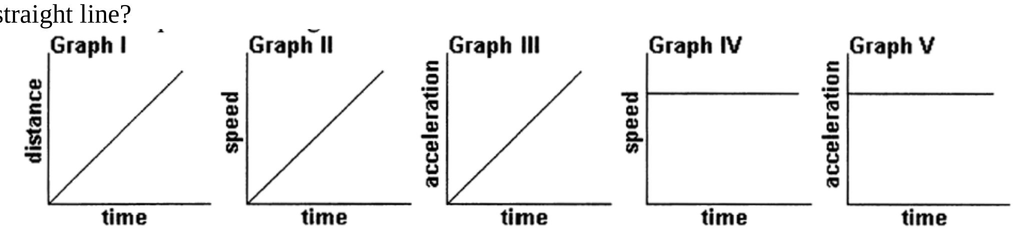
118. What is the acceleration of the toy car at t = 4 s?
(A) –1 m/s^2 (B) 0 m/s^2 (C) 1 m/s^2 (D) 2 m/s^2 (E) 4 m/s^2

119. What was the total displacement of the toy car for the entire 10 second interval shown?
(A) 0 meters (B) 6.5 meters (C) 9 meters (D) 10 meters (E) 11.5 meters

120. An object is thrown upwards with a velocity of 30 m/s near the surface of the earth. After two seconds what would be the direction of the displacement, velocity and acceleration?

	Displacement	velocity	acceleration
(A)	up	up	up
(B)	up	up	down
(C)	up	down	down
(D)	up	down	up
(E)	down	down	down

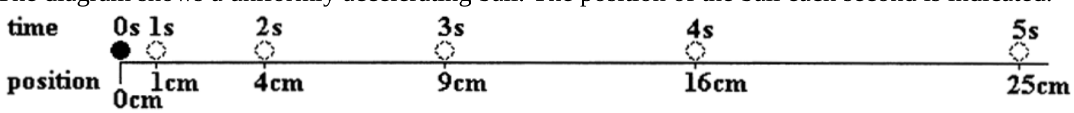
121. Which of the following graphs could correctly represent the motion of an object moving with a constant speed in a straight line?



(A) Graph I only (B) Graphs II and V only (C) Graph II only (D) Graphs I and IV only
(E) All of the above graphs represent constant velocity

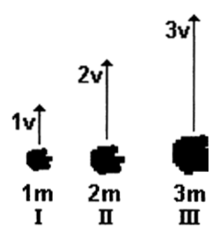
Questions 122 – 123

The diagram shows a uniformly accelerating ball. The position of the ball each second is indicated.

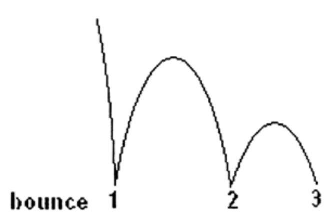


122. What is the average speed of the ball between 3 and 4 seconds?
(A) 3.0 cm/s (B) 7.0 cm/s (C) 3.5 cm/s (D) 12.5 cm/s (E) 4.0 cm/s

123. Which of the following is closest to the acceleration of the ball?
(A) 1 cm/s^2 (B) 4 cm/s^2 (C) 2 cm/s^2 (D) 5 cm/s^2 (E) 3 cm/s^2



124. Three stones of different mass (1 *m*, *2m* & *3m)* are thrown vertically upward with different velocities (l v, 2v & 3v respectively). The diagram indicates the mass and velocity of each stone. Rank from high to low the maximum height of each stone. Assume air resistance is negligible.
(A) I, II, III (B) II, I, III (C) III, II, I (D) I, III, II (E) all reach the same height



125. A rubber ball bounces on the ground as shown. After each bounce, the ball reaches one-half the height of the bounce before it. If the time the ball was in the air between the first and second bounce was 1 second. What would be the time between the second and third bounce?
(A) 0.50 sec (B) 0.71 sec (C) 1.0 sec (D) 1.4 sec (E) 2.0 sec

126. The driver of a car makes an emergency stop by slamming on the car's brakes and skidding to a stop. How far would the car have skidded if it had been traveling twice as fast?
(A) 4 times as far (B) the same distance (C) 2 times as far (D) $\sqrt{2}$ times as far
(E) the mass of the car must be known

127. A pebble is dropped from a high vertical cliff. The collision of the pebble with the ground below is seen 1.50 seconds after the pebble is dropped. With what speed did the pebble hit the ground? Ignore air resistance.
(A) 10 m/s (B) 15 m/s (C) 48.6 m/s (D) 100.4 m/s (E) 343 m/s

128. A snail is moving along a straight line. Its initial position is x_0 = –5 meters and it is moving away from the origin and slowing down. In this coordinate system, the signs of the initial position, initial velocity and acceleration, respectively, are

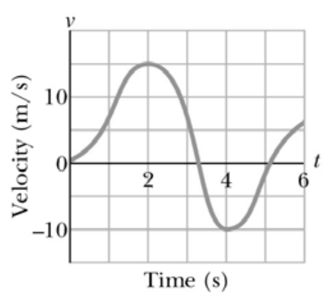
Choice	x_0	v_0	a
(A)	–	+	+
(B)	–	–	+
(C)	–	–	–
(D)	–	+	–
(E)	+	+	+

129. An arrow is shot horizontally toward a target 20 m away. In traveling the first 5 m horizontally, the arrow falls 0.2 m. In traveling the next 5 m horizontally, it will fall an additional
(A) 0.6 m (B) 0.4 m (C) 0.3 m (D) 0.2 m (E) 0.1 m

130. How tall is a tree if the sun is at a 53º angle above the horizon and the shadow is 8.0 meters long?
(A) 4.8 m (B) 10.6 m (C) 6.0 m (D) 13.3 m (E) 8.0 m

131. Three students were arguing about the height of a parking garage. One student suggested that to determine the height of the garage, they simply had to drop tennis balls from the top and time the fall of the tennis balls. If the time for the ball to fall was 1.4 seconds, approximately how tall is the parking garage?
(A) 4.9 m (B) 7.0 m (C) 9.8 m (D) 13.8 m (E) 19.6 m

132. An arrow is shot from a bow at an angle of 40º from the horizontal at a speed of 24.0 m/s. Ignoring air resistance, what is the arrow's maximum height above its launch point?
(A) 5.9 m (B) 11.9 m (C) 16.9 m (D) 23.8 m (E) 28.8 m



133. A car has the velocity versus time curve shown above. Which of the following statements regarding its motion is INCORRECT?
(A) The car is moving fastest at 2.0 s (B) The car is at rest at approximately 5.2 s.
(C) The car is speeding up from t = 0 to t = 2.0 s (D) The car has negative acceleration at t = 4.5 s.
(E) The car has no acceleration at the instant t = 2 s.

134. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) the speed of both rocks is constant while they fall.
(D) they strike the ground more than half a second apart.
(E) they strike the ground with the same kinetic energy.

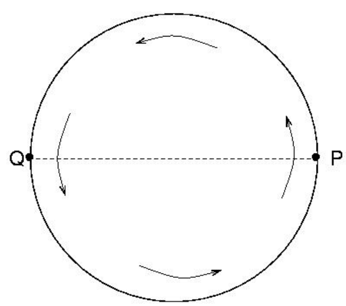
135. A cart is initially moving at 0.5 m/s along a track. The cart comes to rest after traveling 1 m. The experiment is repeated on the same track, but now the cart is initially moving at 1 m/s. How far does the cart travel before coming to rest?
(A) 1 m (B) 2 m (C) 3 m (D) 4 m (E) 8 m

*136. During an interval of time, a tennis ball is moved so that the angle between the velocity and the acceleration of the ball is kept at a constant 120º. Which statement is true about the tennis ball during this interval of time?
(A) Its speed increases and it is changing its direction of travel.
(B) Its speed decreases and it is changing its direction of travel.
(C) Its speed remains constant, but it is changing its direction of travel.
(D) Its speed remains constant and it is not changing its direction of travel.
(E) Its speed decreases and it is not changing its direction of travel.

137. A dog starts from rest and runs in a straight line with a constant acceleration of 2.5 m/s^2. How much time does it take for the dog to run a distance of 10.0 m?
(A) 8.0 s (B) 4.0 s (C) 2.8 s (D) 2.0 s (E) 1.4 s

<u>Questions 138 – 139</u>

A particle continuously moves in a circular path at constant speed in a counterclockwise direction. Consider a time interval during which the particle moves along this circular path from point P to point Q. Point Q is exactly half-way around the circle from Point P.

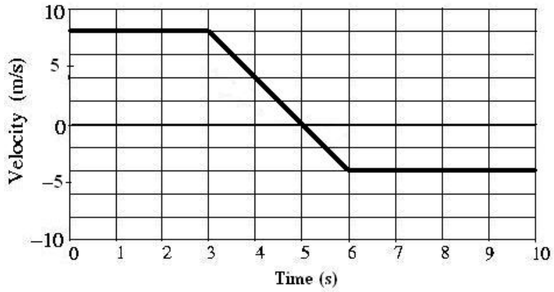


138. What is the direction of the average velocity during this time interval?
(A) → (B) ← (C) ↑ (D) ↓ (E) The average velocity is zero.

*139. What is the direction of the average acceleration during this time interval?
(A) → (B) ← (C) ↑ (D) ↓ (E) The average acceleration is zero.

Questions 140 – 141

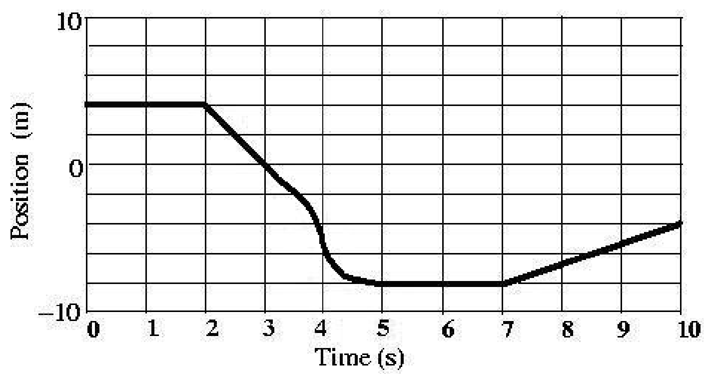
The velocity vs. time graph for the motion of a car on a straight track is shown in the diagram. The thick line represents the velocity. Assume that the car starts at the origin $x = 0$.



140. At which time is the car the greatest distance from the origin?
(A) t = 10 s (B) t = 6 s (C) t = 5s (D) t = 3s (E) t = 0 s

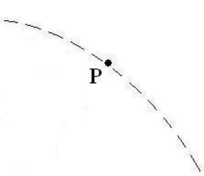
141. What is the average speed of the car for the 10 second interval?
(A) 1.20 m/s (B) 1.40 m/s (C) 3.30 m/s (D) 5.00 m/s (E) 5.40 m/s

142. Consider the motion of an object given by the position vs. time graph shown. For what time(s) is the speed of the object greatest?



(A) At all times from $t = 0.0\ s \rightarrow t = 2.0\ s$ (B) At time $t = 3.0\ s$ (C) At time $t = 4.0\ s$
(D) At all times from $t = 5.0\ s \rightarrow t = 7.0\ s$ (E) At time $t = 8.5\ s$

143. The free fall trajectory of an object thrown horizontally from the top of a building is shown as the dashed line in the figure. Which sets of arrows best correspond to the directions of the velocity and of the acceleration for the object at the point labeled *P* on the trajectory?



	velocity	acceleration
(A)	↘	↓
(B)	→	↓
(C)	→	↖
(D)	↘	↙
(E)	↘	↘

144. A toy car moves 3.0 m to the North in one second. The car then moves at 9.0 m/s due South for two seconds. What is the average speed of the car for this three second trip?
(A) 4.0 m/s (B) 5.0 m/s (C) 6.0 m/s (D) 7.0 m/s (E) 12 m/s

145. A projectile launched from the ground landed a horizontal distance of 120.0 m from its launch point. The projectile was in the air for a time of 4.00 seconds. If the projectile landed at the same vertical position from which it was launched, what was the launch speed of the projectile? Ignore air resistance.
(A) 22.4 m/s (B) 30.0 m/s (C) 36.1 m/s (D) 42.4 m/s (E) 50.0 m/s

146. Two automobiles are 150 kilometers apart and traveling toward each other. One automobile is moving at 60 km/h and the other is moving at 40 km/h. In how many hours will they meet?
(A) 1.5 (B) 1.75 (C) 2.0 (D) 2.5 (E) 3.0

147. A particle moves on the *x*-axis. When the particle's acceleration is positive and increasing
(A) its velocity must be positive (B) its velocity must be negative (C) it must be slowing down
(D) it must be speeding up (E) none of the above must be true.

148. What does one obtain by dividing the distance of 12 Mm by the time of 4 Ts?
(A) 3 nm/s (B) 3 μm/s (C) 3 mm/s (D) 3 km/s (E) 3 Gm/s

149. Is it possible for an object's velocity to increase while its acceleration decreases?
(A) No, this is impossible because of the way in which acceleration is defined
(B) No, because if acceleration is decreasing the object will be slowing down
(C) No, because velocity and acceleration must always be in the same direction
(D) Yes, an example would be a falling object near the surface of the moon
(E) Yes, an example would be a falling object in the presence of air resistance

<u>Questions 150 – 151</u>

During a recent winter storm, bales of hay had to be dropped from an airplane to a herd of cattle below. Assume the airplane flew horizontally at an altitude of 180 m with a constant velocity of 50 m/s and dropped one bale of hay every two seconds. It is reasonable to assume that air resistance will be negligible for this situation.

150. As the bales are falling through the air, what will happen to their distance of separation?
(A) the distance of separation will increase
(B) the distance of separation will decrease
(C) the distance of separation will remain constant
(D) the distance of separation will depend on the mass of the bales
(E) none of the above are always true

151. About how far apart from each other will the bales land on the ground?
(A) 9000 m (B) 300 m (C) 180 m (D) 100 m (E) 50 m

*152. If a boat can travel with a speed of *v* in still water, which of the following trips will take the least amount of time.
(A) traveling a distance of 2*d* in still water
(B) traveling a distance of 2*d* across (perpendicular to) the current in a stream
(C) traveling a distance *d* downstream and returning a distance *d* upstream
(D) traveling a distance *d* upstream and returning a distance *d* downstream
(E) all of the above will take equal times

*153. Suppose two cars are racing on a circular track one kilometer in circumference. The first car can circle the track in 15 seconds at top speed while the second car can circle the track in 12 seconds at top speed. How much lead does the first car need starting the last lap of the race not to lose?
(A) at least 250 m (B) at least 83 m (C) at least 200 m (D) at least 67 m (E) at least 104 m

AP Physics Free Response Practice – Kinematics

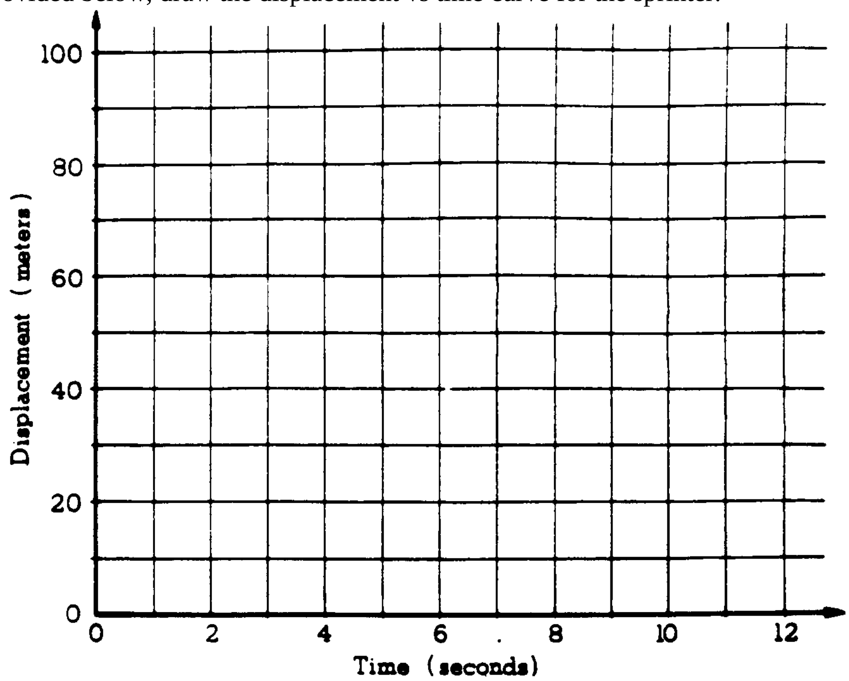
1982B1. The first meters of a 100-meter dash are covered in 2 seconds by a sprinter who starts from rest and accelerates with a constant acceleration. The remaining 90 meters are run with the same velocity the sprinter had after 2 seconds.

a. Determine the sprinter's constant acceleration during the first 2 seconds.

b. Determine the sprinters velocity after 2 seconds have elapsed.

c. Determine the total time needed to run the full 100 meters.

d. On the axes provided below, draw the displacement vs time curve for the sprinter.



2006B2. A world-class runner can complete a 100 m dash in about 10 s. Past studies have shown that runners in such a race accelerate uniformly for a time *t* and then run at constant speed for the remainder of the race. A world-class runner is visiting your physics class. You are to develop a procedure that will allow you to determine the uniform acceleration *a* and an approximate value of *t* for the runner in a 100 m dash. By necessity your experiment will be done on a straight track and include your whole class of eleven students.

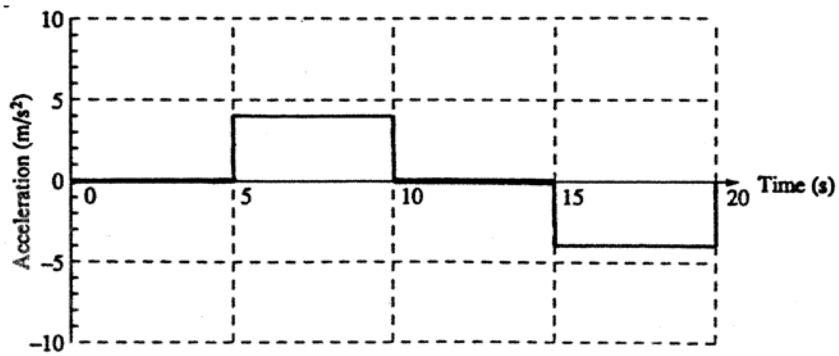
(a) By checking the line next to each appropriate item in the list below, select the equipment, other than the runner and the track, that your class will need to do the experiment.

____Stopwatches ____Tape measures ____ Rulers ____ Masking tape

____Metersticks ____ Starter's pistol ____ String ____ Chalk

(b) Outline the procedure that you would use to determine *a* and *t*, including a labeled diagram of the experimental setup. Use symbols to identify carefully what measurements you would make and include in your procedure how you would use each piece of the equipment you checked in part (a).

(c) Outline the process of data analysis, including how you will identify the portion of the race that has uniform acceleration, and how you would calculate the uniform acceleration.

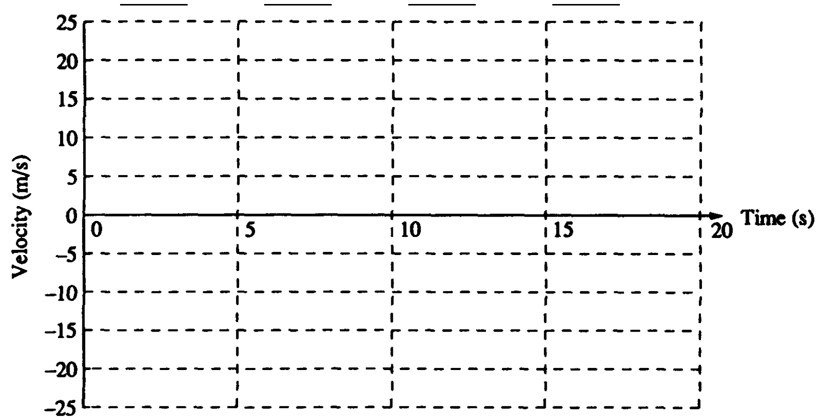


1993B1 (modified) A student stands in an elevator and records his acceleration as a function of time. The data are shown in the graph above. At time $t = 0$, the elevator is at displacement $x = 0$ with velocity $v = 0$. Assume that the positive directions for displacement, velocity, and acceleration are upward.

a. Determine the velocity v of the elevator at the end of each 5-second interval.

i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
v (m/s)	_____	_____	_____	_____



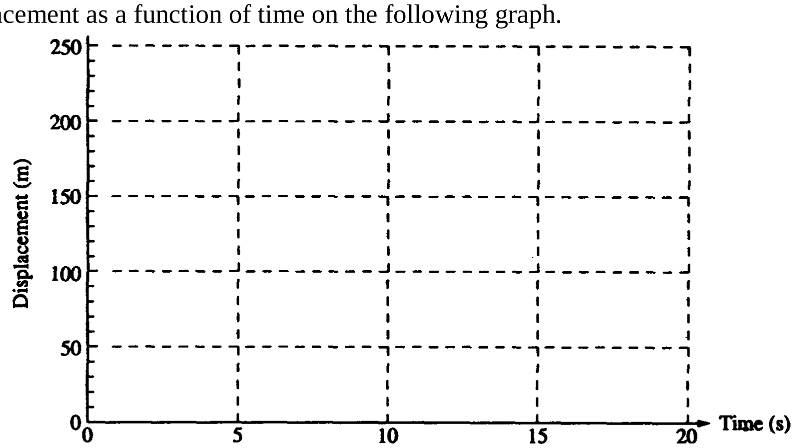
ii. Plot the velocity as a function of time on the following graph.

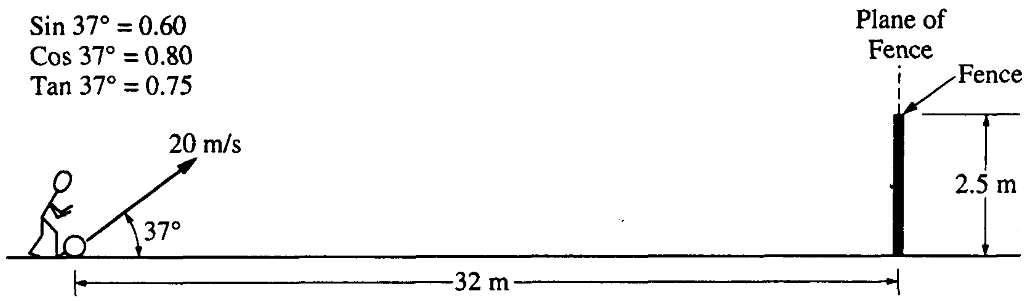
b. Determine the displacement x of the elevator above the starting point at the end of each 5-second interval.

i. Indicate your results by completing the following table.

Time Interval (s)	0–5	5–10	10–15	15–20
x (m)	_____	_____	_____	_____

ii. Plot the displacement as a function of time on the following graph.





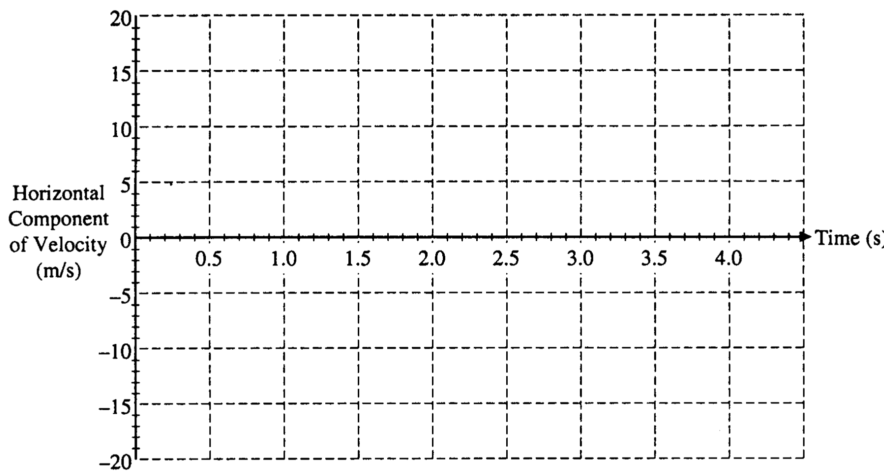
Note: Diagram not drawn to scale.

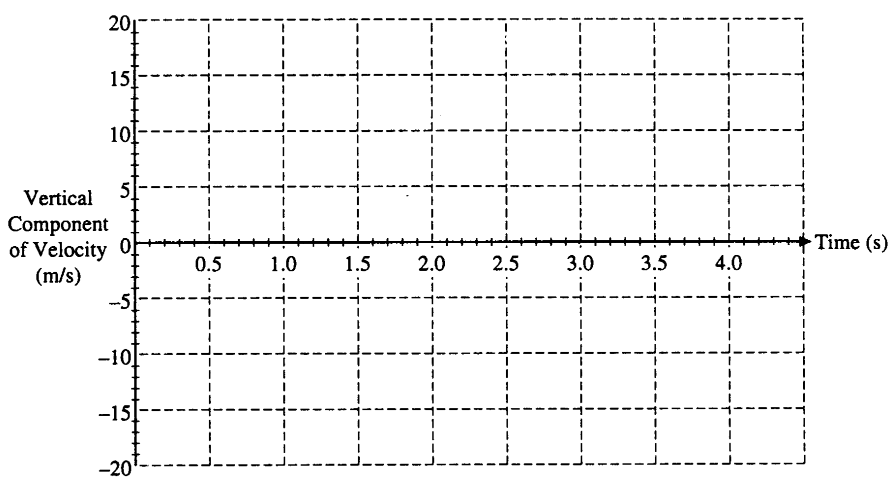
1994B1 (modified) A ball of mass 0.5 kilogram, initially at rest, is kicked directly toward a fence from a point 32 meters away, as shown above. The velocity of the ball as it leaves the kicker's foot is 20 meters per second at an angle of 37° above the horizontal. The top of the fence is 2.5 meters high. The ball hits nothing while in flight and air resistance is negligible.

a. Determine the time it takes for the ball to reach the plane of the fence.

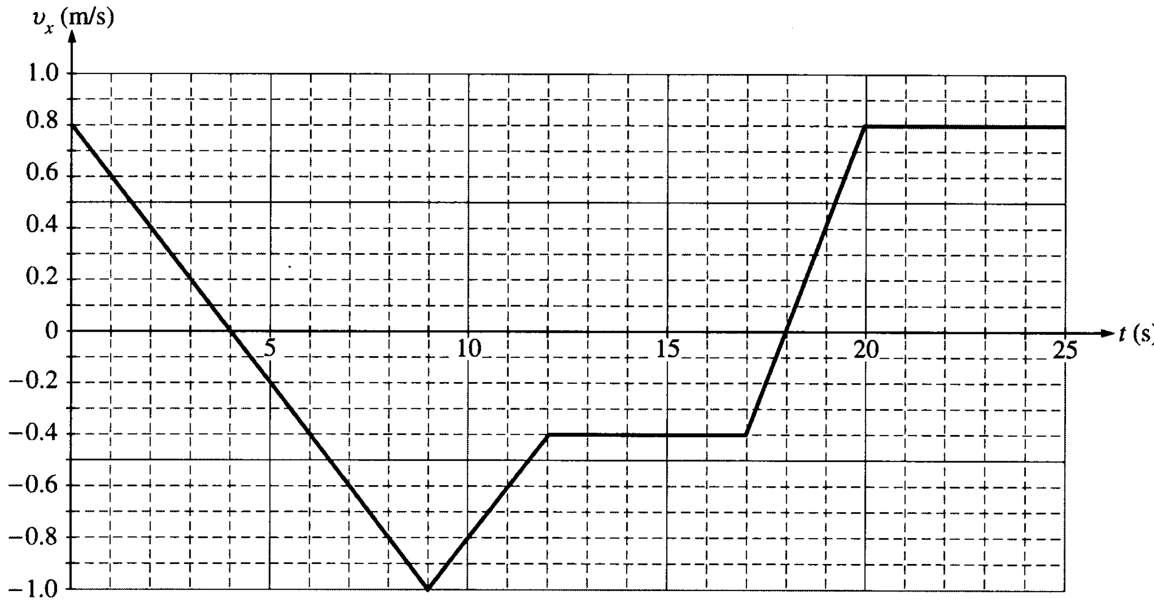
b. Will the ball hit the fence? If so, how far below the top of the fence will it hit? If not, how far above the top of the fence will it pass?

c. On the axes below, sketch the horizontal and vertical components of the velocity of the ball as functions of time until the ball reaches the plane of the fence.





2000B1 (modified) A 0.50 kg cart moves on a straight horizontal track. The graph of velocity *v* versus time *t* for the cart is given below.

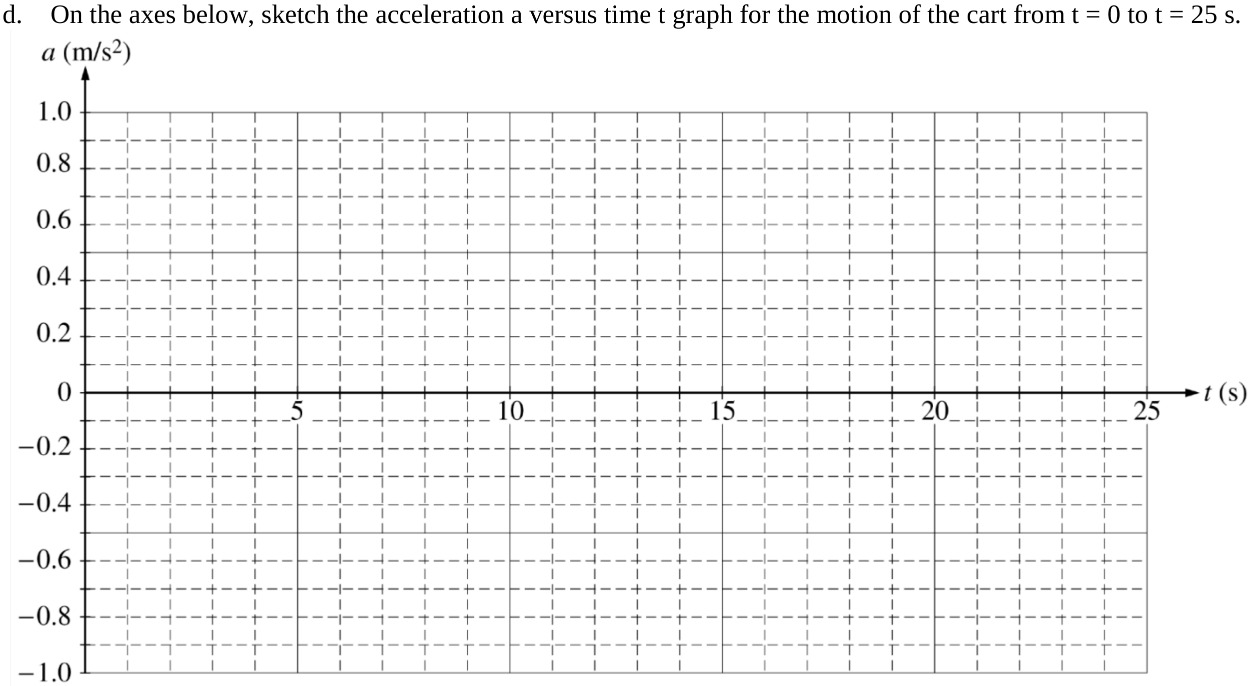


a. Indicate every time t for which the cart is at rest.

b. Indicate every time interval for which the speed (magnitude of velocity) of the cart is increasing.

c. Determine the horizontal position x of the cart at t = 9.0 s if the cart is located at x = 2.0 m at t = 0.

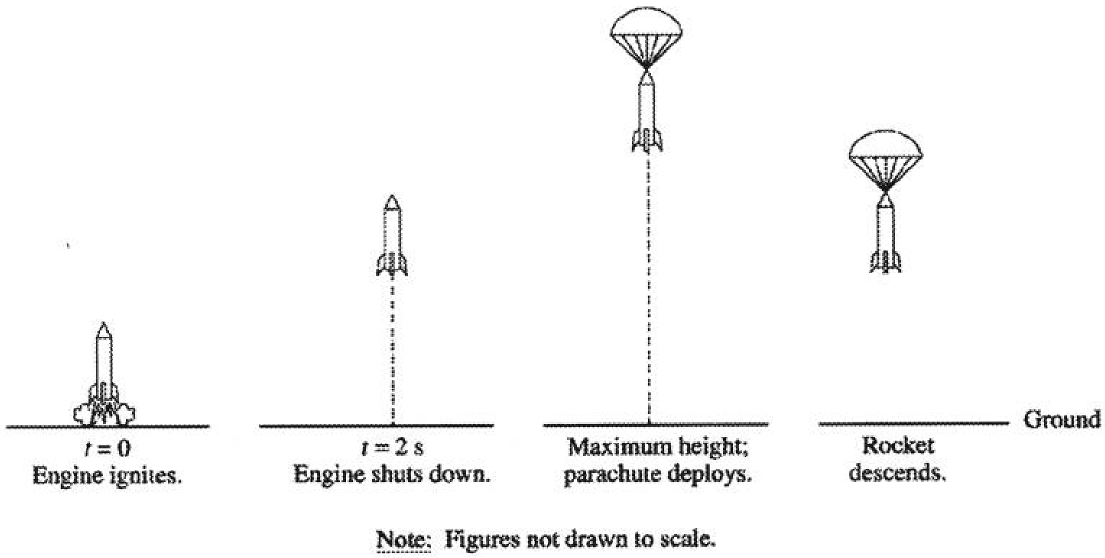
d. On the axes below, sketch the acceleration a versus time t graph for the motion of the cart from t = 0 to t = 25 s.



e. From t = 25 s until the cart reaches the end of the track, the cart continues with constant horizontal velocity. The cart leaves the end of the track and hits the floor, which is 0.40 m below the track. Neglecting air resistance, determine each of the following:

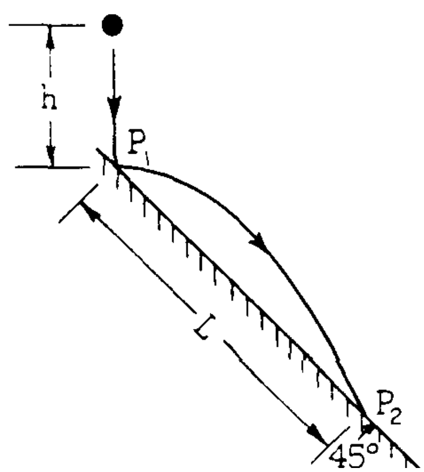
i. The time from when the cart leaves the track until it first hits the floor

ii. The horizontal distance from the end of the track to the point at which the cart first hits the floor



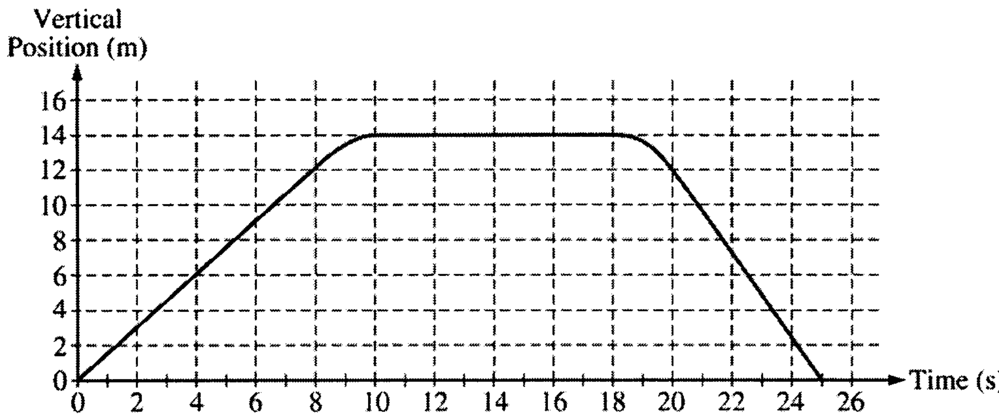
2002B1 (modified) A model rocket is launched vertically with an engine that is ignited at time t = 0, as shown above. The engine provides an upward acceleration of 30 m/s^2 for 2.0 s. Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

a. Determine the speed of the rocket after the 2 s firing of the engine.
b. What maximum height will the rocket reach?
c. At what time after t = 0 will the maximum height be reached?



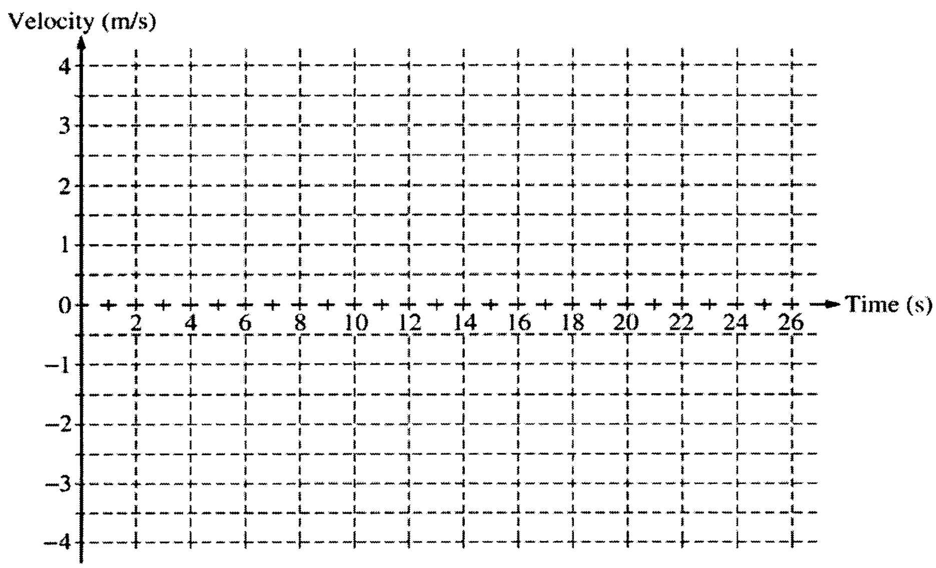
*1979M1 (modified) A ball of mass m is released from rest at a distance h above a frictionless plane inclined at an angle of 45° to the horizontal as shown above. The ball bounces horizontally off the plane at point P_1 with the same speed with which it struck the plane and strikes the plane again at point P_2. In terms of g and h determine each of the following quantities:

a. The speed of the ball just after it first bounces off the plane at P_1.
b. The time the ball is in flight between points P_1 and P_2.
c. The distance L along the plane from P_1 to P_2.
d. The speed of the ball just before it strikes the plane at P_2.

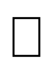


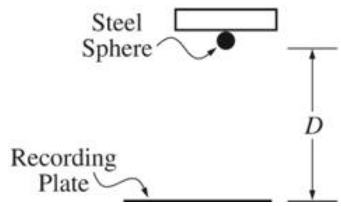
2005B1 (modified) The vertical position of an elevator as a function of time is shown above.

a. On the grid below, graph the velocity of the elevator as a function of time.



b. i. Calculate the average acceleration for the time period t = 8 s to t = 10 s.
   ii. On the box below that represents the elevator, draw a vector to represent the direction of this average acceleration.

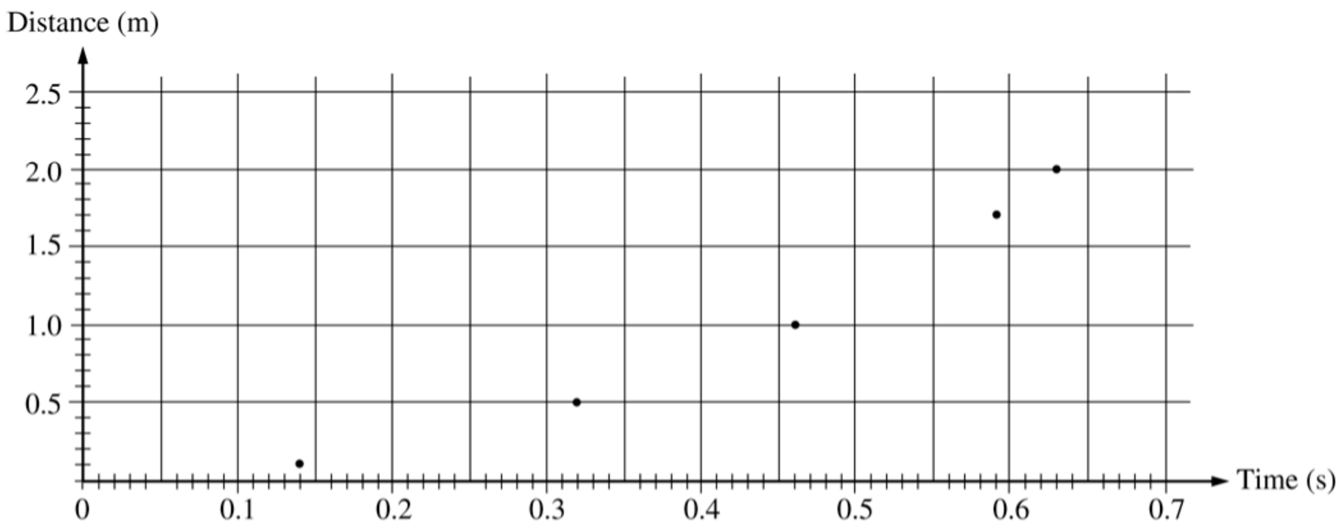




2006Bb1. A student wishing to determine experimentally the acceleration g due to gravity has an apparatus that holds a small steel sphere above a recording plate, as shown above. When the sphere is released, a timer automatically begins recording the time of fall. The timer automatically stops when the sphere strikes the recording plate.

The student measures the time of fall for different values of the distance D shown above and records the data in the table below. These data points are also plotted on the graph.

Distance of Fall (m)	0.10	0.50	1.00	1.70	2.00
Time of Fall (s)	0.14	0.32	0.46	0.59	0.63

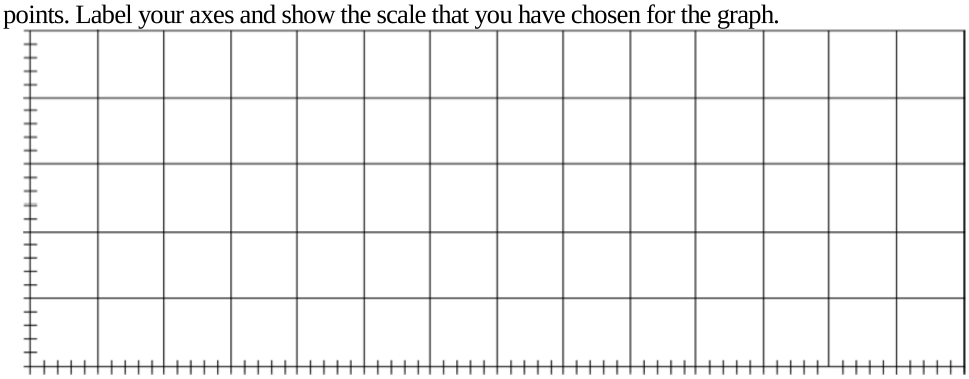


(a) On the grid above, sketch the smooth curve that best represents the student's data

The student can use these data for distance D and time t to produce a second graph from which the acceleration g due to gravity can be determined.

(b) If only the variables D and t are used, what quantities should the student graph in order to produce a linear relationship between the two quantities?

(c) On the grid below, plot the data points for the quantities you have identified in part (b), and sketch the best straight-line fit to the points. Label your axes and show the scale that you have chosen for the graph.



(d) Using the slope of your graph in part (c), calculate the acceleration g due to gravity in this experiment.

(e) State one way in which the student could improve the accuracy of the results if the experiment were to be performed again. Explain why this would improve the accuracy.

ANSWERS - AP Physics Multiple Choice Practice – Kinematics

	Solution	Answer
1.	Total distance = 60 miles, total time = 1.5 hours; average speed = total distance/total time	B
2.	Area bounded by the curve is the displacement By inspection of particle A the positive area between 0 and 1s will be countered by an equal negative area between 1 and 2s.	B
3.	Constant non-zero acceleration would be a straight line with a non-zero slope	D
4.	Area bounded by the curve is the displacement By inspection of particle A the positive area between 0 and 1s will be countered by an equal negative area between 1 and 2s.	A
5.	Area bounded by the curve is the displacement By inspection the negative area between 0 and 1s will be countered by an equal negative area sometime between 1 and 2s.	C
6.	Between 0 and 1 s; $d_1 = vt$; from 1 to 11 seconds; $d_2 = v_0t + ½ at^2$; $d = d_1 + d_2$	C
7.	Since the slope is positive and constant, so is the velocity, therefore the acceleration must be zero	A
8.	For a horizontal projectile, the initial speed does not affect the time in the air. Use $v_{0y} = 0$ with $10\text{ m} = ½ gt^2$	C
9.	The time in the air for a horizontal projectile is dependent on the height and independent of the initial speed. Since the time in the air is the same at speed v and at speed 2v, the distance (d = vt) will be twice as much at a speed of 2v	C
10.	Average velocity = total displacement/total time; magnitude of total displacement = 500 m (3-4-5 triangle) and total time = 150 seconds	B
11.	The acceleration is constant and negative which means the slope of the velocity time graph must have a constant negative slope. (Only one choice has the correct acceleration anyway)	D
12.	From rest, $h = ½ gt^2$	C
13.	At the top of its path, the vertical component of the velocity is zero, which makes the speed at the top a minimum. With symmetry, the projectile has the same speed when at the same height, whether moving up or down.	D
14.	g points down in projectile motion. Always.	E
15.	For a horizontal projectile; $h = ½ gt^2$ (initial vertical component of velocity is zero)	E
16.	At *every* point of a projectiles free-fall, the acceleration is the acceleration due to gravity	E
17.	Average speed = total distance/total time = (8 m – 2 m)/(1 second)	D
18.	For a horizontal projectile; $h = ½ gt^2$ (initial vertical component of velocity is zero)	C
19.	The area under the curve is the displacement. There is more area under the curve for Car X.	A
20.	Area under the curve is the displacement. Car Y is moving faster as they reach the same point.	B
21.	Uniformly accelerated means the speed-time graph should be a stright line with non-zero slope. The corresponding distance-time graph should have an increasing slope (curve upward)	E
22.	From the equation $d = ½ at^2$, displacement is proportional to time squared. Traveling from rest for twice the time gives 4 times the displacement (or 4 m). Since the object already travelled 1 m in the first second, during the time interval from 1 s to 2 s the object travelled the remaining 3 m	C
23.	To travel straight across the river, the upstream component of the boat's velocity must cancel the current. Since the speed of the current is the same as the speed of the boat, the boat must head directly upstream to cancel the current, which leaves no component across the river	E

24. v_{iy} = 200 m/s sin 30º = 100 m/s. At maximum height $v_y = 0$. Use $v_y^2 = v_{iy}^2 + 2gh$ C

25. Acceleration is proportional to Δv. $\Delta v = v_2 - v_1 = v_2 + (-v_1)$ E

26. From a height of 45 m (= ½ gt^2) it takes 3 seconds to strike the ground. In that time, the ball thrown traveled 30 m. v = d/t B

27. 9.8 m/s^2 can be thought of as a change in speed of 9.8 m/s *per second*. A

28. v_i = 0 m/s; v_f = 30 m/s; t = 6 s; $\bar{v} = \frac{v_i + v_f}{2} = \frac{d}{t}$ D

29. velocity of package relative to observer on ground $v_{pg} = v_1 = \searrow$ D

velocity of package relative to pilot $v_{pp} = v_2 = \downarrow$

velocity of pilot relative to ground $v_{po} = \longrightarrow$

Putting these together into a right triangle yields $v_{pg}^2 + v_2^2 = v_1^2$

30. While the object momentarily stops at its peak, it never stops accelerating downward. D

31. Maximum height of a projectile is found from $v_y = 0$ at max height and $v_y^2 = v_{iy}^2 + 2gh$ and gives $h_{max} = v_{iy}^2/2g = (v_i \sin\theta)^2/2g$. Fired straight up, θ = 90º and we have $v_i = \sqrt{2gh}$ C

Plugging this initial velocity into the equation for a 45º angle (sin 45º = $\frac{1}{\sqrt{2}}$) gives

$h_{new} = (\sqrt{2gh}\ \frac{1}{\sqrt{2}})^2/2g = h/2$

32. g points down in projectile motion. Always. C

33. horizontal velocity v_x remains the same thorughout the flight. g remains the same as well. E

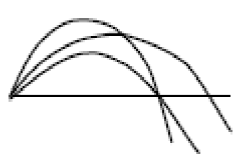
34. A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows an increasing slope, then a constant slope, then a decreasing slope (to zero) D

35. For a dropped object: d = ½ gt^2 D

36. For a horizontal projectile, the initial speed does not affect the time in the air. Use $v_{0y} = 0$ with 10 m = ½ gt^2 to get t = $\sqrt{2}$ For the horzontal part of the motion; v = d/t C

37. A velocity-time graph represents the *slope* of the displacement-time graph. Analyzing the v-t graph shows a constant slope, then a decreasing slope to zero, becoming negative and increasing, then a constant slope. Note this is an analysis of the *values* of v, not the slope of the graph itself A

38. By process of elimination (A and B are unrealistic; C is wrong, air resistance should decrease the acceleration; E is irrelevant) D

39. 

The 45° angle gives the maximum horizontal travel to the original elevation, but the smaller angle causes the projectile to have a greater horizontal component of velocity, so given the additional time of travel allows such a trajectory to advance a greater horizontal distance. In other words given enough time the smaller angle of launch gives a parabola which will eventual cross the parabola of the 45° launch. C

40. The area under the curve of an acceleration-time graph is the change in speed. D

41. In the 4 seconds to reach the ground, the flare travelled 70 m/s × 4 s = 280 m horizontally. D

42. In the 4 seconds to reach the ground, the flare travelled 70 m/s × 4 s = 280 m horizontally. The plane travelled $d = v_i t + ½ at^2$ = (70 m/s)(4 s) + (0.5)(0.75 m/s^2)(4 s)^2 = 280 m + 6 m, or 6 m ahead of the flare. B

43. Positive velocity = positive slope. Negative acceleration = decreasing slope (or downward curvature E

44. The slope of the line represents her velocity. Beginning positive and constant, going to zero, then positive and larger than the initial, then negative while the line returns to the time axis B

45. Dropped from a height of 9.8 m, the life preserver takes (9.8 m = ½ gt^2); t = 1.4 seconds to reach the water. In that 1.4 seconds the swimmer covered (6.0 m – 2.0 m) = 4.0 m meaning the water speed is (4.0 m)/(1.4 s) D

46. The ball takes time *T*/2 to reach height *H*. Using $v_y = 0$ at maximum height and $\bar{v} = \frac{v_i + v_f}{2} = \frac{d}{t}$ gives the initial speed as 4*H*/*T*. In addition from the top H = ½ g(T/2)^2 = gT^2/8. Plugging in a time T/4 gives d = (4H/T)(T/4) + ½ (–g)(T/4)^2 = H – ¼ (gT^2/8) = ¾ H E

47. While the object momentarily stops at its peak, it never stops accelerating downward. Without air resistance, symmetry dictates time up = time down. With air resistance considered, the ball will have a larger average velocity on the way up and a lower average velocity on the way down since it will land with a smaller speed than it was thrown, meaning the ball takes longer to fall. A

48. Total distance = d. Time for first ¾ d is t_1 = (¾d)/v = 3d/4v. Time for second part is t_2 = (¼ d)/(½ v) = 2d/4v. Total time is then $t_1 + t_2$ = 5d/4v. Average speed = d/(5d/4v) B

49. Positive acceleration is an increasing slope (including negative slope increasing toward zero) or upward curvature C

50. With air resistance, the acceleration (the slope of the curve) will decrease toward zero as the ball reached terminal velocity. Note: without air resistance, choice (A) would be correct D

51. Since for the first 4 seconds, the car is accelerating positively the entire time, the car will be moving fastest just beofre slowing down after t = 4 seconds. C

52. The area under the curve represents the change in velocity. The car begins from rest with an increasing positive velocity, after 4 seconds the car begins to slow and the area under the curve from 4 to 8 seconds couters the increase in velocity form 0 to 4 seconds, bringing the car to rest. However, the car never changed direction and was moving away from its original starting position the entire time. E

53. The ball will land with a speed given by the equaion $v^2 = v_i^2 + 2gH$ or $v = \sqrt{2gH}$. Rebounding with ¾ the speed gives a new height of $v_f = 0 = (¾ \sqrt{2gH})^2 + 2(-g)h_{new}$ C

54. The velocity-time graph should represent the slope of the position-time graph and the acceleration-time graph should represent the slope of the velocity-time graph C

55. It's a surprising result, but while both the horizontal and verticla components change at a given height with varying launch angle, the *speed* $(v_x^2 + v_y^2)^{1/2}$ will be independent of α (try it!) C

56. $v_f^2 = v_i^2 + 2ad$ B

57. d_1 = (+7 m/s)(2 s) + ½ (–10 m/s^2)(2 s)^2; d_1 = (–7 m/s)(2 s) + ½ (–10 m/s^2)(2 s)^2 C

58. Range of a projectile $R = (v_i^2 \sin 2\theta)/g$ and maximum range occurs at $\theta = 45°$, which gives $v_i = \sqrt{Rg}$. Maximum height of a projectile is found from $v_y = 0$ at max height and $v_y^2 = v_{iy}^2 + 2gh$ and gives $h_{max} = v_{iy}^2/2g = (v_i \sin\theta)^2/2g$. Maximum range occurs at 45º, which gives $h = (Rg)(\sin 45)^2/2g$ — B

59. $v_f^2 = v_i^2 + 2ad$ — D

60. The diagonal of a face of the cube is $3\sqrt{2}$ m. The diagonal across the cube itself is the hypoteneuse of this face diagonal and a cube edge: $\sqrt{(3\sqrt{2})^2 + 3^2}$ — A

61. Instantaneous velocity is the slope of the line at that point — A

62. Displacement is the area under the curve. Maximum displacement is just before the car turns around at 2.5 seconds. — D

63. Range of a projectile $R = (v_i^2 \sin 2\theta)/g$ and maximum range occurs at $\theta = 45°$, which gives $v_i = \sqrt{Rg}$. Using $\theta = 30°$ gives $R_{new} = R \sin 60°$ — A

64. (advanced question!) The time for one bounce is found from $-v = v + (-g)t$ which gives $t = 2v/g$. We are summing the time for all bounces, while the velocity (and hence the time) converge in a geometric series with the ratio $v_{n+1}/v_n = r < 1$ to $\frac{1}{1-r}$ — A

65. The acceleration is the slope of the curve at 90 seconds. — B

66. From the equation $d = ½ at^2$, displacement is proportional to time squared. Traveling from rest for twice the time gives 4 times the displacement (or 4 h). Since the object already travelled h in the first second, during the time interval from 1 s to 2 s the object travelled the remaining 3h — C

67. $d = v_i t + ½ gt^2$ — D

68. The relative speed between the coyote and the prairie dog is 14.5 m/s. To cover the 45 m distance between them will take $t = d/v = (45 \text{ m})/(14.5 \text{ m/s})$ — B

69. For the first part of the trip (the thrust): $d_1 = v_i t + ½ at^2 = 0 \text{ m} + ½ (50 \text{ m/s}^2)(2 \text{ s})^2 = 100 \text{ m}$
For the second part, we first find the velocity after the thrust $v = at = 100$ m/s and at the maximum height $v_f = 0$, so to find d_2 we use $v_f^2 = v_i^2 + 2ad_2$ which gives $d_2 = 510$ m — C

70. Total displacement west = 1100 m; total displacement south = 400 m. Use the Pythagorean theorem. — C

71. For a horizontal projectile ($v_{iy} = 0$ m/s) to fall 1 m takes (using $1 \text{ m} = ½ gt^2$) 0.45 seconds. To travel 30 m in this time requires a speed of $d/t = (30 \text{ m})/(0.45 \text{ s})$ — D

72. Maximum height of a projectile is found from $v_y = 0$ m/s at max height and $(0 \text{ m/s})^2 = v^2 + 2gh$ and gives $h = v^2/2g$
The height at which the projectile is moving with half the speed is found from $(½v)^2 = v^2 + 2(-g)d$ which gives $d = 3v^2/8g = 0.75$ h — E

73. Looking at choices A, D and E eliminates the possibility of choices B and C (each ball increases its speed by 9.8 m/s each second, negating those choices anyway). Since ball A is moving faster than ball B at all times, it will continue to pull away from ball B (the relative speed between the balls separates them). — E

74. Since they all have the same horizontal component of the shell's velocity, the shell that spends the longest time in the air will travel the farthest. That is the shell launched at the largest angle (mass is irrelevant). — D

75. This is merely asking for the horizontal range of a horizontal projectile. The time in the air is found from the height using $h = \frac{1}{2} gt^2$ which gives $t = \sqrt{\frac{2h}{g}}$. The range is found using d = vt — E

76. Flying into the wind the airliners speed relative to the ground is 500 km/h – 100 km/h = 400 km/h and a 3000 km trip will take t = d/v = 7.5 hours. Flying with the wind the airliners speed relative to the ground is 500 km/h + 100 km/h = 600 km/h and a 3000 km trip will take t = d/v = 5 hours making the total time 12.5 hours. — C

77. The horizontal component of the velocity is 28.3 m/s cos 60º = 14.15 m/s. If the ball is in the air for 5 seconds the horizontal displacement is $x = v_x t$ — D

78. since (from rest) $d = \frac{1}{2} gt^2$, distance is proportional to time squared. An object falling for twice the time will fall four times the distance. — D

79. $\bar{v} = \frac{v_i + v_f}{2} = \frac{d}{t}$ — A

80. v_f = –40 m/s (negative since it is moving down when landing). Use $v_f = v_i + (-g)t$ — B

81. For a horizontal projectile (v_{iy} = 0 m/s) to fall 0.05 m takes (using $0.05\text{ m} = \frac{1}{2} gt^2$) 0.1 seconds. To travel 20 m in this time requires a speed of d/t = (20 m)/(0.1 s) — D

82. 9.8 m/s^2 means the speed changes by 9.8 m/s each second — B

83. Once released, the package is in free-fall (subject to gravity only) — D

84. For the first part v = at = 8.0 m/s and $d = \frac{1}{2} at^2$ = 40 m. In the second part of the trip, the speed remains at 8 m/s, and travels an additional d = vt = 80 m — C

85. The definition of termincal velocity is the velocity at which the force of air friction balances the weight of the object and the object *no longer accelerates*. — B

86. To reach a speed of 30 m/s when dropped takes (using v = at) about 3 seconds. The distance fallen after three seconds is found using $d = \frac{1}{2} at^2$ — C

87. 9.8 m/s^2 means the speed changes by 9.8 m/s each second (in the downward direction) — D

88. Total distance = 800 km. Times are (400 km)/(80 km/h) = 5 hours and (400 km)/(100 km/h) = 4 hours. Average speed = total distance divided by total time. — B

89. $\Delta v = at$ — B

90. 9.8 m/s^2 means the speed changes by 9.8 m/s each second — D

91. Velocity is a vector, speed is a scalar — B

92. Choices A, B, C and E all refer to vectors — D

93. Falling on the Moon is no different conceptually than falling on the Earth — C

94. Since the line is above the t axis for the entire flight, the duck is always moving in the positive (forward) direction, until it stops at point D — D

95. One could analyze the graphs based on slope, but more simply, the graph of position versus time should represent the actual path followed by the ball as seen on a platform moving past you at constant speed. — C

96. Other than the falling portions (a = –9.8 m/s^2) the ball should have a "spike" in the acceleration when it bounces due to the rapid change of velocity from downward to upward. — B

97. The same average speed would be indicated by the same distance travelled in the time interval — C

98. At t_3, car #1 is ahead of car #2 and at t_4, car #1 is behind car #2. They were in the same position somewhere in between — D

99. Average speed = (total distance)/(total time). Cars #2 and #3 travelled the same distance. — B

100. If you look at the distance covered in each time interval you should nitce a pattern: 2 m, 6 m, 10 m, 14 m, 18 m; making the distance in the next second 22 m. — C

101. Instantaneous speed is the slope of the line at that point. — B

102. A non-zero accleeration is inidcated by a curve in the line — E

103. Net displacement north = 300 miles sin 30º = 150 miles — C
Net displacement east = (300 miles cos 30º – 600 miles) = – 340 miles, or 340 miles west.
Angle north of west is $\tan^{-1}\left(\frac{150 miles}{340 miles}\right)$

104. Maximum height of a projectile is found from v_y = 0 m/s at max height and $(0\text{ m/s})^2 = v^2 + 2gh$ and gives $h = v^2/2g$. At twice the initial speed, the height will be 4 times as much — C

105. Average speed = total distance divided by total time = (0.48 m)/(0.2 s) — E

106. $d = \frac{1}{2}at^2$ (use any point) — E

107. $v = v_i + at$ — B

108. Acceleration is the slope of the line segment — C

109. Displacement is the area under the line — E

110. v_i = 30 m, v_f = 0, d = 45 m; $\bar{v} = \frac{v_i + v_f}{2} = \frac{d}{t}$ — D

111. In a vacuum, there is no air resistance and hence no terminal velocity. It will continue to accelerate. — E

112. A projectile launched at a smaller angle does not go as high and will fall to the ground first. — B

113. $v_x = v_i \cos\theta$ — C

114. Velocity is the slope of the line. — D

115. Positive acceleration is an upward curvature — D

116. Average acceleration = $\Delta v/\Delta t$ — E

117. $d = \frac{1}{2}at^2$ — E

118. Acceleration is the slope of the line segment — C

119. Displacement is the area between the line and the t-axis. Area is negative when the line is below the t-axis. — B

120. After two seconds, the object would be above it's original position, still moving upward, but the acceleration due to gravity is always pointing down — B

121. Constant speed is a constant slope on a position-time graph, a horizontal line on a velocity time graph or a zero value on an acceleration-time graph — D

122. Average speed = total distance divided by total time = (7 cm)/(1 s) — B

123. $d = \frac{1}{2}at^2$ (use any point) — C

124.	Maximum height of a projectile is found from v_y = 0 m/s at max height and $(0\ m/s)^2 = v^2 + 2gh$ and gives $h = v^2/2g$. Mass is irrelevant. Largest initial speed = highest.	C
125.	Using $d = ½ at^2$ shows the height is proportional to the time squared. ½ the maximum height is $\frac{1}{\sqrt{2}}$ times the time.	B
126.	Stopping distance is found using $v_f = 0 = v_i^2 + 2ad$ which gives $d = v_i^2/2a$ where stopping distance is proportional to initial speed squared.	A
127.	$v_f = v_i + gt$	B
128.	Moving away from the origin will maintain a negative position and velocity. Slowing down indicates the acceleration is opposite in direction to the velocity.	B
129.	The arrow travels equal horizontal distances in equal amounts of time. The distance fallen is proportional to time squared. The arrow will have fallen a total of 0.8 m in the next 5 m horizontally, or an *additional* 0.6 m.	A
130.	Tan 53º = h/(8 m)	B
131.	$d = ½ at^2$	C
132.	Maximum height of a projectile is found from $v_y = 0$ at max height and $v_y^2 = v_{iy}^2 + 2gh$ and gives $h_{max} = v_{iy}^2/2g = (v_i \sin\theta)^2/2g$	B
133.	Acceleration is the slope of the line	D
134.	Since the first rock is always traveling faster, the relative distance between them is always increasing.	A
135.	Stopping distance is found using $v_f = 0 = v_i^2 + 2ad$ which gives $d = v_i^2/2a$ where stopping distance is proportional to initial speed squared.	B
136.	At an angle of 120º, there is a component of the acceleration perpendicular to the velocity causing the direction to change and a component in the opposite direction of the velocity, causing it to slow down.	B
137.	$d = ½ at^2$	C
138.	The displacement is directly to the left. The average velocity is proportional to the displacement	B
139.	The velocity is initially pointing up, the final velocity points down. The acceleration is in the same direction as $\Delta v = v_f + (-v_i)$	D
140.	The car is the greatest distance just before it reverses direction at 5 seconds.	C
141.	Average speed = (total *distance*)/(total time), the total distance is the magnitude of the area under the line (the area below the t-axis is considered positive)	D
142.	Speed is the slope of the line.	C
143.	velocity is pointing tangent to the path, acceleration (gravity) is downward.	A
144.	Average speed = (total *distance*)/(total time)	D
145.	To travel 120 m horizontally in 4 s gives v_x = 30 m/s. The time to reach maximum height was 2 seconds and v_y = 0 at the maximum height which gives v_{iy} = 20 m/s. $v_i = \sqrt{v_x^2 + v_{iy}^2}$	C
146.	The relative speed between the two cars is $v_1 - v_2$ = (60 km/h) – (–40 km/h) = 100 km/h. They will meet in $t = d/v_{relative}$ = 150 km/100 km/h	A

147. Acceleration is independent of velocity (you can accelerate in any direction while traveling in a ny direction). E

148. 12/4 =3, now the units: M = 10^6, T = 10^{12}: M/T = 10^{-6} = micro (μ) B

149. Acceleration is independent of velocity (you can accelerate in any direction while traveling in a ny direction). If the accleration is in the same direction as the velocity, the object is speeding up. E

150. As the first bales dropped will always be traveling faster than the later bales, their relative velocity will cause their separation to always iincrease. A

151. Horizontally, the bales will all travel at the speed of the plane, as gravity will not affect their horizontal motion. D = vt = (50 m/s)(2 seconds apart) D

152. Traveling in still water will take a time t = d/v = 2d/v. Traveling perpendicularly across the stream requires the boat to head at an angle into the current, causing the relative velocity of the boat to the shore to be less than when in still water and therefore take a longer time. Since this eliminates choice E and choices D and C are identical, that leaves A as the only single option. A

If you really want proof:
To show C and D take longer, we have the following (let the current be moving with speed w):

traveling downstream; $v_{rel} = v + w$ and time = $\frac{d}{v+w}$

traveling upstream; $v_{rel} = v - w$ and time = $\frac{d}{v-w}$

total time = $\frac{d}{v+w} + \frac{d}{v-w} = \frac{2dv}{v^2 - w^2} = \frac{2d}{v - \frac{w^2}{v}} > \frac{2d}{v}$

153. When the first car starts the last lap, it will finish the race in 15 seconds from that point. In 15 seconds, the second car will travel (1 km/12 s) × 15 s = 1250 m so the first car must be at least 250 m ahead when starting the last lap to win the race. A

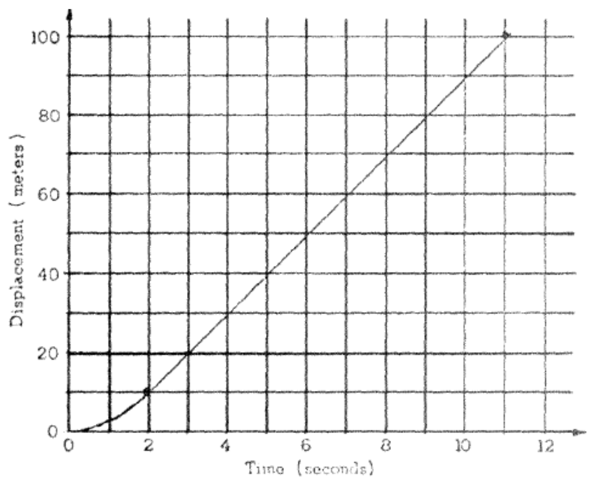
1982B1

a. For the first 2 seconds, while acceleration is constant, $d = ½ at^2$
Substituting the given values d = 10 meters, t = 2 seconds gives $a = 5\ m/s^2$

b. The velocity after accelerating from rest for 2 seconds is given by v = at, so v = 10 m/s

c. The displacement, time, and constant velocity for the last 90 meters are related by d = vt.
To cover this distance takes t = d/v = 9 s. The total time is therefore 9 + 2 = 11 seconds

d.



2006B2

Two general approaches were used by most of the students.
Approach A: Spread the students out every 10 meters or so. The students each start their stopwatches as the runner starts and measure the time for the runner to reach their positions.

Analysis variant 1: Make a position vs. time graph. Fit the parabolic and linear parts of the graph and establish the position and time at which the parabola makes the transition to the straight line.
Analysis variant 2: Use the position and time measurements to determine a series of average velocities ($v_{avg} = \Delta x/\Delta t$) for the intervals. Graph these velocities vs. time to obtain a horizontal line and a line with positive slope. Establish the position and time at which the sloped and horizontal lines intersect.
Analysis variant 3: Use the position and time measurements to determine a series of average accelerations ($\Delta x = v_0 t + ½ at^2$. Graph these accelerations vs. time to obtain two horizontal lines, one with a nonzero value and one at zero acceleration. Establish the position and time at which the acceleration drops to zero.

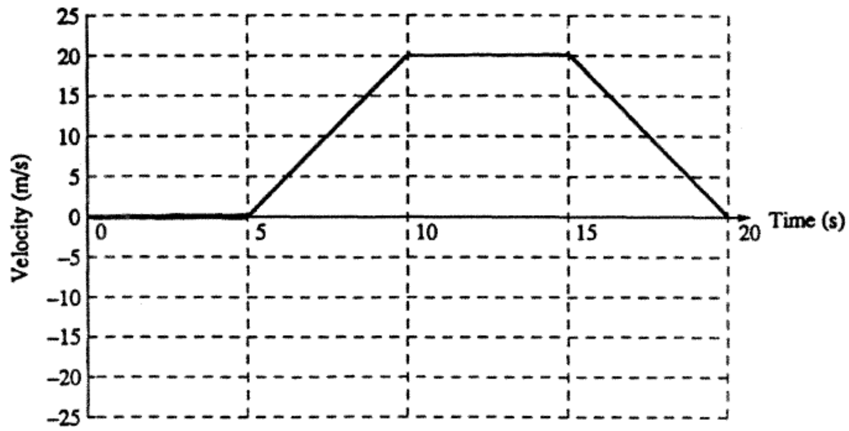
Approach B: Concentrate the students at intervals at the end of the run, in order to get a very precise value of the constant speed v_f, or at the beginning in order to get a precise value for *a*. The total distance *D* is given by $a = ½ a t_u^2 + v_f(T - t_u)$, where *T* is the total measured run time. In addition $v_f = a t_u$ These equations can be solved for a and t_u (if v_f is measured directly) or v_f and t_u (if *a* is measured directly). Students may have also defined and used distances, speeds, and times for the accelerated and constant-speed portions of the run in deriving these relationships.

1993B1

a. i. Use the kinematic equation applicable for constant acceleration: $v = v_0 + at$. For each time interval, substitute the initial velocity for that interval, the appropriate acceleration from the graph and a time of 5 seconds.

5 seconds: *v* = 0 + (0)(5 s) = 0
10 seconds: *v* = 0 + (4 *m/s*^{2}*)(5* s) = 20 *mls*
15 seconds: v = 20 *mls* + (0)(5 s) = 20 *mls*
20 seconds: v = 20 *mls* + (-4 *m/s*^{2}*)(5* s) = 0

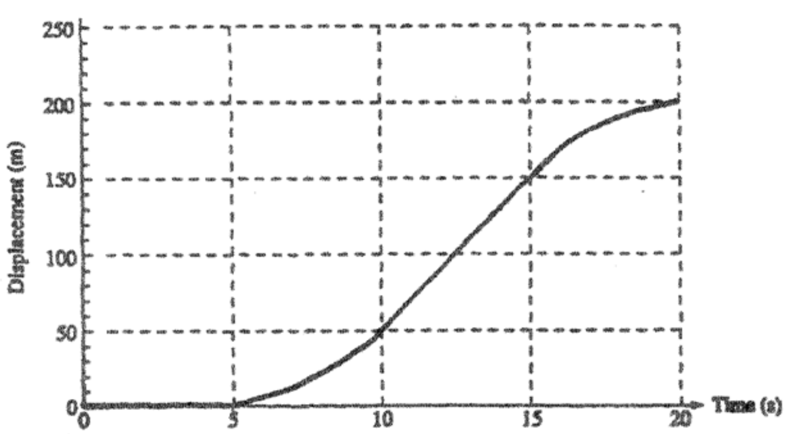
ii.



b. i. Use the kinematic equation applicable for constant acceleration, $x = x_0 + v_0t + ½ at^2$. For each time interval, substitute the initial position for that interval, the initial velocity for that interval from part (a), the appropriate acceleration, and a time of 5 seconds.

5 seconds: x = 0 + (0)(5 s) + ½ (0)(5 s)^2 = 0
10 seconds: x = 0 + (0)(5 s) + ½ (4 m/s^2)(5 s)^2 = 50 m
15 seconds: x = 50 m + (20 m/s)(5 s) + ½ (0)(5 s)^2 = 150 m
20 seconds: x = 150 m + (20 m/s)(5 s) + ½ (–4 m/s^2)(5 s)^2 = 200 m

ii.

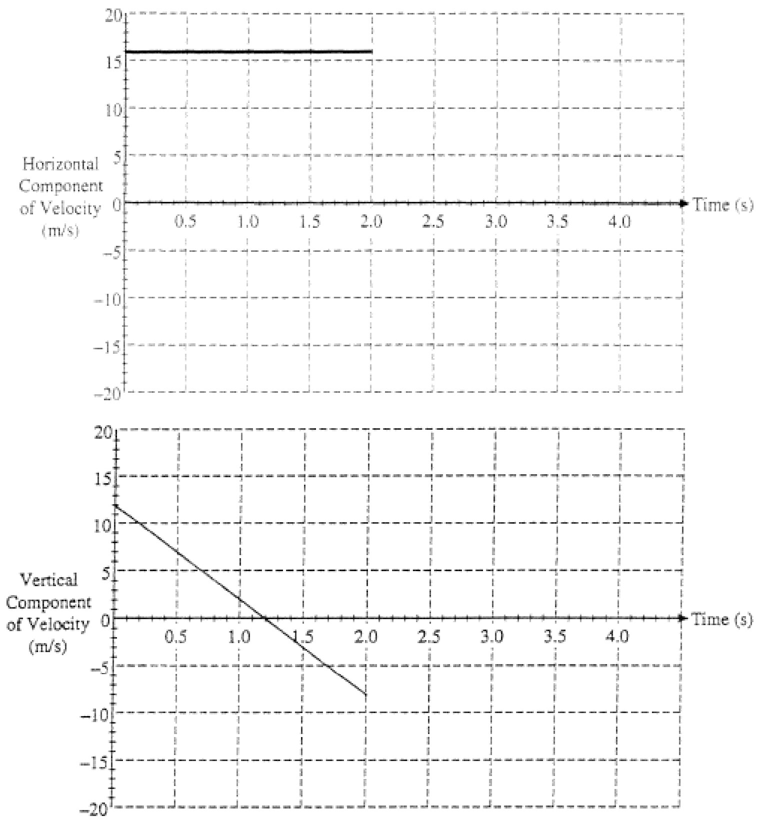


1994B1

a. The horizontal component of the velocity is constant so $v_x t = d$ where $v_x = v_0 \cos\theta = 16$ m/s
$t = d/v = 2$ s

b. The height of the ball during its flight is given by $y = v_{0y}t + ½ gt^2$ where $v_{0y} = v_0 \sin\theta = 12$ m/s and $g = –9.8$ m/s^2 which gives at $t = 2$ s, $y = 4.4$ m. The fence is 2.5 m high so the ball passes above the fence by 4.4 m – 2.5 m = 1.9 m

c.

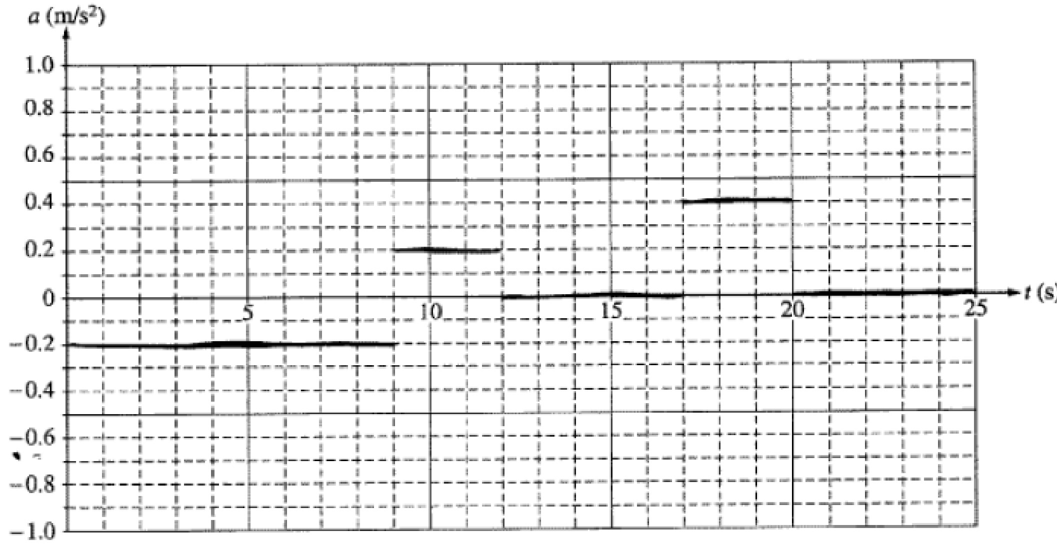


2000B1

a. The car is at rest where the line crosses the t axis. At t = 4 s and 18 s.

b. The speed of the cart increases when the line moves away from the t axis (larger values of v, positive or negative). This occurs during the intervals t = 4 to 9 seconds and t = 18 to 20 seconds.

c. The change in position is equal to the area under the graph. From 0 to 4 seconds the area is positive and from 4 to 9 seconds the area is negative. The total area is –0.9 m. Adding this to the initial position gives $x = x_0 + \Delta x$ = 2.0 m + (–0.9 m) = 1.1 m d.



e. i. $y = ½ gt^2$ (v_{0y} = 0 m/s) gives t = 0.28 seconds.
   ii. $x = v_x t$ = 0.22 m

2002B1

a. $v_1 = v_0 + at$ = 60 m/s
b. The height of the rocket when the engine stops firing $y_1 = ½ at^2$ = 60 m
   To determine the extra height after the firing stops, use v_f^2 = 0 m/s = $v_1^2 + 2(–g)y_2$ giving y_2 = 180 m
   total height = $y_1 + y_2$ = 240 m
c. To determine the time of travel from when the engine stops firing use v_f = 0 m/s = $v_1 + (–g)t_2$ giving t_2 = 6 s.
   The total time is then 2 s + 6 s = 8 seconds

1979M1

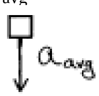
a. The speed after falling a height h is found from $v_f^2 = v_i^2 + 2gh$, where v_i = 0 m/s giving $v_f = \sqrt{2gh}$
b/c. During the flight from P_1 to P_2 the ball maintains a horizontal speed of $\sqrt{2gh}$ and travels a horizontal distance of $\frac{L}{\sqrt{2}}$ thus (using d = vt) we have $\frac{L}{\sqrt{2}} = \sqrt{2gh}$ t. During the same time t the ball travels the same distance vertically given by $\frac{L}{\sqrt{2}} = \frac{1}{2}gt^2$. Setting these expressions equal gives us $\sqrt{2gh}$ t = ½ gt^2. Solving for t and substituting into the expression of L gives $t = \sqrt{8h/g}$ and $L = 4\sqrt{2}h$
d. During the flight from P_1 to P_2 the ball maintains a horizontal speed of $\sqrt{2gh}$ and the vertical speed at P_2 can be found from $v_y = v_i + at$ where v_i = 0, a = g and t is the time found above. Once v_x and v_y are known the speed is $\sqrt{v_x^2 + v_y^2}$ giving $v = \sqrt{10gh}$

2005B1

a.

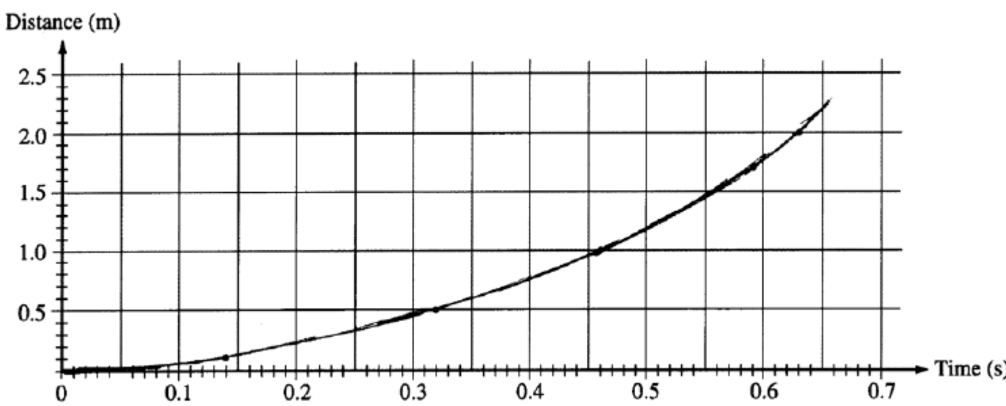
b. i. $a_{avg} = \Delta v/\Delta t = (0 – 1.5\ m/s)/(\ 2\ s) = – 0.75\ m/s^2$

ii.



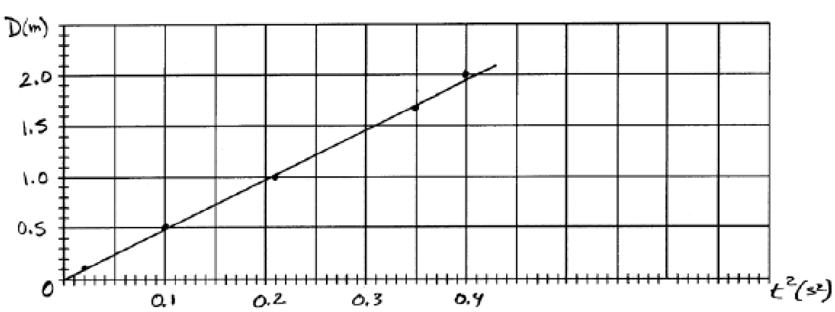
2006Bb1

a.



b. Distance and time are related by the equation $D = ½\ gt^2$. To yield a straight line, the quantities that should be graphed are D and t^2 or $\sqrt{D}$ and t.

c.

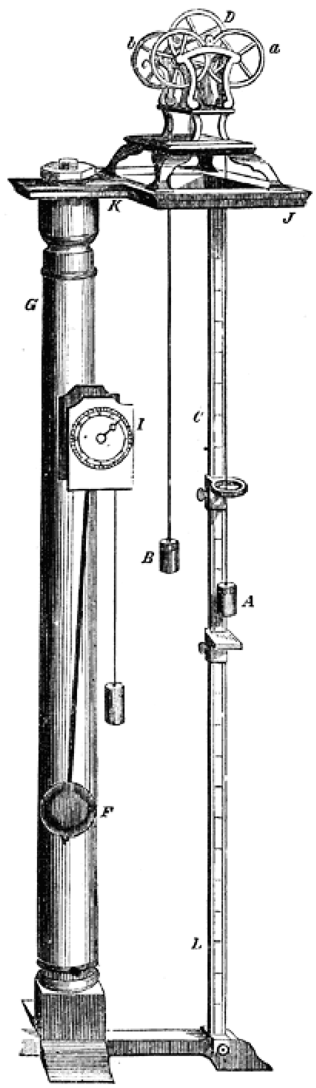


d. The slope of the graph of D vs. t^2 is ½ g. The slope of the line shown is $4.9\ m/s^2$ giving $g = 9.8\ m/s^2$

e. (example) Do several trials for each value of *D* and take averages. This reduces personal and random error.

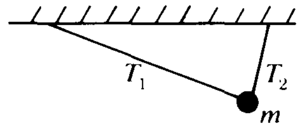
Chapter 2

Dynamics



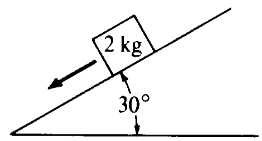
AP Physics Multiple Choice Practice – Dynamics

SECTION A – Linear Dynamics



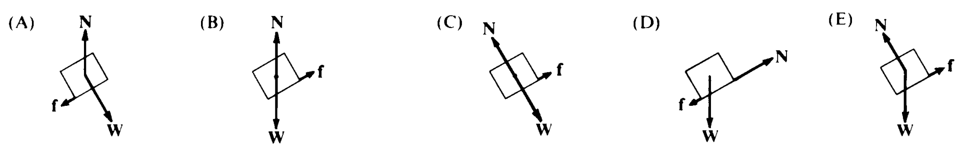
1. A ball of mass m is suspended from two strings of unequal length as shown above. The magnitudes of the tensions T_1 and T_2 in the strings must satisfy which of the following relations?
(A) $T_1 = T_2$ (B) $T_1 > T_2$ (C) $T_1 < T_2$ (D) $T_1 + T_2 = mg$ (E) $T_1 - T_2 = mg$

Questions 2 – 3

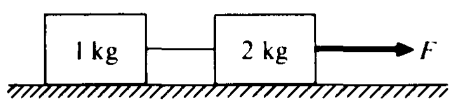


A 2-kilogram block slides down a 30° incline as shown above with an acceleration of 2 meters per second squared.

2. Which of the following diagrams best represents the gravitational force W. the frictional force f, and the normal force N that act on the block?

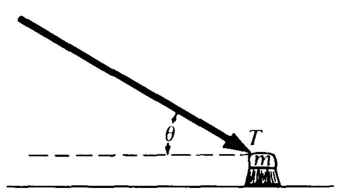


3. The magnitude of the frictional force along the plane is most nearly
(A) 2.5 N (B) 5 N (C) 6 N (D) 10 N (E) 16 N

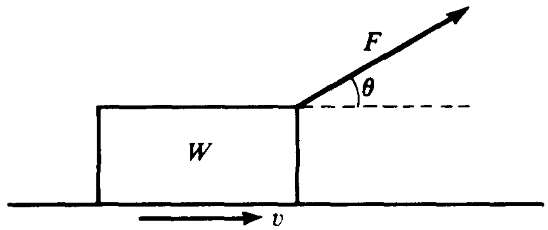


4. When the frictionless system shown above is accelerated by an applied force of magnitude the tension in the string between the blocks is (A) 2F (B) F (C) 2/3 F (D) ½ F (E) 1/3 F

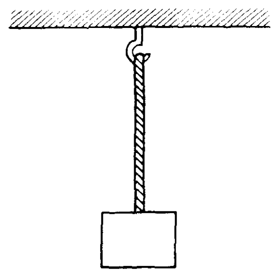
5. A ball falls straight down through the air under the influence of gravity. There is a retarding force F on the ball with magnitude given by $F = bv$, where v is the speed of the ball and b is a positive constant. The magnitude of the acceleration, a of the ball at any time is equal to which of the following?
(A) $g - b$ (B) $g - bv/m$ (C) $g + bv/m$ (D) g/b (E) bv/m



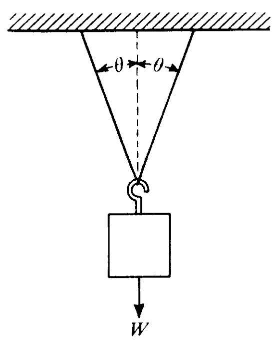
6. A push broom of mass m is pushed across a rough horizontal floor by a force of magnitude T directed at angle θ as shown above. The coefficient of friction between the broom and the floor is μ. The frictional force on the broom has magnitude
(A) $\mu(mg + T\sin\theta)$ (B) $\mu(mg - T\sin\theta)$ (C) $\mu(mg + T\cos\theta)$ (D) $\mu(mg - T\cos\theta)$ (E) μmg



7. A block of weight W is pulled along a horizontal surface at constant speed v by a force F, which acts at an angle of θ with the horizontal, as shown above. The normal force exerted on the block by the surface has magnitude
(A) W – F cos θ (B) W – F sin θ (C) W (D) W + F sin θ (E) W + F cos θ

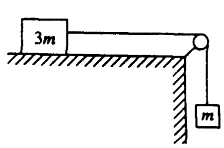


8. A uniform rope of weight 50 newtons hangs from a hook as shown above. A box of weight 100 newtons hangs from the rope. What is the tension in the rope?
(A) 50 N throughout the rope (B) 75 N throughout the rope (C) 100 N throughout the rope
(D) 150 N throughout the rope (E) It varies from 100 N at the bottom of the rope to 150 N at the top.



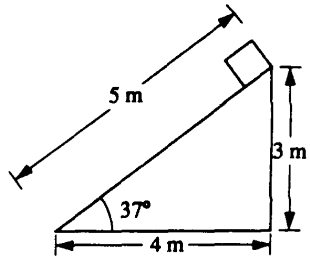
9. When an object of weight W is suspended from the center of a massless string as shown above, the tension at any point in the string is
(A) 2Wcosθ (B) ½Wcosθ (C) Wcosθ (D) W/(2cosθ) (E) W/(cosθ)

*10. An ideal spring obeys Hooke's law, F = –kx. A mass of 0.50 kilogram hung vertically from this spring stretches the spring 0.075 meter. The value of the force constant for the spring is most nearly
(A) 0.33 N/m (B) 0.66 N/m (C) 6.6 N/m (D) 33 N/m (E) 66 N/m



11. A block of mass 3m can move without friction on a horizontal table. This block is attached to another block of mass m by a cord that passes over a frictionless pulley, as shown above. If the masses of the cord and the pulley are negligible, what is the magnitude of the acceleration of the descending block?
(A) Zero (B) g/4 (C) g/3 (D) 2g/3 (E) g

Questions 12 – 13

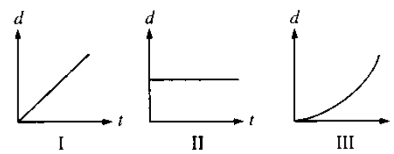


A plane 5 meters in length is inclined at an angle of 37°, as shown above. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.

12. The mass of the block is most nearly
(A) 1.0 kg (B) 1.2 kg (C) 1.6 kg (D) 2.0 kg (E) 2.5 kg

13. The magnitude of the normal force exerted on the block by the plane is most nearly
(A) 10 N (B) 12 N (C) 16 N (D) 20 N (E) 33 N

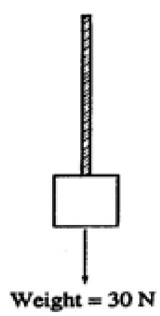
14. Three forces act on an object. If the object is in translational equilibrium, which of the following must be true?
I. The vector sum of the three forces must equal zero.
II. The magnitudes of the three forces must be equal.
III. All three forces must be parallel.
(A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, and III



15. Three objects can only move along a straight, level path. The graphs above show the position *d* of each of the objects plotted as a function of time *t*. The sum of the forces on the object is zero in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

*16.For which of the following motions of an object must the acceleration always be zero?

I. Any motion in a straight line
II. Simple harmonic motion
III. Any motion in a circle
(A) I only (B) II only (C) III only (D) Either I or III, but not II
(E) None of these motions guarantees zero acceleration.



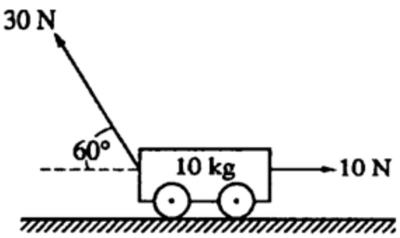
17. A rope of negligible mass supports a block that weighs 30 N, as shown above. The breaking strength of the rope is 50 N. The largest acceleration that can be given to the block by pulling up on it with the rope without breaking the rope is most nearly
(A) 6 m/s^2 (B) 6.7 m/s^2 (C) 10 m/s^2 (D) 15 m/s^2 (E) 16.7 m/s^2

Questions 18 – 19

A horizontal, uniform board of weight 125 N and length 4 m is supported by vertical chains at each end. A person weighing 500 N is sitting on the board. The tension in the right chain is 250 N.

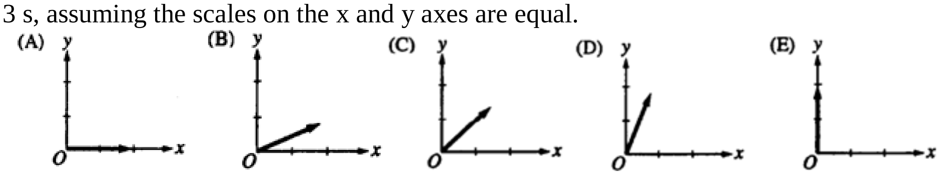
18. What is the tension in the left chain?
(A) 250 N (B) 375 N (C) 500 N (D) 625 N (E) 875 N

*19. How far from the left end of the board is the person sitting?
(A) 0.4 m (B) 1.5 m (C) 2 m (D) 2.5 m (E) 3 m

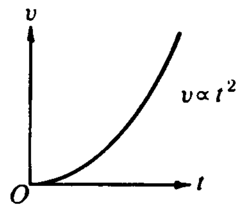


20. The cart of mass 10 kg shown above moves without frictional loss on a level table. A 10 N force pulls on the cart horizontally to the right. At the same time, a 30 N force at an angle of 60° above the horizontal pulls on the cart to the left. What is the magnitude of the horizontal acceleration of the cart?
(A) 0.5 m/s^2 (B) 1.6 m/s^2 (C) 2.0 m/s^2 (D) 2.5 m/s^2 (E) 2.6 m/s^2

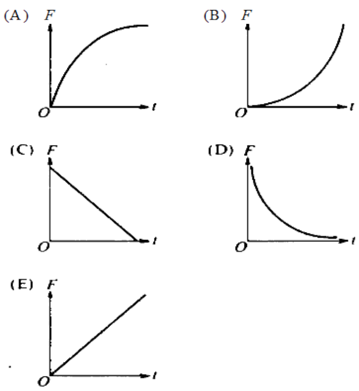
21. An object of mass *m* is initially at rest and free to move without friction in any direction in the *xy*-plane. A constant net force of magnitude *F* directed in the *+x* direction acts on the object for 1 s. Immediately thereafter a constant net force of the same magnitude *F* directed in the *+y* direction acts on the object for 1 s. After this, no forces act on the object. Which of the following vectors could represent the velocity of the object at the end of 3 s, assuming the scales on the x and y axes are equal.

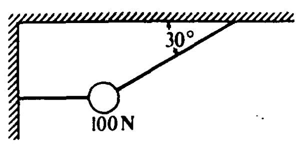


22. Two people are pulling on the ends of a rope. Each person pulls with a force of 100 N. The tension in the rope is:
(A) 0 N (B) 50 N (C) 100 N (D) 141 N (E) 200 N

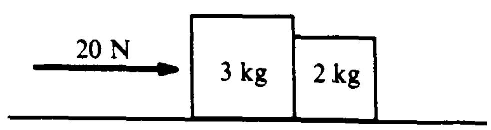


23. The parabola above is a graph of speed v as a function of time t for an object. Which of the following graphs best represents the magnitude F of the net force exerted on the object as a function of time t?

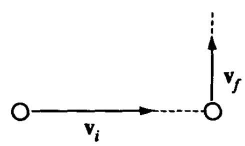




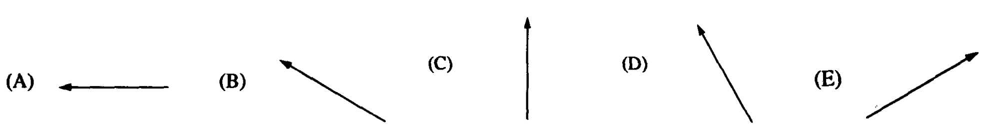
24. A 100-newton weight is suspended by two cords as shown above. The tension in the slanted cord is
(A) 50 N (B) 100 N (C) 150 N (D) 200 N (E) 250 N

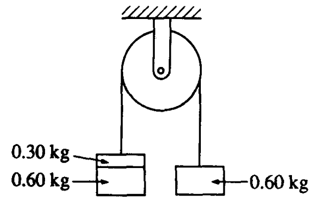


25. Two blocks are pushed along a horizontal frictionless surface by a force of 20 newtons to the right, as shown above. The force that the 2-kilogram block exerts on the 3-kilogram block is
(A) 8 newtons to the left (B) 8 newtons to the right (C) 10 newtons to the left
(D) 12 newtons to the right (E) 20 newtons to the left



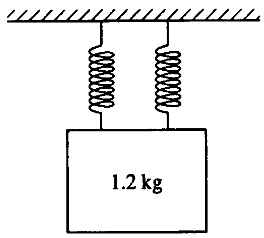
26. A ball initially moves horizontally with velocity v_i, as shown above. It is then struck by a stick. After leaving the stick, the ball moves vertically with a velocity v_f, which is smaller in magnitude than v_i. Which of the following vectors best represents the direction of the average force that the stick exerts on the ball?





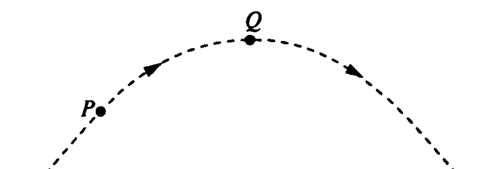
27. Two 0.60-kilogram objects are connected by a thread that passes over a light, frictionless pulley, as shown above. The objects are initially held at rest. If a third object with a mass of 0.30 kilogram is added on top of one of the 0.60-kilogram objects as shown and the objects are released, the magnitude of the acceleration of the 0.30-kilogram object is most nearly

(A) 10.0 m/s^2 (B) 6.0 m/s^2 (C) 3.0 m/s^2 (D) 2.0 m/s^2 (E) 1.0 m/s^2

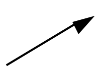
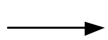
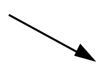
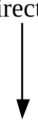
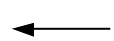


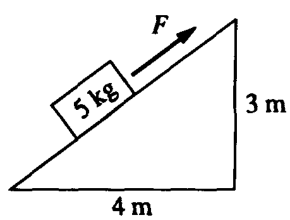
*28. Two identical massless springs are hung from a horizontal support. A block of mass 1.2 kilograms is suspended from the pair of springs, as shown above. When the block is in equilibrium, each spring is stretched an additional 0.15 meter. The force constant of each spring is most nearly

(A) 40 N/m (B) 48 N/m (C) 60 N/m (D) 80 N/m (E) 96 N/m



29. A ball is thrown and follows a parabolic path, as shown above. Air friction is negligible. Point Q is the highest point on the path. Which of the following best indicates the direction of the net force on the ball at point P ?

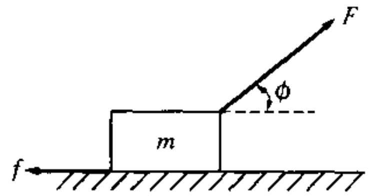
(A)  (B)  (C)  (D)  (E) 



30. A block of mass 5 kilograms lies on an inclined plane, as shown above. The horizontal and vertical supports for the plane have lengths of 4 meters and 3 meters, respectively. The coefficient of friction between the plane and the block is 0.3. The magnitude of the force F necessary to pull the block up the plane with constant speed is most nearly

(A) 30 N (B) 42 N (C) 49 N (D) 50 N (E) 58 N

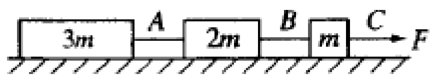
Questions 31 – 32



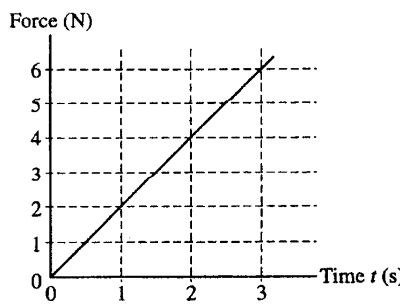
A block of mass m is accelerated across a rough surface by a force of magnitude F that is exerted at an angle ϕ with the horizontal, as shown above. The frictional force on the block exerted by the surface has magnitude *f*.

31. What is the acceleration of the block?
(A) F/*m* (B) (Fcosϕ)/m (C) (F–*f*)/*m* (D) (Fcosϕ–*f*)/*m* (E) (Fsinϕ–*mg*)/*m*

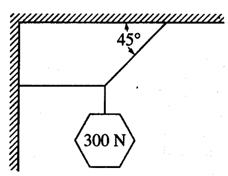
32. What is the coefficient of friction between the block and the surface?
(A) *f*/*mg* (B) *mg*/*f* (C) (*mg*–Fcosϕ)/*f* (D) *f*/(*mg*–Fcosϕ) (E) *f*/(*mg*–Fsinϕ)



33. Three blocks of masses 3*m*, 2*m*, and*s* are connected to strings *A*, *B*, and *C* as shown above. The blocks are pulled along a rough surface by a force of magnitude *F* exerted by string *C*. The coefficient of friction between each block and the surface is the same. Which string must be the strongest in order not to break?
(A) *A* (B) *B* (C) *C* (D) They must all be the same strength.
(E) It is impossible to determine without knowing the coefficient of friction.



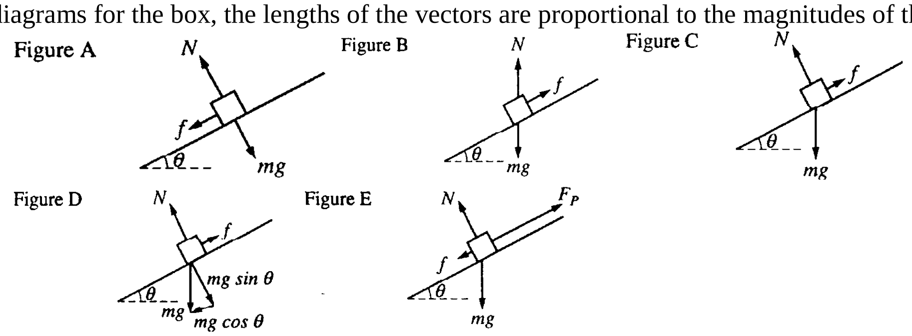
34. A block of mass 3 kg, initially at rest, is pulled along a frictionless, horizontal surface with a force shown as a function of time t by the graph above. The acceleration of the block at t = 2 s is
(A) 3/4 m/s^2 (B) 4/3 m/s^2 (C) 2 m/s^2 (D) 8 m/s^2 (E) 12 m/s^2



35. An object weighing 300 N is suspended by means of two cords, as shown above. The tension in the horizontal cord is
(A) 0 N (B) 150 N (C) 210 N (D) 300 N (E) 400 N

Questions 36 – 38

A small box is on a ramp tilted at an angle θ above the horizontal. The box may be subject to the following forces: frictional (f) ,gravitational (mg), pulling or pushing (F_P) and normal *(I)*. In the following free-body diagrams for the box, the lengths of the vectors are proportional to the magnitudes of the forces.

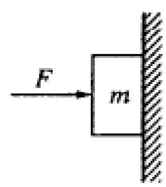


36. Which figure best represents the free-body diagram for the box if it is accelerating up the ramp?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

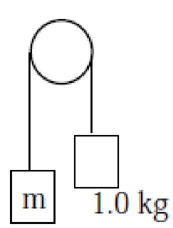
37. Which figure best represents the free-body diagram for the box if it is at rest on the ramp?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

38. Which figure best represents the free-body diagram for the box if it is sliding down the ramp at constant speed?
(A) Figure A (B) Figure B (C) Figure C (D) Figure D (E) Figure E

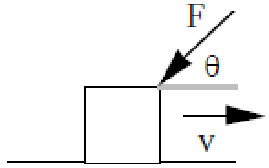
39. Two blocks of masses M and m, with $M > m$, are connected by a light string. The string passes over a frictionless pulley of negligible mass so that the blocks hang vertically. The blocks are then released from rest. What is the acceleration of the block of mass M ?
(A) g (B) $\frac{M-m}{M}g$ (C) $\frac{M+m}{M}g$ (D) $\frac{M+m}{M-m}g$ (E) $\frac{M-m}{M+m}g$



40. A horizontal force F pushes a block of mass m against a vertical wall. The coefficient of friction between the block and the wall is μ. What value of F is necessary to keep the block from slipping down the wall?
(A) mg (B) μ mg (C) mg/μ (D) mg(1 - μ) (E) mg(1 + μ)

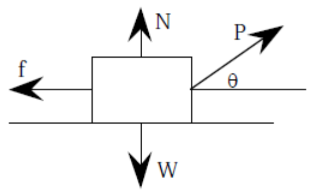


41. One end of a massless rope is attached to a mass m; the other end is attached to a mass of 1.00 kg. The rope is hung over a massless frictionless pulley as shown in the accompanying figure. Mass m accelerates downward at 5.0 m/s^2. What is m?
(A) 3.0 kg (B) 2.0 kg (C) 1.5 kg D. 1.0 kg (E) 0.5 kg



42. As shown in the accompanying figure, a force *F* is exerted at an angle of θ. The block of weight *mg* is initially moving the right with speed *v*. The coefficient of friction between the rough floor and the block is μ. The frictional force acting on the block is:
(A) μ*mg* to the left. (B) μ*mg* to the right. (C) μ*mg* – *F* sin θ to the left. (D) μ(*mg* – *F* cos θ) to the right.
(E) μ(*mg* + *F* sin θ) to the left.

43. The "reaction" force does not cancel the "action" force because:
(A) The action force is greater than the reaction force. (B) The action force is less than the reaction force.
(C) They act on different bodies. (D) They are in the same direction.
(E) The reaction exists only after the action force is removed.

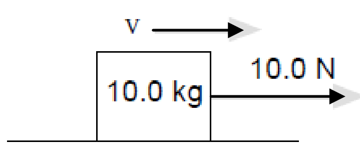


44. A student pulls a wooden box along a rough horizontal floor at constant speed by means of a force *P* as shown to the right. Which of the following must be true?
(A) P > f and N < W.
(B) P > f and N = W.
(C) P = f and N > W.
(D) P = f and N = W.
(E) P < f and N = W.

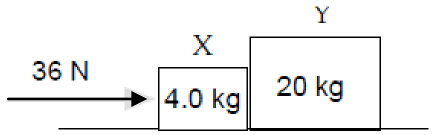
45. A block with initial velocity 4.0 m/s slides 8.0 m across a rough horizontal floor before coming to rest. The coefficient of friction is:
(A) 0.80 (B) 0.40 (C) 0.20 (D) 0.10 (E) 0.05

46. A car whose mass is 1500 kg is accelerated uniformly from rest to a speed of 20 m/s in 10 s. The magnitude of the net force accelerating the car is:
(A) 1000 N (B) 2000 N (C) 3000 N (D) 20000 N (E) 30000 N

47. An 800-kg elevator accelerates downward at 2.0 m/s^2. The force exerted by the cable on the elevator is:
(A) 1.6 kN down (B) 1.6 kN up (C) 6.4 kN up (D) 8.0 kN down (E) 9.6 kN down

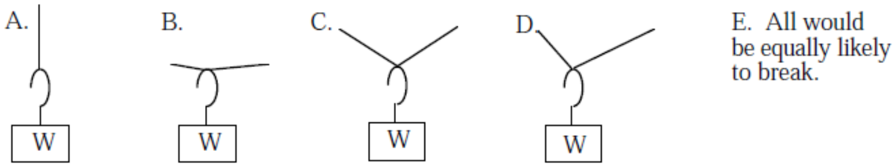


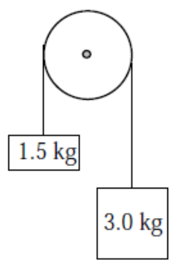
48. The 10.0 kg box shown in the figure to the right is sliding to the right along the floor. A horizontal force of 10.0 N is being applied to the right. The coefficient of kinetic friction between the box and the floor is 0.20. The box is moving with:
(A) acceleration to the left. (B) centripetal acceleration. (C) acceleration to the right.
(D) constant speed and constant velocity. (E) constant speed but not constant velocity.



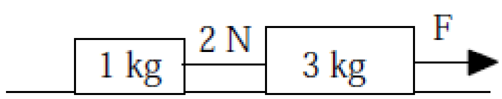
49. Two blocks *X* and *Y* are in contact on a horizontal frictionless surface. A 36 N constant force is applied to *X* as shown to the right. The force exerted by *X* on *Y* is:
(A) 1.5 N (B) 6.0 N (C) 29 N (D) 30 N (E) 36 N

50. Assume the objects in the following diagrams have equal mass and the strings holding them in place are identical. In which case would the string be most likely to break?





51. A string with masses of 1.5 kg and 3.0 kg on its ends is hung over a frictionless, massless pulley as shown to the right. What is the approximate magnitude of the acceleration of the masses?
(A) 1.5 m/s^2 (B) 3.0 m/s^2 (C) 3.3 m/s^2 (D) 6.7 m/s^2 (E) 10 m/s^2

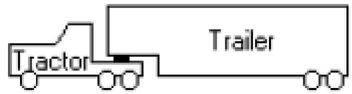


52. Two blocks of mass 1.0 kg and 3.0 kg are connected by a string which has a tension of 2.0 N. A force *F* acts in the direction shown to the right. Assuming friction is negligible, what is the value of *F*?
(A) 1.0 N (B) 2.0 N (C) 4.0 N (D) 6.0 N (E) 8.0 N

53. An object in equilibrium has three forces, F_1 of 30 N, F_2 of 50 N, and F_3 of 70 N, acting on it. The magnitude of the resultant of F_1 and F_2 is
(A) 10 N (B) 20 N (C) 40 N (D) 70 N (E) 80 N

54. A 50-kg student stands on a scale in an elevator. At the instant the elevator has a downward acceleration of 1.0 m/s^2 and an upward velocity of 3.0 m/s, the scale reads approximately
(A) 350 N (B) 450 N (C) 500 N (D) 550 N (E) 650 N

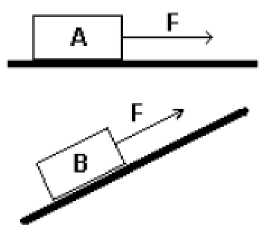
55. If the net force on an object were doubled while at the same time the mass of the object was halved, then the acceleration of the object is
(A) 1/4 as great. (B) 1/2 as great. (C) 2 times greater. (D) 4 times greater. (E) unchanged



56. A tractor-trailer truck is traveling down the road. The mass of the trailer is 4 times the mass of the tractor. If the tractor accelerates forward, the force that the trailer applies on the tractor is
(A) 4 times greater than the force of the tractor on the trailer.
(B) 2 times greater than the force of the tractor on the trailer.
(C) equal to the force of the tractor on the trailer.
(D) ¼ the force of the tractor on the trailer.
(E) zero since the tractor is pulling the trailer forward.

57. Two boxes are accelerated to the right on a frictionless horizontal surface as shown. The larger box has a mass of 9 kilograms and the smaller box has a mass of 3 kilograms. If a 24 newton horizontal force pulls on the larger box, with what force does the larger box pull on the smaller box?
(A) 3 N (B) 6 N (C) 8 N (D) 18 N (E) 24 N

58. What happens to the inertia of an object when its velocity is doubled?
(A) the object's inertia becomes 2 times greater
(B) the object's inertia becomes 2 times greater
(C) the object's inertia becomes 4 times greater
(D) the object's inertia becomes 8 times greater
(E) the object's inertia is unchanged



59. A wooden box is first pulled across a horizontal steel plate as shown in the diagram A. The box is then pulled across the same steel plate while the plate is inclined as shown in diagram B. How does the force required to overcome friction in the inclined case (B) compare to the horizontal case (A)?
(A) the frictional force is the same in both cases
(B) the inclined case has a greater frictional force
(C) the inclined case has less frictional force
(D) the frictional force increases with angle until the angle is 90º, then drops to zero
(E) more information is required

60. An object near the surface of the earth with a weight of 100 newtons is accelerated at 4 m/s^2. What is the net force on the object?
(A) 25 N (B) 40 N (C) 250 N (D) 400 N (E) 2500 N

Questions 61 – 62

A car of mass *m* slides across a patch of ice at a speed *v* with its brakes locked. It then hits dry pavement and skids to a stop in a distance *d*. The coefficient of kinetic friction between the tires and the dry road is μ.

61. If the car had a mass of 2*m*, it would have skidded a distance of
(A) 0.5 *d* (B) *d* (C) 1.41 *d* (D) 2 *d* (E) 4 *d*

62. If the car had a speed of 2*v*, it would have skidded a distance of
(A) 0.5 *d* (B) *d* (C) 1.41 *d* (D) 2 *d* (E) 4 *d*

63. A 500-gram ball moving at 15 m/s slows down uniformly until it stops. If the ball travels 15 meters, what was the average net force applied while it was coming to a stop?
(A) 0.37 newtons (B) 3.75 newtons (C) 37.5 newtons (D) 375 newtons (E) 3750 newtons

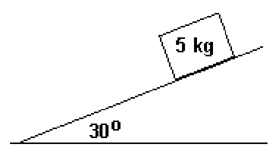
64. A block rests on a flat plane inclined at an angle of 30º with respect to the horizontal. What is the minimum coefficient of friction necessary to keep the block from sliding?
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{4}$ (E) $\frac{2}{\sqrt{3}}$

65. A force of 6 newtons and a force of 10 newtons can be combine to form a resultant with a magnitude of which of the following
(A) 0 newtons (B) 2 newtons (C) 8 newtons (D) 20 newtons (E) 60 newtons

66. The order of magnitude of the weight of an apple is:
(A) 10^{-4} N (B) 10^{-2} N (C) 10^{-1} N (D) 10^{0} N (E) 10^{1} N

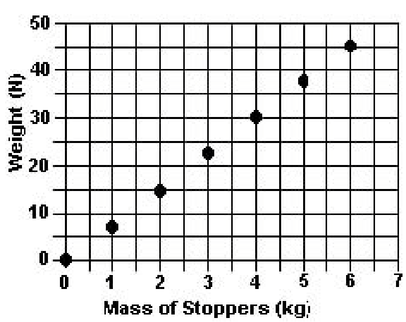
Questions 67 – 68

A 5 kg block rests on a flat plane inclined at an angle of 30° to the horizon as shown in the diagram below.

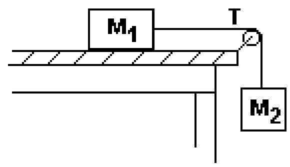


67. What would be the acceleration of the block down the plane assuming the force of friction is negligible?
(A) 0.5 m/s^2 (B) 0.87 m/s^2 (C) 5 m/s^2 (D) 8.7 m/s^2 (E) 10 m/s^2

68. If the block is placed on a second plane (where friction is significant) inclined at the same angle, it will begin to accelerate at 2.0 m/s^2. What is the force of friction between the block and the second inclined plane?
(A) 10 N (B) 15 N (C) 25 N (D) 43.3 N (E) 50 N



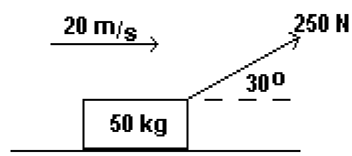
69. The graph at left shows the relationship between the mass of a number of rubber stoppers and their resulting weight on some far-off planet. The slope of the graph is a representation of the:
(A) mass of a stopper
(B) density of a stopper
(C) volume of a stopper
(D) acceleration due to gravity
(E) number of stoppers for each unit of weight



70. Two masses, m_1 and m_2, are connected by a cord and arranged as shown in the diagram with m_1 sliding along on a frictionless surface and m_2 hanging from a light frictionless pulley. What would be the mass of the falling mass, m_2, if both the sliding mass, m_1, and the tension, T, in the cord were known?

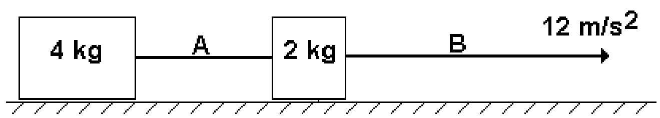
(A) $\frac{1}{(g-1)}$ (B) $\frac{m_1 g - T}{g}$ (C) $\frac{1}{2}Tg$ (D) $\frac{m_1(T-g)}{(gm_1 - T)}$ (E) $\frac{Tm_1}{(gm_1 - T)}$

71. A box with a mass of 50 kg is dragged across the floor by a rope which makes an angle of 30º with the horizontal. Which of the following would be closest to the coefficient of kinetic friction between the box and the floor if a 250 newton force on the rope is required to move the crate at a constant speed of 20 m/s as shown in the diagram?
(A) 0.26 (B) 0.33 (C) 0.44 (D) 0.59 (E) 0.77



Questions 72 – 74

A 2 kg mass and a 4 kg mass on a horizontal frictionless surface are connected by a massless string A. They are pulled horizontally across the surface by a second string B with a constant acceleration of 12 m/s^2.



72. What is the magnitude of the force of string B on the 2 kg mass?
(A) 72 N (B) 48 N (C) 24 N (D) 6 N (E) 3 N

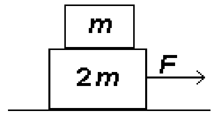
73. What is the magnitude of the force of string A on the 4 kg mass?
(A) 72 N (B) 48 N (C) 24 N (D) 6 N (E) 3 N

74. What is the magnitude of the net force on the 2 kg mass?
(A) 72 N (B) 48 N (C) 24 N (D) 6 N (E) 3 N

75. A mass is suspended from the roof of a lift (elevator) by means of a spring balance. The lift (elevator) is moving upwards and the readings of the spring balance are noted as follows:

Speeding up: R_U Constant speed: R_C Slowing down: R_D

Which of the following is a correct relationship between the readings?
(A) $R_U > R_C$ (B) $R_U = R_D$ (C) $R_C = R_D$ (D) $R_C < R_D$ (E) $R_U < R_D$



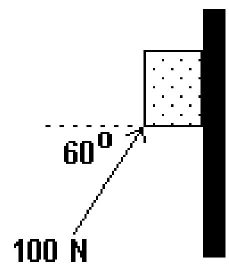
76. A small box of mass *m* is placed on top of a larger box of mass *2m* as shown in the diagram at right. When a force *F* is applied to the large box, both boxes accelerate to the right with the same acceleration. If the coefficient of friction between all surfaces is μ, what would be the force accelerating the smaller mass?

(A) $\frac{F}{3} - mg\mu$ (B) $F - 3mg\mu$ (C) $F - mg\mu$ (D) $\frac{F - mg\mu}{3}$ (E) $\frac{F}{3}$

77. The S.I. unit of force is named the newton in honor of Sir Isaac Newton's contributions to physics. Which of the following combination of units is the equivalent of a newton?

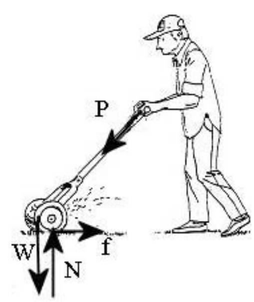
(A) kg (B) $\text{kg}\frac{\text{m}}{\text{s}}$ (C) $\text{kg}\frac{\text{m}^2}{\text{s}}$ (D) $\text{kg}\frac{\text{m}}{\text{s}^2}$ (E) $\text{kg}\frac{\text{m}^2}{\text{s}^2}$

78. A 6.0 kg block initially at rest is pushed against a wall by a 100 N force as shown. The coefficient of kinetic friction is 0.30 while the coefficient of static friction is 0.50. What is true of the friction acting on the block after a time of 1 second?
(A) Static friction acts upward on the block.
(B) Kinetic friction acts upward on the block
(C) No friction acts on the block
(D) Kinetic friction acts downward on the block.
(E) Static friction acts downward on the block.



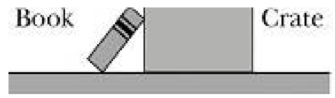
79. A homeowner pushes a lawn mower across a horizontal patch of grass with a constant speed by applying a force P. The arrows in the diagram correctly indicate the directions but not necessarily the magnitudes of the various forces on the lawn mower. Which of the following relations among the various force magnitudes, *W, f, N, P* is CORRECT?

(A) $P > f$ and $N > W$
(B) $P < f$ and $N = W$
(C) $P > f$ and $N < W$
(D) $P = f$ and $N > W$
(E) none of the above



*80. A mass, M, is at rest on a frictionless surface, connected to an ideal horizontal spring that is unstretched. A person extends the spring 30 cm from equilibrium and holds it at this location by applying a 10 N force. The spring is brought back to equilibrium and the mass connected to it is now doubled to 2M. If the spring is extended back 30 cm from equilibrium, what is the necessary force applied by the person to hold the mass stationary there?
(A) 20.0 N (B) 14.1 N (C) 10.0 N (D) 7.07 N (E) 5.00 N

81. A baseball is thrown by a pitcher with a speed of 35 m/s. The batter swings and hits the ball. The magnitude of the force that the ball exerts on the bat is always
(A) zero as it is only the bat that exerts a force on the ball.
(B) equal to the gravitational force acting on the ball.
(C) larger than the force the bat exerts on the ball.
(D) smaller than the force the bat exerts on the ball.
(E) equal to the force that the bat exerts on the ball.



82. A book leans against a crate on a table. Neither is moving. Which of the following statements concerning this situation is CORRECT?
(A) The force of the book on the crate is less than that of crate on the book.
(B) Although there is no friction acting on the crate, there must be friction acting on the book or else it will fall.
(C) The net force acting on the book is zero.
(D) The direction of the frictional force acting on the book is in the same direction as the frictional force acting on the crate.
(E) The Newton's Third Law reaction force to the weight of the book is the normal force from the table.

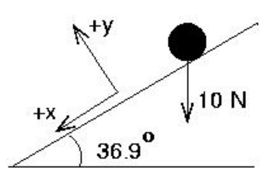
83. A crate of toys remains at rest on a sleigh as the sleigh is pulled up a hill with an increasing speed. The crate is not fastened down to the sleigh. What force is responsible for the crate's increase in speed up the hill?
(A) the contact force (normal force) of the ground on the sleigh
(B) the force of static friction of the sleigh on the crate
(C) the contact force (normal force) of the sleigh on the crate
(D) the gravitational force acting on the sleigh
(E) no force is needed

84. A student weighing 500 N stands on a bathroom scale in the school's elevator. When the scale reads 520 N, the elevator must be
(A) accelerating upward. (B) accelerating downward. (C) moving upward at a constant speed.
(D) moving downward at a constant speed. (E) at rest.

85. In which one of the following situations is the net force constantly zero on the object?
(A) A mass attached to a string and swinging like a pendulum.
(B) A stone falling freely in a gravitational field.
(C) An astronaut floating in the International Space Station.
(D) A snowboarder riding down a steep hill.
(E) A skydiver who has reached terminal velocity.

86. A box slides to the right across a horizontal floor. A person called Ted exerts a force T to the right on the box. A person called Mario exerts a force M to the left, which is half as large as the force T. Given that there is friction f and the box accelerates to the right, rank the sizes of these three forces exerted on the box.
(A) $f < M < T$ (B) $M < f < T$ (C) $M < T < f$ (D) $f = M < T$ (E) It cannot be determined.

87. You hold a rubber ball in your hand. The Newton's third law companion force to the force of gravity on the ball is the force exerted by which object onto what other object?
(A) ball on the hand (B) Earth on the ball (C) ball on the Earth (D) Earth on your hand (E) hand on the ball

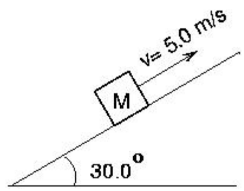


88. An object on an inclined plane has a gravitational force of magnitude 10 N acting on it from the Earth. Which of the following gives the correct components of this gravitational force for the coordinate axes shown in the figure? The y-axis is perpendicular to the incline's surface while the x-axis is parallel to the inclined surface.

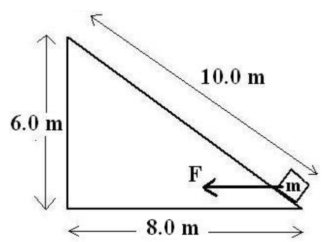
	x-component	y-component
(A)	$+6\ N$	$-8N$
(B)	$+8N$	$-6N$
(C)	$-6N$	$+8N$
(D)	$-8N$	$+6\ N$
(E)	$0\ N$	$+10\ N$

89. A spaceman of mass 80 kg is sitting in a spacecraft near the surface of the Earth. The spacecraft is accelerating upward at five times the acceleration due to gravity. What is the force of the spaceman on the spacecraft?
(A) 4800 N (B) 4000 N (C) 3200 N (D) 800 N (E) 400 N

90. A 22.0 kg suitcase is dragged in a straight line at a constant speed of 1.10 m/s across a level airport floor by a student on the way to Mexico. The individual pulls with a 1.0×10^2 N force along a handle which makes an upward angle of 30.0 degrees with respect to the horizontal. What is the coefficient of kinetic friction between the suitcase and the floor?
(A) $\mu_k = 0.013$ (B) $\mu_k = 0.394$ (C) $\mu_k = 0.509$ (D) $\mu_k = 0.866$ (E) $\mu_k = 1.055$

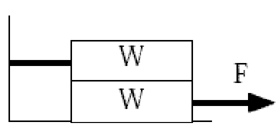


91. A person pushes a block of mass $M = 6.0 kg$ with a constant speed of 5.0 m/s straight up a flat surface inclined 30.0° above the horizontal. The coefficient of kinetic friction between the block and the surface is $\mu = 0.40$. What is the net force acting on the block?
(A) 0 N (B) 21 N (C) 30 N (D) 51 N (E) 76 N

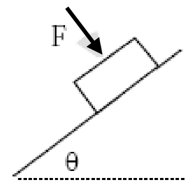


92. In the figure above, a box moves with speed 5.0 m/s at the bottom of a rough, fixed inclined plane. The box slides with constant acceleration to the top of the incline as it is being pushed directly to the left with a constant force of F = 240 N. The box, of mass m = 20.0 kg, has a speed of 2.50 m/s when it reaches the top of the incline. What is the magnitude of the acceleration of the box as it slides up the incline?
(A) 12.0 m/s^2 (B) 10.0 m/s^2 (C) 5.88 m/s^2 (D) 1.88 m/s^2 (E) 0.938 m/s^2

93. A 20.0 kg box remains at rest on a horizontal surface while a person pushes directly to the right on the box with a force of 60 N. The coefficient of kinetic friction between the box and the surface is $\mu_k = 0.20$. The coefficient of static friction between the box and the surface is $\mu_s = 0.60$. What is the magnitude of the force of friction acting on the box during the push?
(A) 200 N (B) 120 N (C) 60 N (D) 40 N (E) 0 N

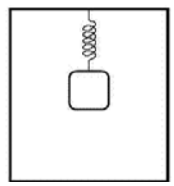


*94.Two identical blocks of weight W are placed one on top of the other as shown in the diagram above. The upper block is tied to the wall. The lower block is pulled to the right with a force F. The coefficient of static friction between all surfaces in contact is μ. What is the largest force F that can be exerted before the lower block starts to slip?
(A) μW (B) $3\mu W/2$ (C) $2\mu W$ (D) $5\mu W/2$ (E) $3\mu W$



95. A force F is used to hold a block of mass m on an incline as shown in the diagram (see above). The plane makes an angle of θ with the horizontal and F is perpendicular to the plane. The coefficient of friction between the plane and the block is μ. What is the minimum force, F, necessary to keep the block at rest?
(A) μmg (B) $mg\cos\theta$ (C) $mg\sin\theta$ (D) $mg\sin\theta/\mu$ (E) $mg(\sin\theta - \mu\cos\theta)/\mu$

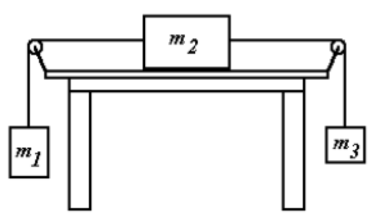
*96. A mass m is resting at equilibrium suspended from a vertical spring of natural length L and spring constant k inside a box as shown.



The box begins accelerating upward with acceleration a. How much closer does the equilibrium position of the mass move to the bottom of the box?
(A) $\frac{a}{g}L$ (B) $\frac{g}{a}L$ (C) $\frac{m(g+a)}{k}$ (D) $\frac{m(g-a)}{k}$ (E) $\frac{ma}{k}$

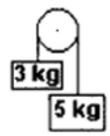
*97. When the speed of a rear-drive car is increasing on a horizontal road, what is the direction of the frictional force on the tires?
(A) backward on the front tires and forward on the rear tires.
(B) forward on the front tires and backward on the rear tires.
(C) forward on all tires.
(D) backward on all tires.
(E) zero.

*98. A ball of mass m is launched into the air. Ignore air resistance, but assume that there is a wind that exerts a constant force F_o in the $-x$ direction. In terms of F_o and the acceleration due to gravity g, at what angle above the positive x-axis must the ball be launched in order to come back to the point from which it was launched?
(A) $\tan^{-1}(F_0/mg)$ (B) $\tan^{-1}(mg/F_0)$ (C) $\sin^{-1}(F_0/mg)$ (D) the angle depends on the launch speed
(E) no such angle is possible



99. Given the three masses as shown in the diagram above, if the coefficient of kinetic friction between the large mass (m_2) and the table is μ, what would be the upward acceleration of the small mass (m_3)? The mass and friction of the cords and pulleys are small enough to produce a negligible effect on the system.
(A) $m_1g/(m_1 + m_2 + m_3)$ (B) $g(m_1 + m_2\mu)/(m_1 + m_2 + m_3)$ (C) $g\mu(m_1 + m_2 + m_3)/(m_1 - m_2 - m_3)$
(D) $g\mu(m_1 - m_2 - m_3)/(m_1 + m_2 + m_3)$ (E) $g(m_1 - \mu m_2 - m_3)/(m_1 + m_2 + m_3)$

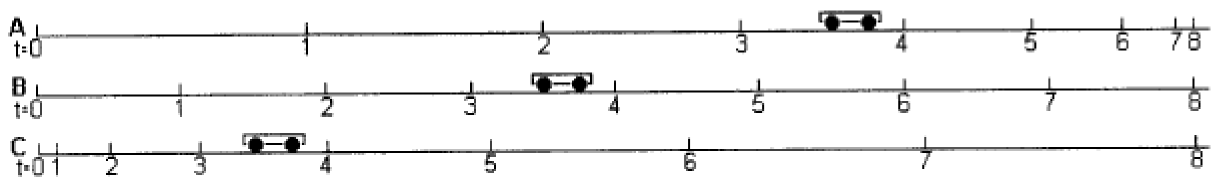
100. Two masses 5.0 and 7.0 kg are originally at rest on a frictionless surface. The masses are connected by a light cord. A second cord is attached to the 7.0 kg mass and pulled with a horizontal force of 30 N. What is the tension in the cord that connects the two masses?
(A) 5 N (B) 7 N (C) 12.5 N (D) 17.5 N (E) 30 N



101. Two masses are connected by a light cord which is looped over a light frictionless pulley. If one mass is 3.0 kg and the second mass is 5.0 kg, what is the downward acceleration of the heavier mass? Assume air resistance is negligible.
(A) 9.8 m/s^2 (B) 8.4 m/s^2 (C) 6.3 m/s^2 (D) 3.8 m/s^2 (E) 2.5 m/s^2

<u>Questions 102 – 103</u>

Three identical laboratory carts A, B, and C are each subject to a constant force F_A, F_B, and F_C, respectively. One or more of these forces may be zero. The diagram below shows the position of each cart at each second of an 8.0 second interval.

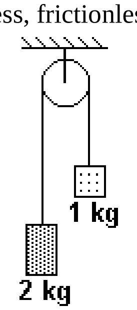


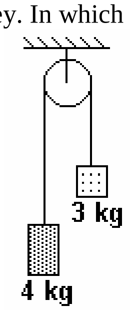
102. Which car has the greatest average velocity during the interval?
(A) A (B) B (C) C (D) all three average velocities are equal (E) not enough information is provided

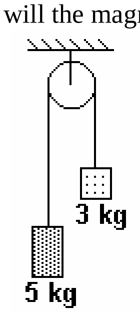
103. How does the magnitude of the force acting on each car compare?
(A) $F_A > F_B > F_C$ (B) $F_A = F_C > F_B$ (C) $F_A > F_C = F_B$ (D) $F_A = F_B > F_C$ (E) not enough information provided

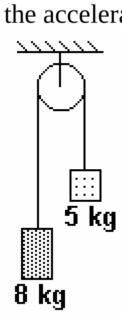
104. A skydiver is falling at terminal velocity before opening her parachute. After opening her parachute, she falls at a much smaller terminal velocity. How does the total upward force before she opens her parachute compare to the total upward force after she opens her parachute?
(A) The ratio of the forces is equal to the ratio of the velocities.
(B) The ratio of the forces is equal to the inverse ratio of the velocities.
(C) the upward force with the parachute will depend on the size of the parachute.
(D) The upward force before the parachute will be greater because of the greater velocity.
(E) The upward force in both cases must be the same.

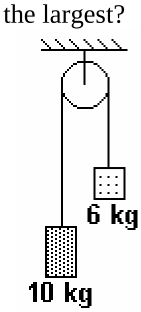
105. Each of the diagrams below represents two weights connected by a massless string which passes over a massless, frictionless pulley. In which diagram will the magnitude of the acceleration be the largest?

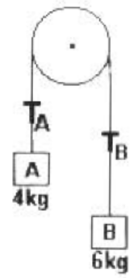
(A) 

(B) 

(C) 

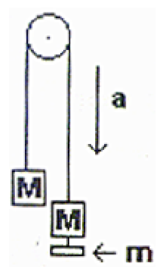
(D) 

(E) 

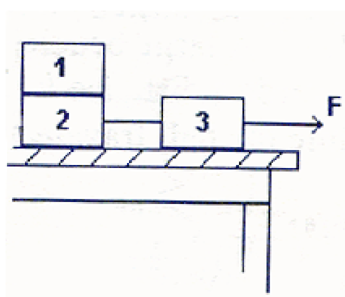


106.A simple Atwood's machine is shown in the diagram above. It is composed of a frictionless lightweight pulley with two cubes connected by a light string. If cube A has a mass of 4.0 kg and cube B has a mass of 6.0 kg, the system will move such that cube B accelerates downwards. What would be the tension in the two parts of the string between the pulley and the cubes?
(A) T_A = 47 N ; T_B = 71 N (B) T_A = 47 N ; T_B = 47 N (C) T_A = 47 N ; T_B = 42 N
(D) T_A = 39 N ; T_B = 59 N (E) T_A = 39 N ; T_B = 39 N

107.If a net force *F* applied to an object of mass *m* will produce an acceleration of *a*, what is the mass of a second object which accelerates at 5*a* when acted upon by a net force of 2*F*?
(A) (2/5)*m* (B) 2*m* (C) (5/2)*m* (D) 5*m* (E) 10*m*

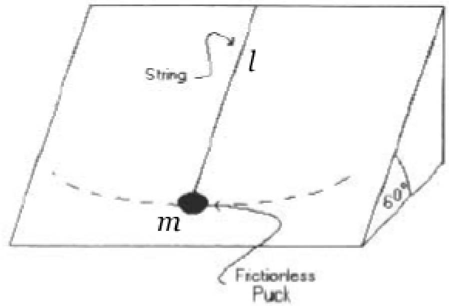


108.A simple Atwood's machine remains motionless when equal masses *M* are placed on each end of the chord. When a small mass *m* is added to one side, the masses have an acceleration *a*. What is *M*? You may neglect friction and the mass of the cord and pulley.
(A) $\frac{m(g-a)}{2a}$ (B) $\frac{2m(g-a)}{a}$ (C) $\frac{2m(g+a)}{a}$ (D) $\frac{m(g+a)}{2a}$ (E) $\frac{m(a-g)}{2a}$

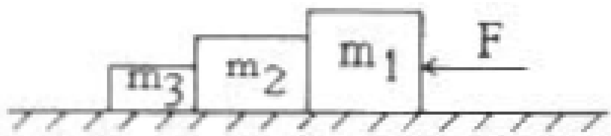


*109.Block 1 is stacked on top of block 2. Block 2 is connected by a light cord to block 3, which is pulled along a frictionless surface with a force *F* as shown in the diagram. Block 1 is accelerated at the same rate as block 2 because of the frictional forces between the two blocks. If all three blocks have the same mass *m*, what is the minimum coefficient of static friction between block 1 and block 2?
(A) 2F/mg (B) 2F/3mg (C) F/mg (D) 3F/2mg (E) F/3mg

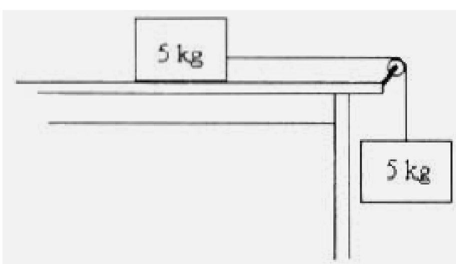
110.An object originally traveling at a velocity, v_0, is accelerated to a velocity, *v*, in a time, *t*, by a constant force, *F*. What would be the mass of the object?
(A) $\frac{v-v_0}{Ft}$ (B) $\frac{Ft}{v-v_0}$ (C) $\frac{F(v-v_0)}{t}$ (D) $\frac{F}{vt}$ (E) $\frac{F}{v_0 t}$



111. A frictionless air puck of mass m is placed on a plane surface inclined at an angle of 60° with respect to the horizontal. A string of length l is attached to the puck at one end and the upper edge of the inclined plane at the other to constrain the movement of the puck. What would be the magnitude of the normal force from the plane acting on the puck?
(A) $mg(\sin 60°)$ (B) $mg(\cos 30°)$ (C) $mg(\tan 30°)$ (D) $\frac{mg}{\tan 60}$ (E) None of these



*112. Three blocks (m_1, m_2, and m_3) are sliding at a constant velocity across a rough surface as shown in the diagram above. The coefficient of kinetic friction between each block and the surface is μ. What would be the force of m_1 on m_2?
(A) $(m_2 + m_3)g\mu$ (B) $F - (m_2 - m_3)g\mu$ (C) $(m_1 + m_2 + m_3)g\mu$ (D) F (E) $m_1 g\mu - (m_2 + m_3)g\mu$

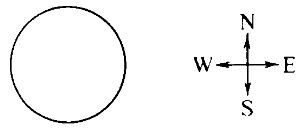


113. Two 5 kg masses are attached to opposite ends of a long massless cord which passes tautly over a massless frictionless pulley. The upper mass is initially held at rest on a table 50 cm from the pulley. The coefficient of kinetic friction between this mass and the table is 0.2. When the system is released, its resulting acceleration is closest to which of the following?
(A) 9.8 m/s^2 (B) 7.8 m/s^2 (C) 4.9 m/s^2 (D) 3.9 m/s^2 (E) 1.9 m/s^2

SECTION B – Circular Motion

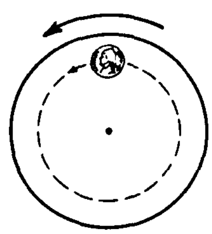
1. When a person stands on a rotating merry-go-round, the frictional force exerted on the person by the merry-go-round is
   (A) greater in magnitude than the frictional force exerted on the person by the merry-go-round
   (B) opposite in direction to the frictional force exerted on the merry-go-round by the person
   (C) directed away from the center of the merry-go-round
   (D) zero if the rate of rotation is constant
   (E) independent of the person's mass

2. A ball attached to a string is whirled around in a horizontal circle having a radius r. If the radius of the circle is changed to 4r and the same centripetal force is applied by the string, the new speed of the ball is which of the following?
   (A) One-quarter the original speed (B) One-half the original speed
   (C) The same as the original speed (D) Twice the original speed
   (E) Four times the original speed



View of Track from Above

3. A racing car is moving around the circular track of radius 300 meters shown above. At the instant when the car's velocity is directed due east, its acceleration is directed due south and has a magnitude of 3 meters per second squared. When viewed from above, the car is moving
   (A) clockwise at 30 m/s (B) clockwise at 10 m/s (C) counterclockwise at 30 m/s
   (D) counterclockwise at 10 m/s (E) with constant velocity

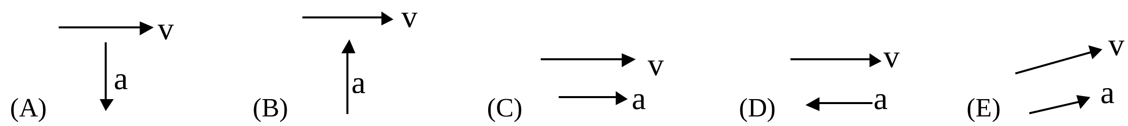


View from Above

4. The horizontal turntable shown above rotates at a constant rate. As viewed from above, a coin on the turntable moves counterclockwise in a circle as shown. Which of the following vectors best represents the direction of the frictional force exerted on the coin by the turntable when the coin is in the position shown?
   (A) ← (B) → (C) ↙ (D) ↓ (E) ↑

5. In which of the following situations would an object be accelerated?
   I. It moves in a straight line at constant speed.
   II. It moves with uniform circular motion.
   III. It travels as a projectile in a gravitational field with negligible air resistance.
   (A) I only (B) III only (C) I and II only (D) II and III only (E) I, II, and III.

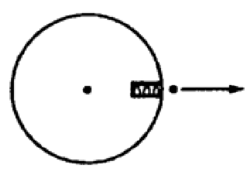
6. An automobile moves at constant speed down one hill and up another hill along the smoothly curved surface shown above. Which of the following diagrams best represents the directions of the velocity and the acceleration of the automobile at the instant that it is at the lowest position. as shown?



7. A car initially travels north and then turns to the left along a circular curve. This causes a package on the seat of the car to slide toward the right side of the car. Which of the following is true of the net force on the package while it is sliding?
(A) The force is directed away from the center of the circle.
(B) The force is directed north.
(C) There is not enough force directed north to keep the package from sliding.
(D) There is not enough force tangential to the car's path to keep the package from sliding.
(E) There is not enough force directed toward the center of the circle to keep the package from sliding.

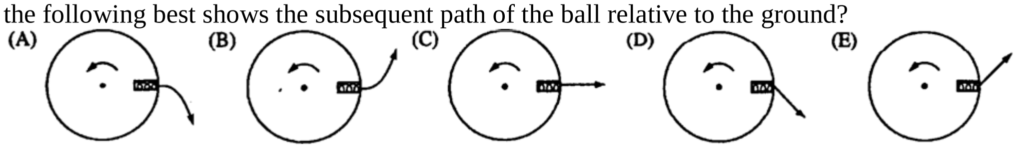
8. A child has a toy tied to the end of a string and whirls the toy at constant speed in a horizontal circular path of radius *R*. The toy completes each revolution of its motion in a time period *T*. What is the magnitude of the acceleration of the toy?

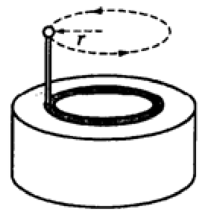
(A) Zero (B) $\frac{4\pi^2 R}{T^2}$ (C) $\frac{\pi R}{T^2}$ (D) g (E) $2\pi g$



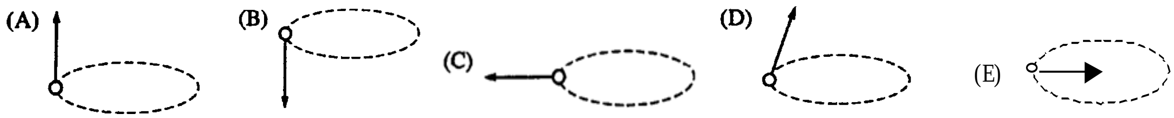
Top View

9. A compressed spring mounted on a disk can project a small ball. When the disk is not rotating, as shown in the top view above, the ball moves radially outward. The disk then rotates in a counterclockwise direction as seen from above, and the ball is projected outward at the instant the disk is in the position shown above. Which of the following best shows the subsequent path of the ball relative to the ground?

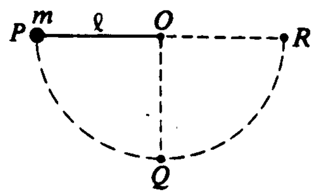




10. A steel ball supported by a stick rotates in a circle of radius *r*, as shown above. The direction of the net force acting on the ball when it is in the position shown is indicated by which of the following?



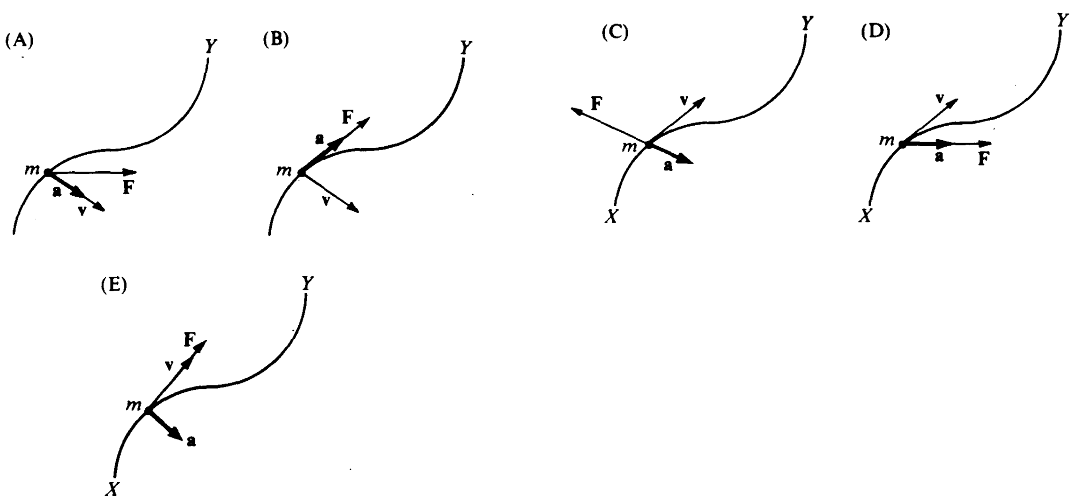
11. Inside a washing machine, the radius of the cylinder where the clothes sit is 0.50 m. In one of its settings the machine spins the cylinder at 2.0 revolutions per second. What is the acceleration of an item of clothing?
(A) 0.080 m/s^2 (B) 1.6 m/s^2 (C) 8.0 m/s^2 (D) 79 m/s^2 (E) 25 m/s^2

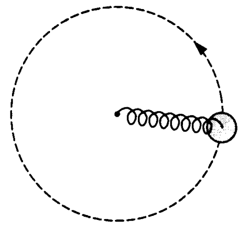


12. A ball of mass m is attached to the end of a string of length Q as shown above. The ball is released from rest from position P. where the string is horizontal. It swings through position Q. where the string is vertical, and then to position R. where the string is again horizontal. What are the directions of the acceleration vectors of the ball at positions Q and R?

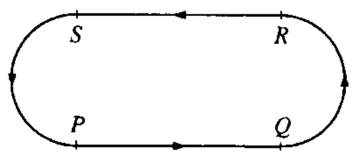
Position Q	Position R
(A) Downward	Downward
(B) Downward	To the right
(C) Upward	Downward
(D) Upward	To the left
(E) To the right	To the left

13. A mass m moves on a curved path from point X to point Y. Which of the following diagrams indicates a possible combination of the net force F on the mass, and the velocity v and acceleration a of the mass at the location shown?

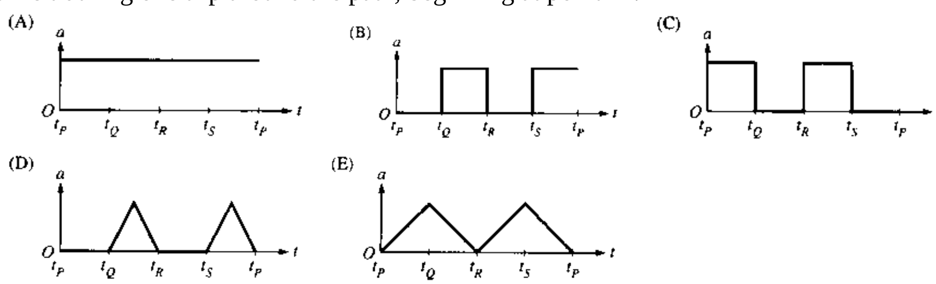




*14. A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view above. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible. What is the centripetal force on the disc?
(A) 0.3 N (B) 3N (C) 10 N (D) 300 N (E) 1,000 N

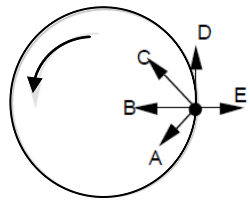


15. A figure of a dancer on a music box moves counterclockwise at constant speed around the path shown above. The path is such that the lengths of its segments, *PQ*, *QR*, *RS*, and *SP*, are equal. Arcs *QR* and *SP* are semicircles. Which of the following best represents the magnitude of the dancer's acceleration as a function of time t during one trip around the path, beginning at point *P* ?



16. A car travels forward with constant velocity. It goes over a small stone, which gets stuck in the groove of a tire. The initial acceleration of the stone, as it leaves the surface of the road, is
(A) vertically upward (B) horizontally forward (C) horizontally backward
(D) zero (E) upward and forward, at approximately 45° to the horizontal

17. A car is traveling on a road in hilly terrain, see figure to the right. Assume the car has speed v and the tops and bottoms of the hills have radius of curvature R. The driver of the car is most likely to feel weightless:
(A) at the top of a hill when $v > \sqrt{gR}$ (B) at the bottom of a hill when $v > \sqrt{gR}$
(C) going down a hill when $v = \sqrt{gR}$ (D) at the top of a hill when $v < \sqrt{gR}$
(E) at the bottom of a hill when $v < \sqrt{gR}$

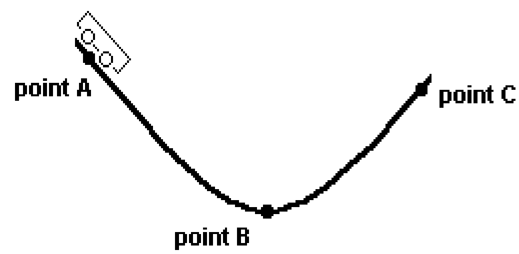


18. An object shown in the accompanying figure moves in uniform circular motion. Which arrow best depicts the net force acting on the object at the instant shown?
(A) A (B) B (C) C (D) D (E) E

19. A child whirls a ball at the end of a rope, in a uniform circular motion. Which of the following statements is **NOT** true?
(A) The speed of the ball is constant (B) The velocity is of the ball is constant (C) The radius is constant
(D) The magnitude of the ball's acceleration is constant
(E) The acceleration of the ball is directed radially inwards towards the center

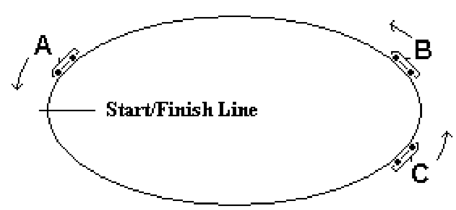
20. An astronaut in an orbiting space craft attaches a mass *m* to a string and whirls it around in uniform circular motion. The radius of the circle is *r*, the speed of the mass is *v*, and the tension in the string is *F*. If the mass, radius, and speed were all to double the tension required to maintain uniform circular motion would be
(A) *F*/2 (B) *F* (C) 2*F* (D) 4*F* (E) 8*F*

21. Assume the roller coaster cart rolls frictionlessly along the curved track from point A to point C under the influence of gravity. What would be the direction of the cart's acceleration at point B?
(A) upward
(B) downward
(C) forward
(D) backward
(E) no acceleration



<u>Questions 22 – 23</u>

The diagram below is a snapshot of three cars all moving counterclockwise during a one lap race on an elliptical track.



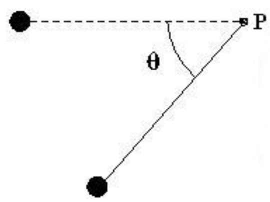
22. Which car has had the lowest average speed during the race so far?
(A) car A
(B) car B
(C) car C
(D) all three cars have had the same average speed
(E) cannot be determined with information provided

23. Which car at the moment of the snapshot MUST have a net force acting on it?
(A) car A
(B) car B
(C) car C
(D) all three cars have net forces acting on them
(E) cannot be determined with information provided

24. A centripetal force of 5.0 newtons is applied to a rubber stopper moving at a constant speed in a horizontal circle. If the same force is applied, but the radius is made smaller, what happens to the speed, *v*, and the frequency, *f*, of the stopper?
(A) *v* increases & *f* increases
(B) *v* decreases & *f* decreases
(C) *v* increases & *f* decreases
(D) *v* decreases & *f* increases
(E) neither changes

25. What is the centripetal acceleration of an object (mass = 50 g) on the end of an 80-cm string rotating at a constant rate of 4 times a second?
(A) 25 m/s^2 (B) 32 m/s^2 (C) 100 m/s^2 (D) 500 m/s^2 (E) 2500 m/s^2

26. What net force is necessary to keep a 1.0 kg puck moving in a circle of radius 0.5 m on a horizontal frictionless surface with a speed of 2.0 m/s?
(A) 0 N (B) 2.0 N (C) 4.0 N (D) 8.0 N (E) 16 N



27. Astronauts on the Moon perform an experiment with a simple pendulum that is released from the horizontal position at rest. At the moment shown in the diagram with 0° <θ < 90°, the total acceleration of the mass may be directed in which of the following ways?
(A) straight to the right (B) straight to the left (C) straight upward (D) straight downward
(E) straight along the connecting string toward point P (the pivot)

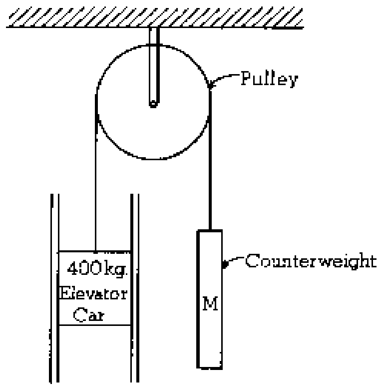
28. A 4.0 kg mass is attached to one end of a rope 2 m long. If the mass is swung in a vertical circle from the free end of the rope, what is the tension in the rope when the mass is at its highest point if it is moving with a speed of 5 m/s?
(A) 5.4 N (B) 10.8 N (C) 21.6 N (D) 50 N (E) 65.4 N

29. A ball of mass *m* is fastened to a string. The ball swings at constant speed in a vertical circle of radius *R* with the other end of the string held fixed. Neglecting air resistance, what is the difference between the string's tension at the bottom of the circle and at the top of the circle?
(A) 1·*mg* (B) 2·*mg* (C) 4·*mg* (D) 6·*mg* (E) 8·*mg*

30. An object weighing 4 newtons swings on the end of a string as a simple pendulum. At the bottom of the swing, the tension in the string is 6 newtons. What is the magnitude of the centripetal acceleration of the object at the bottom of the swing?
(A) 0 (B) 0.5 g (C) g (D) 1.5 g (E) 2.5 g

31. Riders in a carnival ride stand with their backs against the wall of a circular room of diameter 8.0 m. The room is spinning horizontally about an axis through its center at a rate of 45 rev/min when the floor drops so that it no longer provides any support for the riders. What is the minimum coefficient of static friction between the wall and the rider required so that the rider does not slide down the wall?
(A) 0.0012 (B) 0.056 (C) 0.11 (D) 0.53 (E) 8.9

SECTION A – Linear Dynamics

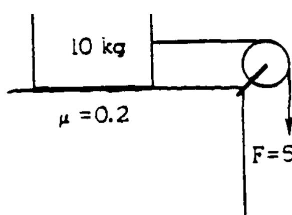


1976B1. The two guide rails for the elevator shown above each exert a constant friction force of 100 newtons on the elevator car when the elevator car is moving upward with an acceleration of 2 meters per second squared. The pulley has negligible friction and mass. Assume g = 10 m/sec^2.

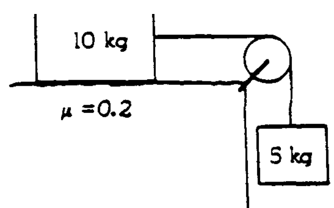
a. On the diagram below, draw and label all forces acting on the elevator car. Identify the source of each force.

b. Calculate the tension in the cable lifting the 400-kilogram elevator car during an upward acceleration of 2 m/sec^2. (Assume g 10 m/sec^2.)

c. Calculate the mass M the counterweight must have to raise the elevator car with an acceleration of 2 m/sec^2.



Case I



Case II

1979B2. A 10-kilogram block rests initially on a table as shown in cases I and II above. The coefficient of sliding friction between the block and the table is 0.2. The block is connected to a cord of negligible mass, which hangs over a massless frictionless pulley. In case I a force of 50 newtons is applied to the cord. In case II an object of mass 5 kilograms is hung on the bottom of the cord. Use g = 10 meters per second squared.

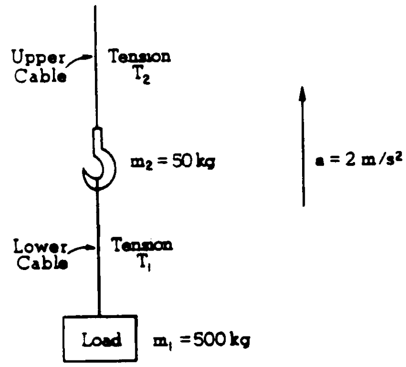
a. Calculate the acceleration of the 10-kilogram block in case I.

b. On the diagrams below, draw and label all the forces acting on each block in case II

10 kg

5 kg

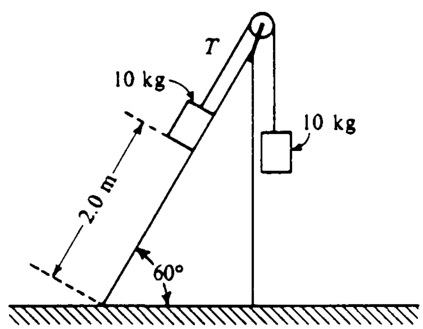
c. Calculate the acceleration of the 10-kilogram block in case II.



1982B2. A crane is used to hoist a load of mass m_1 = 500 kilograms. The load is suspended by a cable from a hook of mass m_2 = 50 kilograms, as shown in the diagram above. The load is lifted upward at a constant acceleration of 2 m/s^2.

a. On the diagrams below draw and label the forces acting on the hook and the forces acting on the load as they accelerate upward

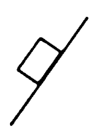
b. Determine the tension T_1 in the lower cable and the tension T_2 in the upper cable as the hook and load are accelerated upward at 2 m/s^2. Use g = 10 m/s^2.



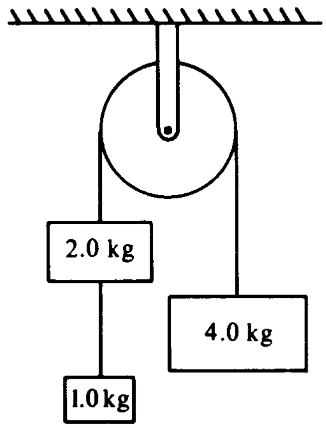
1985B2 (modified) Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use g = 10 m/s^2, sin 60° = 0.87, and cos 60° = 0.50.

a. What is the tension T in the string?

b. On the diagram below, draw and label all the forces acting on the box that is on the plane.



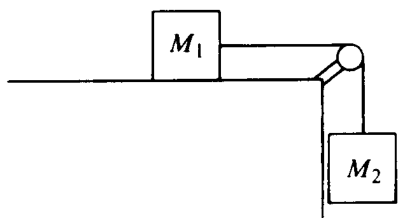
c. Determine the magnitude of the frictional force acting on the box on the plane.



1986B1. Three blocks of masses 1.0, 2.0, and 4.0 kilograms are connected by massless strings, one of which passes over a frictionless pulley of negligible mass, as shown above. Calculate each of the following.

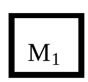
a. The acceleration of the 4-kilogram block
b. The tension in the string supporting the 4-kilogram block
c. The tension in the string connected to the l-kilogram block

---



1987B1. In the system shown above, the block of mass M_1 is on a rough horizontal table. The string that attaches it to the block of mass M_2 passes over a frictionless pulley of negligible mass. The coefficient of kinetic friction μ_k between M_1 and the table is less than the coefficient of static friction μ_s

a. On the diagram below, draw and identify all the forces acting on the block of mass M_1.



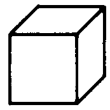
b. In terms of M_1 and M_2 determine the minimum value of μ_s that will prevent the blocks from moving.

The blocks are set in motion by giving M_2 a momentary downward push. In terms of M_1, M_2, μ_k, and g, determine each of the following:

c. The magnitude of the acceleration of M_1
d. The tension in the string.

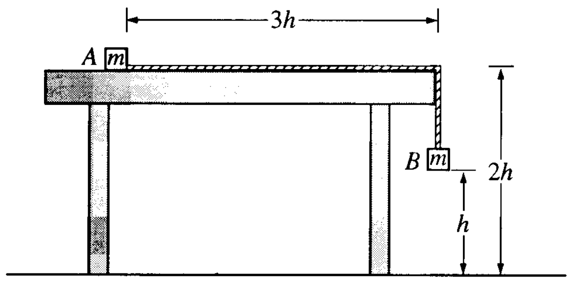
1988B1. A helicopter holding a 70-kilogram package suspended from a rope 5.0 meters long accelerates upward at a rate of 5.2 m/s^2. Neglect air resistance on the package.

a. On the diagram below, draw and label all of the forces acting on the package.



b. Determine the tension in the rope.

c. When the upward velocity of the helicopter is 30 meters per second, the rope is cut and the helicopter continues to accelerate upward at 5.2 m/s^2. Determine the distance between the helicopter and the package 2.0 seconds after the rope is cut.



1998B1 Two small blocks, each of mass m, are connected by a string of constant length 4h and negligible mass. Block A is placed on a smooth tabletop as shown above, and block B hangs over the edge of the table. The tabletop is a distance 2h above the floor. Block B is then released from rest at a distance h above the floor at time t = 0. Express all algebraic answers in terms of h, m, and g.

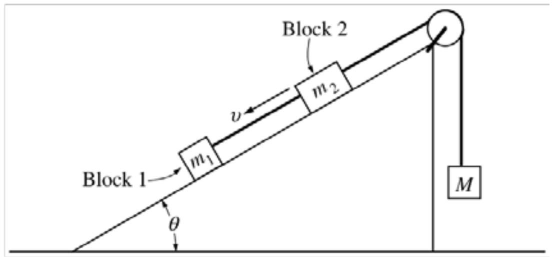
a. Determine the acceleration of block B as it descends.

b. Block B strikes the floor and does not bounce. Determine the time $t = t_1$ at which block B strikes the floor.

c. Describe the motion of block A from time t = 0 to the time when block B strikes the floor.

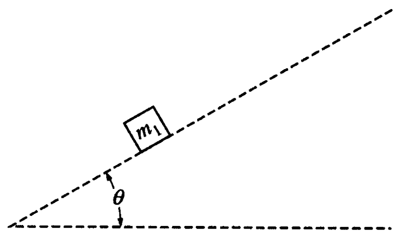
d. Describe the motion of block A from the time block B strikes the floor to the time block A leaves the table.

e. Determine the distance between the landing points of the two blocks.



2000B2. Blocks 1 and 2 of masses m_1 and m_2, respectively, are connected by a light string, as shown above. These blocks are further connected to a block of mass M by another light string that passes over a pulley of negligible mass and friction. Blocks l and 2 move with a constant velocity v down the inclined plane, which makes an angle θ with the horizontal. The kinetic frictional force on block 1 is f and that on block 2 is 2f.

a. On the figure below, draw and label all the forces on block m_1.

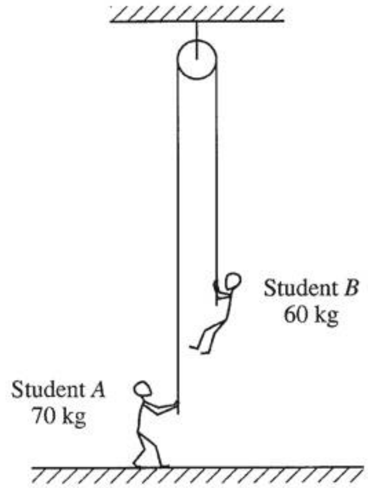


Express your answers to each of the following in terms of m_1, m_2, g, θ, and f.

b. Determine the coefficient of kinetic friction between the inclined plane and block 1.

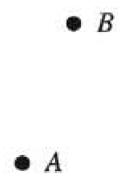
c. Determine the value of the suspended mass M that allows blocks 1 and 2 to move with constant velocity down the plane.

d. The string between blocks 1 and 2 is now cut. Determine the acceleration of block 1 while it is on the inclined plane.



2003B1 A rope of negligible mass passes over a pulley of negligible mass attached to the ceiling, as shown above. One end of the rope is held by Student A of mass 70 kg, who is at rest on the floor. The opposite end of the rope is held by Student B of mass 60 kg, who is suspended at rest above the floor. Use $g = 10\ m/s^2$.

a. On the dots below that represent the students, draw and label free-body diagrams showing the forces on Student A and on Student B.



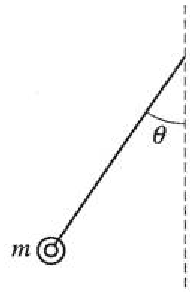
b. Calculate the magnitude of the force exerted by the floor on Student A.

Student B now climbs up the rope at a constant acceleration of $0.25\ m/s^2$ with respect to the floor.

c. Calculate the tension in the rope while Student B is accelerating.

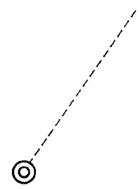
d. As Student B is accelerating, is Student A pulled upward off the floor? Justify your answer.

e. With what minimum acceleration must Student B climb up the rope to lift Student A upward off the floor?



2003Bb1 (modified) An airplane accelerates uniformly from rest. A physicist passenger holds up a thin string of negligible mass to which she has tied her ring, which has a mass *m*. She notices that as the plane accelerates down the runway, the string makes an angle θ with the vertical as shown above.

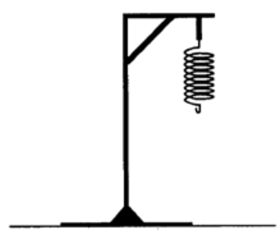
a. In the space below, draw a free-body diagram of the ring, showing and labeling all the forces present.



The plane reaches a takeoff speed of 65 m/s after accelerating for a total of 30 s.

b. Determine the minimum length of the runway needed.

c. Determine the angle θ that the string makes with the vertical during the acceleration of the plane before it leaves the ground.

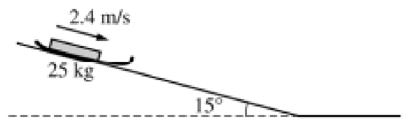


*1996B2 (modified) A spring that can be assumed to be ideal hangs from a stand, as shown above. You wish to determine experimentally the spring constant k of the spring.

a. i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?

Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.

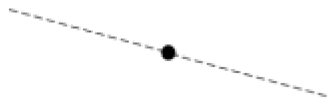
b. i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



B2007B1. An empty sled of mass 25 kg slides down a muddy hill with a constant speed of 2.4 m/s. The slope of the hill is inclined at an angle of 15° with the horizontal as shown in the figure above.

a. Calculate the time it takes the sled to go 21 m down the slope.

b. On the dot below that represents the sled, draw/label a free-body diagram for the sled as it slides down the slope



c. Calculate the frictional force on the sled as it slides down the slope.

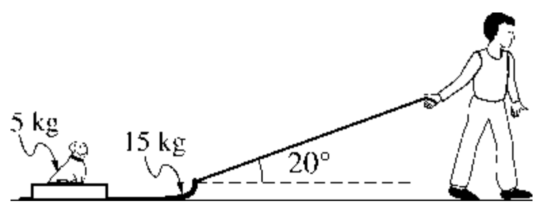
d. Calculate the coefficient of friction between the sled and the muddy surface of the slope.

e. The sled reaches the bottom of the slope and continues on the horizontal ground. Assume the same coefficient of friction.

i. In terms of velocity and acceleration, describe the motion of the sled as it travels on the horizontal ground.

ii. On the axes below, sketch a graph of speed *v* versus time *t* for the sled. Include both the sled's travel down the slope and across the horizontal ground. Clearly indicate with the symbol t_ℓ the time at which the sled leaves the slope.

B2007b1 (modified) A child pulls a 15 kg sled containing a 5.0 kg dog along a straight path on a horizontal surface. He exerts a force of 55 N on the sled at an angle of 20° above the horizontal, as shown in the figure. The coefficient of friction between the sled and the surface is 0.22.

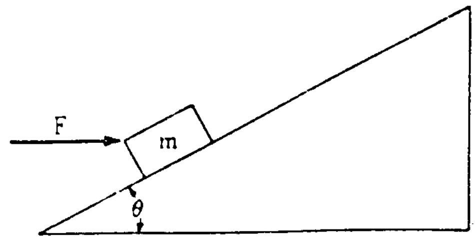


a. On the dot below that represents the sled-dog system, draw and label a free-body diagram for the system as it is pulled along the surface.

b. Calculate the normal force of the surface on the system.

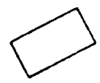
c. Calculate the acceleration of the system.

d. At some later time, the dog rolls off the side of the sled. The child continues to pull with the same force. On the axes below, sketch a graph of speed *v* versus time *t* for the sled. Include both the sled's travel with and without the dog on the sled. Clearly indicate with the symbol t_r the time at which the dog rolls off.



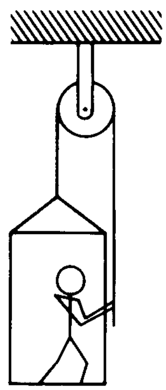
1981M1. A block of mass m, acted on by a force of magnitude F directed horizontally to the right as shown above, slides up an inclined plane that makes an angle θ with the horizontal. The coefficient of sliding friction between the block and the plane is μ.

a. On the diagram of the block below, draw and label all the forces that act on the block as it slides up the plane.



b. Develop an expression in terms of m, θ, F, μ, and g, for the block's acceleration up the plane.

c. Develop an expression for the magnitude of the force F that will allow the block to slide up the plane with constant velocity. What relation must θ and μ satisfy in order for this solution to be physically meaningful?



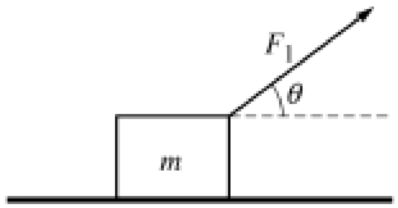
1986M1. The figure above shows an 80-kilogram person standing on a 20-kilogram platform suspended by a rope passing over a stationary pulley that is free to rotate. The other end of the rope is held by the person. The masses of the rope and pulley are negligible. You may use $g = 10\ m/s^2$. Assume that friction is negligible, and the parts of the rope shown remain vertical.

a. If the platform and the person are at rest, what is the tension in the rope?

The person now pulls on the rope so that the acceleration of the person and the platform is $2\ m/s^2$ upward.

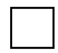
b. What is the tension in the rope under these new conditions?

c. Under these conditions, what is the force exerted by the platform on the person?



2007M1. A block of mass m is pulled along a rough horizontal surface by a constant applied force of magnitude F_1 that acts at an angle θ to the horizontal, as indicated above. The acceleration of the block is a_1. Express all algebraic answers in terms of m, F_1, θ , a_1, and fundamental constants.

a. On the figure below, draw and label a free-body diagram showing all the forces on the block.



b. Derive an expression for the normal force exerted by the surface on the block.

c. Derive an expression for the coefficient of kinetic friction μ between the block and the surface.

d. On the axes below, sketch graphs of the speed v and displacement x of the block as functions of time t if the block started from rest at $x = 0$ and $t = 0$.

e. If the applied force is large enough, the block will lose contact with the surface. Derive an expression for the magnitude of the greatest acceleration a_{max} that the block can have and still maintain contact with the ground.

1996M2. A 300-kg box rests on a platform attached to a forklift, shown above. Starting from rest at time = 0, the box is lowered with a downward acceleration of 1.5 m/s^2

a. Determine the upward force exerted by the horizontal platform on the box as it is lowered.

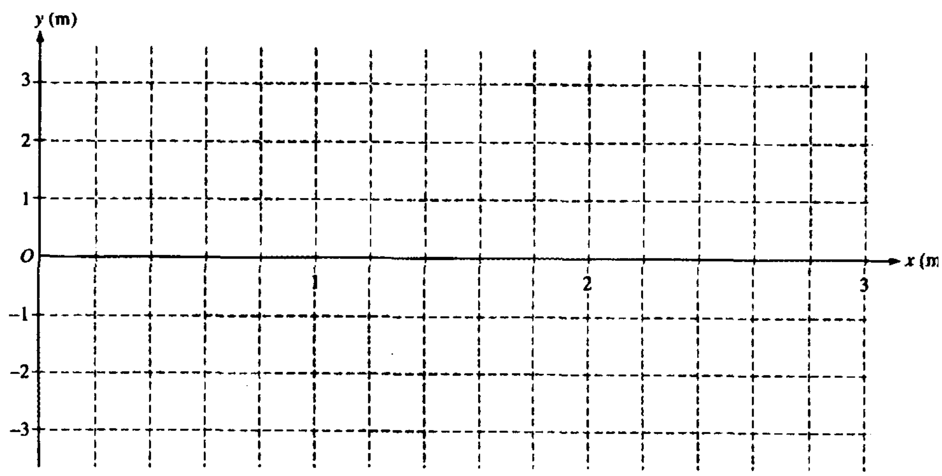
At time t = 0, the forklift also begins to move forward with an acceleration of 2 m/s^2 while lowering the box as described above. The box does not slip or tip over.

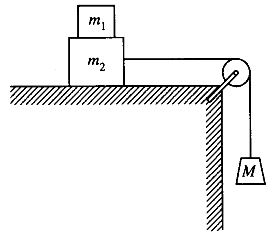
b. Determine the frictional force on the box.

c. Given that the box does not slip, determine the minimum possible coefficient of friction between the box and the platform.

d. Determine an equation for the path of the box that expresses y as a function of x (and <u>not</u> of t), assuming that, at time t = 0, the box has a horizontal position x = 0 and a vertical position y = 2 m above the ground, with zero velocity.

e. On the axes below sketch the path taken by the box





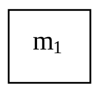
1998M3. Block 1 of mass m_1 is placed on block 2 of mass m_2 which is then placed on a table. A string connecting block 2 to a hanging mass M passes over a pulley attached to one end of the table, as shown above. The mass and friction of the pulley are negligible. The coefficients of friction between blocks 1 and 2 and between block 2 and the tabletop are nonzero and are given in the following table.

	Coefficient Between Blocks 1 and 2	Coefficient Between Block 2 and the Tabletop
Static	μ_{s1}	μ_{s2}
Kinetic	μ_{k1}	μ_{k2}

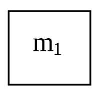
Express your answers in terms of the masses, coefficients of friction, and g, the acceleration due to gravity.

a. Suppose that the value of M is small enough that the blocks remain at rest when released. For each of the following forces, determine the magnitude of the force and draw a vector on the block provided to indicate the direction of the force if it is nonzero.

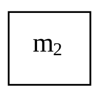
i. The normal force N_1 exerted on block 1 by block 2



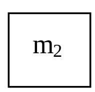
ii. The friction force f_1 exerted on block 1 by block 2



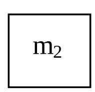
iii. The force T exerted on block 2 by the string



iv. The normal force N_2 exerted on block 2 by the tabletop



v. The friction force f_2 exerted on block 2 by the tabletop



b. Determine the largest value of M for which the blocks can remain at rest.

c. Now suppose that M is large enough that the hanging block descends when the blocks are released. Assume that blocks 1 and 2 are moving as a unit (no slippage). Determine the magnitude *a* of their acceleration.

d. Now suppose that M is large enough that as the hanging block descends, block 1 is slipping on block 2. Determine each of the following.

i. The magnitude a_1 of the acceleration of block 1

ii. The magnitude a_2 of the acceleration of block 2

*2005M1 (modified) A ball of mass M is thrown vertically upward with an initial speed of v_o. It experiences a force of air resistance given by $F = -kv$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_o, and fundamental constants.

a. Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases ____ decreases ____ remains the same

Justify your answer.

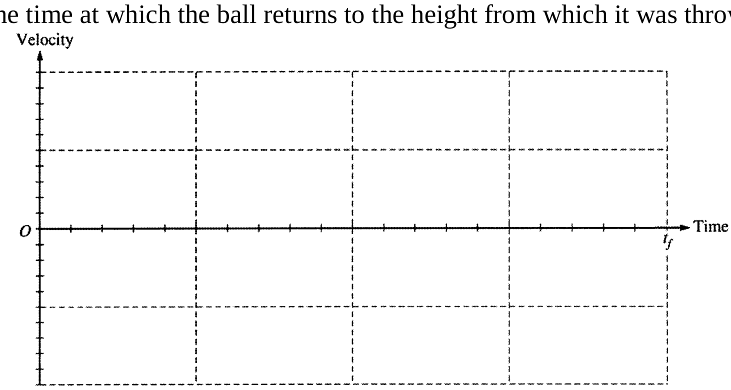
b. Determine the terminal speed of the ball as it moves downward.

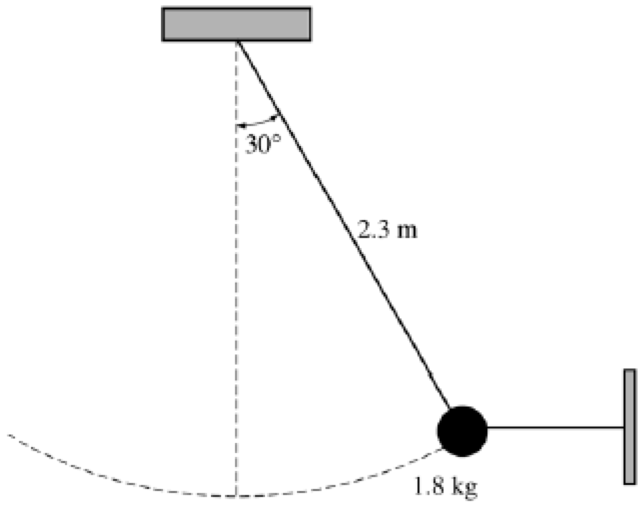
c. Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____ longer to rise ____ longer to fall

Justify your answer.

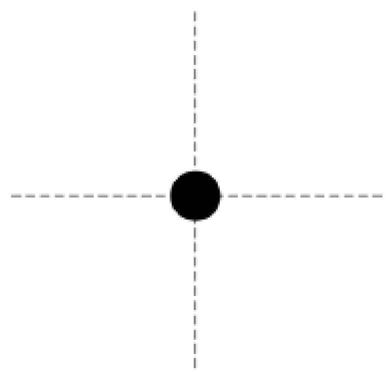
d. On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



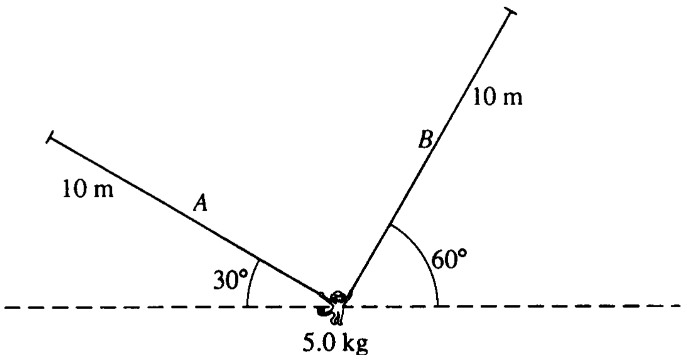


2005B2. A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.

(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.

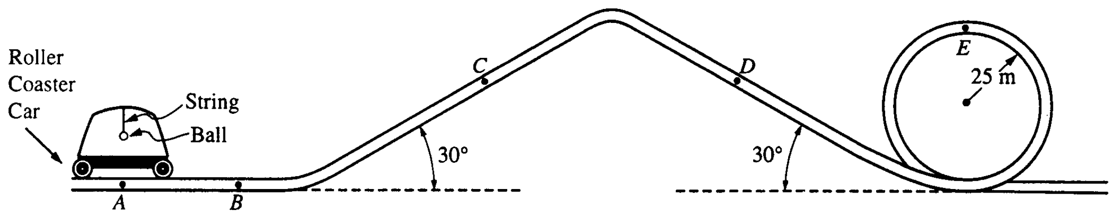


sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.



Note: Figure not drawn to scale.

1995B3. Part of the track of an amusement park roller coaster is shaped as shown above. A safety bar is oriented lengthwise along the top of each car. In one roller coaster car, a small 0.10-kilogram ball is suspended from this bar by a short length of light, inextensible string.

a. Initially, the car is at rest at point A.

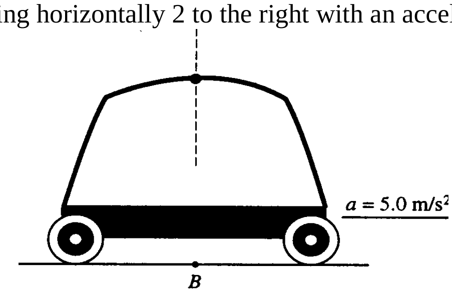
i. On the diagram below, draw and label all the forces acting on the 0.10-kilogram ball.

ii. Calculate the tension in the string.

The car is then accelerated horizontally, goes up a 30° incline, goes down a 30° incline, and then goes around a vertical circular loop of radius 25 meters. For each of the four situations described in parts (b) to (e), do all three of the following. In each situation, assume that the ball has stopped swinging back and forth.

1) Determine the horizontal component T_h of the tension in the string in newtons and record your answer in the space provided.
2)Determine the vertical component T_v of the tension in the string in newtons and record your answer in the space provided.
3)Show on the adjacent diagram the approximate direction of the string with respect to the vertical. The dashed line shows the vertical in each situation.

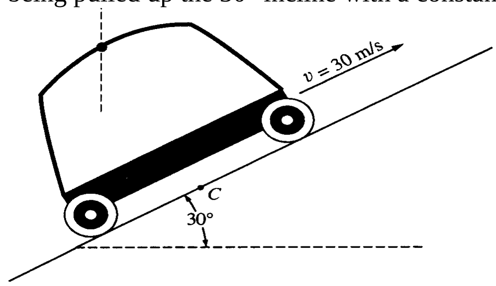
b. The car is at point B moving horizontally 2 to the right with an acceleration of 5.0 m/s .



T_h =_________________________

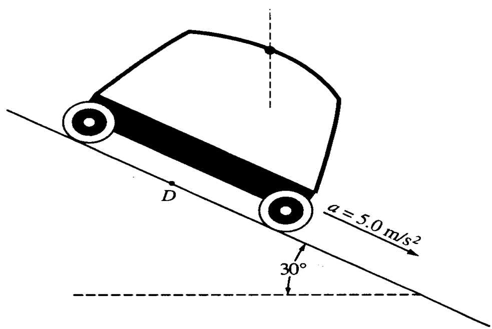
T_v = _________________________

c. The car is at point C and is being pulled up the 30° incline with a constant speed of 30 m/s.



T_h = ___________________________

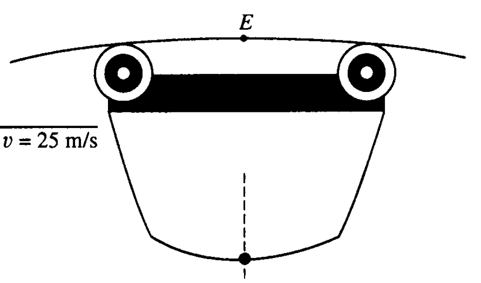
T_v = ___________________________



d. The car is at point D moving down the incline with an acceleration of 5.0 m/s^2 .

T_h =___________________________

T_v = ___________________________

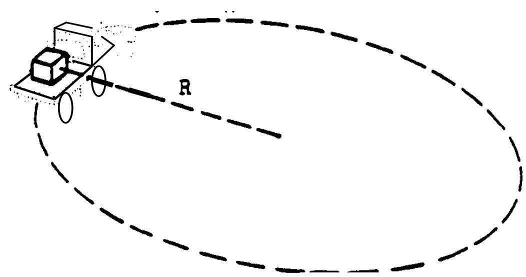


e. The car is at point E moving upside down with an instantaneous speed of 25 m/s and no tangential acceleration at the top of the vertical loop of radius 25 meters.

T_h =___________________________

T_v = ___________________________

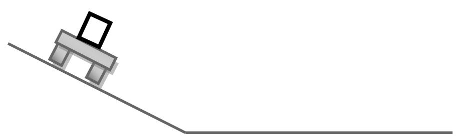
SECTION B – Circular Motion



1977 B2. A box of mass M, held in place by friction, rides on the flatbed of a truck which is traveling with constant speed v. The truck is on an unbanked circular roadway having radius of curvature R.

a. On the diagram provided above, indicate and clearly label all the force vectors acting on the box.

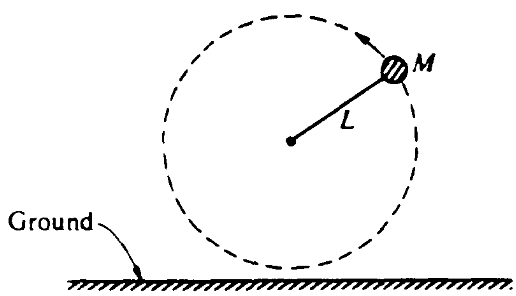
b. Find what condition must be satisfied by the coefficient of static friction μ between the box and the truck bed. Express your answer in terms of v, R, and g.



If the roadway is properly banked, the box will still remain in place on the truck for the same speed v even when the truck bed is frictionless.

c. On the diagram above indicate and clearly label the two forces acting on the box under these conditions

d. Which, if either, of the two forces acting on the box is greater in magnitude?



1984B1. A ball of mass M attached to a string of length L moves in a circle in a vertical plane as shown above. At the top of the circular path, the tension in the string is twice the weight of the ball. At the bottom, the ball just clears the ground. Air resistance is negligible. Express all answers in terms of M, L, and g.

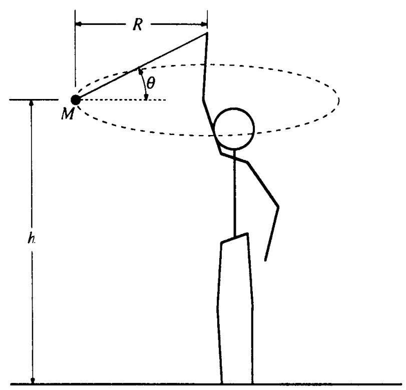
a. Determine the magnitude and direction of the net force on the ball when it is at the top.

b. Determine the speed v_o of the ball at the top.

The string is then cut when the ball is at the top.

c. Determine the time it takes the ball to reach the ground.

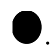
d. Determine the horizontal distance the ball travels before hitting the ground.



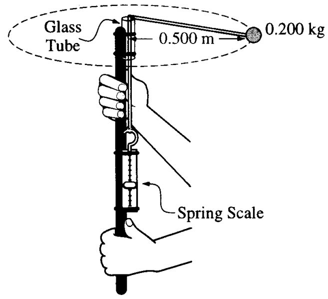
1989B1. An object of mass M on a string is whirled with increasing speed in a horizontal circle, as shown above. When the string breaks, the object has speed v_o and the circular path has radius R and is a height h above the ground. Neglect air friction.

a. Determine the following, expressing all answers in terms of h, v_o, and g.
   i. The time required for the object to hit the ground after the string breaks
   ii. The horizontal distance the object travels from the time the string breaks until it hits the ground
   iii. The speed of the object just before it hits the ground

b. On the figure below, draw and label all the forces acting on the object when it is in the position shown in the diagram above.



c. Determine the tension in the string just before the string breaks. Express your answer in terms of M, R, v_o, & g.



1997B2 (modified) To study circular motion, two students use the hand-held device shown above, which consists of a rod on which a spring scale is attached. A polished glass tube attached at the top serves as a guide for a light cord attached the spring scale. A ball of mass 0.200 kg is attached to the other end of the cord. One student swings the teal around at constant speed in a horizontal circle with a radius of 0.500 m. Assume friction and air resistance are negligible.

a. Explain how the students, by using a timer and the information given above, can determine the speed of the ball as it is revolving.

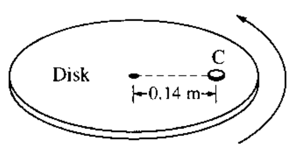
b. The speed of the ball is determined to be 3.7 m/s. Assuming that the cord is horizontal as it swings, calculate the expected tension in the cord.

c. The actual tension in the cord as measured by the spring scale is 5.8 N. What is the percent difference between this measured value of the tension and the value calculated in part b.?

The students find that, despite their best efforts, they cannot swing the ball so that the cord remains exactly horizontal.

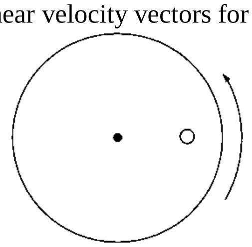
d. i. On the picture of the ball below, draw vectors to represent the forces acting on the ball and identify the force that each vector represents.

ii. Explain why it is not possible for the ball to swing so that the cord remains exactly horizontal.
iii. Calculate the angle that the cord makes with the horizontal.



1999B5 A coin C of mass 0.0050 kg is placed on a horizontal disk at a distance of 0.14 m from the center, as shown above. The disk rotates at a constant rate in a counterclockwise direction as seen from above. The coin does not slip, and the time it takes for the coin to make a complete revolution is 1.5 s.

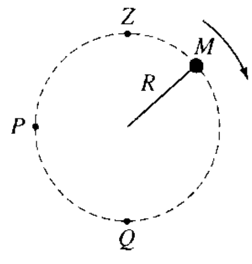
a. The figure below shows the disk and coin as viewed from above. Draw and label vectors on the figure below to show the instantaneous acceleration and linear velocity vectors for the coin when it is at the position shown.



b. Determine the linear speed of the coin.

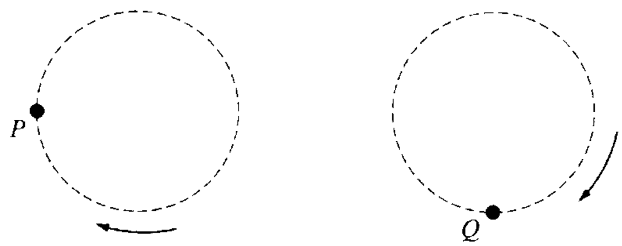
c. The rate of rotation of the disk is gradually increased. The coefficient of static friction between the coin and the disk is 0.50. Determine the linear speed of the coin when it just begins to slip.

d. If the experiment in part (c) were repeated with a second, identical coin glued to the top of the first coin, how would this affect the answer to part (c)? Explain your reasoning.

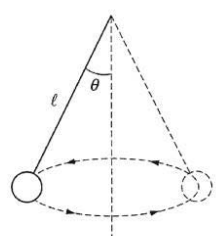


2001B1. A ball of mass M is attached to a string of length R and negligible mass. The ball moves clockwise in a vertical circle, as shown above. When the ball is at point P, the string is horizontal. Point Q is at the bottom of the circle and point Z is at the top of the circle. Air resistance is negligible. Express all algebraic answers in terms of the given quantities and fundamental constants.

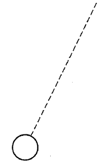
a. On the figures below, draw and label all the forces exerted on the ball when it is at points P and Q, respectively.



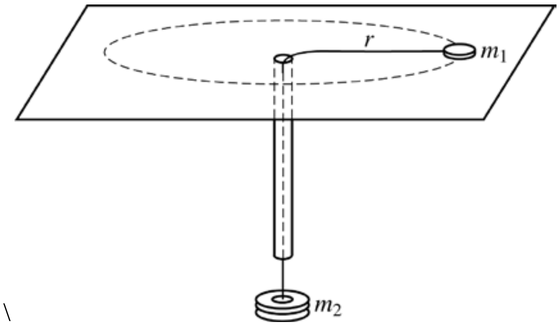
b. Derive an expression for v_{min} the minimum speed the ball can have at point Z without leaving the circular path.
c. The maximum tension the string can have without breaking is T_{max} Derive an expression for v_{max}, the maximum speed the ball can have at point Q without breaking the string.
d. Suppose that the string breaks at the instant the ball is at point P. Describe the motion of the ball immediately after the string breaks.



2002B2B A ball attached to a string of length *l* swings in a horizontal circle, as shown above, with a constant speed. The string makes an angle θ with the vertical, and *T* is the magnitude of the tension in the string. Express your answers to the following in terms of the given quantities and fundamental constants.
a. On the figure below, draw and label vectors to represent all the forces acting on the ball when it is at the position shown in the diagram. The lengths of the vectors should be consistent with the relative magnitudes of the forces.



b. Determine the mass of the ball.
c. Determine the speed of the ball.
d. Determine the frequency of revolution of the ball.
e. Suppose that the string breaks as the ball swings in its circular path. Qualitatively describe the trajectory of the ball after the string breaks but before it hits the ground.



\

2009Bb1 An experiment is performed using the apparatus above. A small disk of mass m_1 on a frictionless table is attached to one end of a string. The string passes through a hole in the table and an attached narrow, vertical plastic tube. An object of mass m_2 is hung at the other end of the string. A student holding the tube makes the disk rotate in a circle of constant radius r, while another student measures the period P.

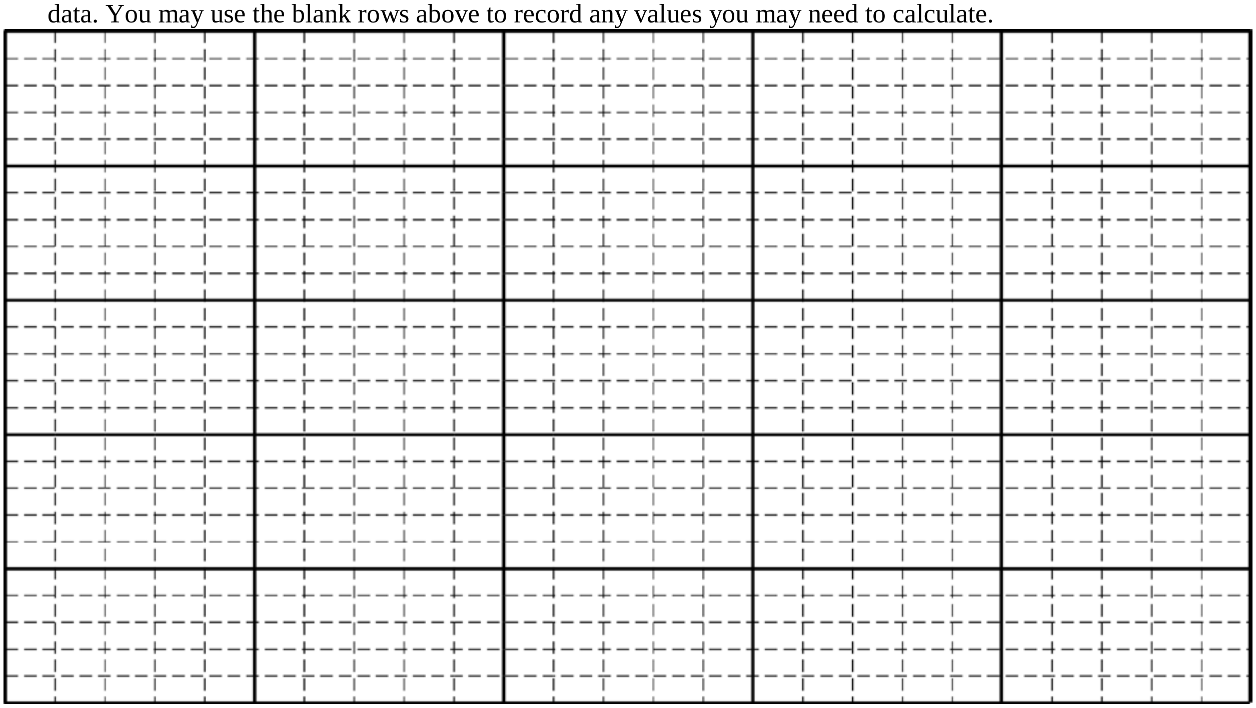
a. Derive the equation $P = 2\pi\sqrt{\frac{m_1 r}{m_2 g}}$ that relates P and m_2.

The procedure is repeated, and the period P is determined for four different values of m_2, where m_1 = 0.012 kg and r = 0.80 m. The data, which are presented below, can be used to compute an experimental value for g.

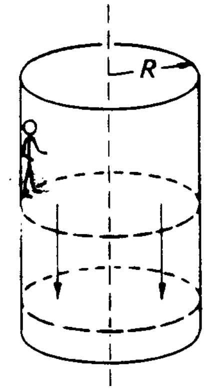
m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75

b. What quantities should be graphed to yield a straight line with a slope that could be used to determine g?

c. On the grid below, plot the quantities determined in part (b), label the axes, and draw the best-fit line to the data. You may use the blank rows above to record any values you may need to calculate.

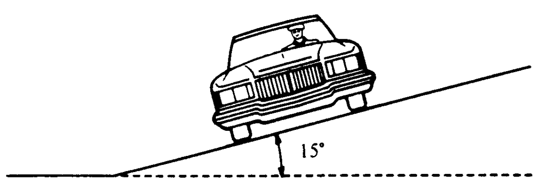


d. Use your graph to calculate the experimental value of g.



*1984M1 (modified) An amusement park ride consists of a rotating vertical cylinder with rough canvas walls. The floor is initially about halfway up the cylinder wall as shown above. After the rider has entered and the cylinder is rotating sufficiently fast, the floor is dropped down, yet the rider does not slide down. The rider has mass of 50 kilograms, The radius R of the cylinder is 5 meters, the frequency of the cylinder when rotating is $1/\pi$ revolutions per second, and the coefficient of static friction between the rider and the wall of the cylinder is 0.6.

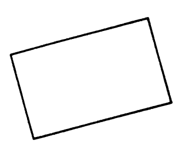
a. On the diagram above, draw and identify the forces on the rider when the system is rotating and the floor has dropped down.
b. Calculate the centripetal force on the rider when the cylinder is rotating and state what provides that force.
c. Calculate the upward force that keeps the rider from falling when the floor is dropped down and state what provides that force.
d. At the same rotational speed, would a rider of twice the mass slide down the wall? Explain your answer.



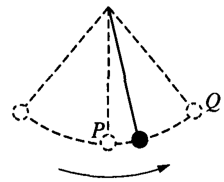
1988M1. A highway curve that has a radius of curvature of 100 meters is banked at an angle of 15° as shown above.

a. Determine the vehicle speed for which this curve is appropriate if there is no friction between the road and the tires of the vehicle.

On a dry day when friction is present, an automobile successfully negotiates the curve at a speed of 25 m/s.

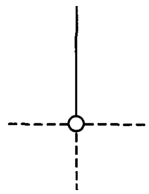


b. On the diagram above, in which the block represents the automobile, draw and label all of the forces on the automobile.
c. Determine the minimum value of the coefficient of friction necessary to keep this automobile from sliding as it goes around the curve.

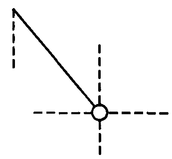


1998B6 A heavy ball swings at the end of a string as shown above, with negligible air resistance. Point P is the lowest point reached by the ball in its motion, and point Q is one of the two highest points.

a. On the following diagrams draw and label vectors that could represent the velocity and acceleration of the ball at points P and Q. If a vector is zero, explicitly state this fact. The dashed lines indicate horizontal and vertical directions.

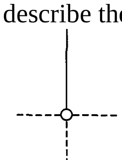


i. Point P

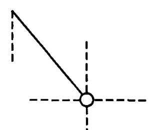


ii. Point Q

b. After several swings, the string breaks. The mass of the string and air resistance are negligible. On the following diagrams, sketch the path of the ball if the break occurs when the ball is at point P or point Q. In each case, briefly describe the motion of the ball after the break.



i. Point P



ii. Point Q

ANSWERS - AP Physics Multiple Choice Practice – Dynamics

SECTION A – Linear Dynamics

	Solution	Answer
1.	As T_2 is more vertical, it is supporting more of the weight of the ball. The horizontal components of T_1 and T_2 are equal.	C
2.	Normal force is perpendicular to the incline, friction acts up, parallel to the incline (opposite the motion of the block), gravity acts straight down.	E
3.	$\Sigma F = ma$; $mg\sin\theta - f = ma$	C
4.	The "diluted" force between objects is the applied force times the ratio of the mass behind the rope to the total mass being pulled. This can be derived from $a = F/m_{total}$ and $F_T = m_{behind\ the\ rope}a$	E
5.	$\Sigma F = ma$; $mg - bv = ma$	B
6.	$F_f = \mu F_N$ where F_N is found from $\Sigma F_y = 0 = (F_N - mg - T\sin\theta)$	A
7.	$\Sigma F_y = 0 = F\sin\theta + F_N - W$	B
8.	The bottom of the rope supports the box, while the top of the rope must support the rope itself and the box.	E
9.	The vertical components of the tension in the rope are two equal upward components of $T\cos\theta$, which support the weight. $\Sigma F_y = 0 = 2T\cos\theta - W$	D
10.	$F = mg = kx$ (the negative sign merely indicates the direction of the spring force relative to the displacement)	E
11.	$\Sigma F_{external} = m_{total}a$; mg is the only force acting from outside the system of masses so we have $mg = (4m)a$	B
12.	$W = mg$	D
13.	$F_N = mg\cos\theta$, $\cos\theta$ = adjacent/hypotenuse = 4/5	C
14.	Three vectors add to zero if they form the sides of a triangle, there is no requirement they be equal or parallel, though it is possible.	A
15.	Any curvature of the line in a d-t graph indicates a non-zero acceleration	C
16.	Motion in a straight line does not mean the speed is constant. Simple harmonic motion is a constantly changing velocity and can only occur with an acceleration. Motion in a circle requires centripetal acceleration.	E
17.	$\Sigma F = ma$; $F_T - mg = ma$; Let F_T = 50 N (the maximum possible tension) amd m = W/g = 3 kg	B
18.	The sum of the tensions in the chains (250 N + T_{left}) must support the weight of the board and the person (125 N + 500 N)	B
19.	From symmetry, each chain supports half of the weight of the board (62.5 N), The weight of the person is then split between the chains with the left chain holding 375 N – 62.5 N = 312.5 N and the right chain supporting 250 N – 62.5 N = 187.5 N or 3/5 of the tension in the left chain. This means if the person sits a distance x from the left end, they sit a distance (5/3)x from the right end. This gives x + (5/3)x = 4 m	B
20.	$\Sigma F = ma$; 10 N – (30 N cos60º) = (10 kg)a	A

21. Since the same force acts for the same time in each direction, the velocity in each direction is the same. The vector should then point at a 45º angle in the first quadrant. C

22. Consider that no part of the system is in motion, this means at each end of the rope, a person pulling with 100 N of force is reacted to with a tension in the rope of 100 N. C

23. As v is proportional to t^2 and a is proportional to $\Delta v/t$, this means a should be proportional to t E

24. $\Sigma F_y = 0 = T\sin 30^\circ - mg$ D

25. F = ma gives 20 N = (5 kg)a or an acceleration of 4 m/s^2. The 2 kg block is accelerating due to the contact force from the 3 kg block $F_{contact} = ma = (2\text{ kg})(4\text{ m/s}^2) = 8$ N. The 2 kg pushes back on the 3 kg block with a force equal in magnitude and opposite in direction. A

26. The direction of the force is the same as the direction of the acceleration, which is proportional to $\Delta v = v_f + (-v_i)$ B

27. $\Sigma F_{external} = m_{total}a$ gives $(0.90\text{ kg} \times 10\text{ m/s}^2) - (0.60\text{ kg} \times 10\text{ m/s}^2) = (1.5\text{ kg})a$ D

28. Each spring supports half of the weight, or 6 N. F = kx A

29. gravity acts downward D

30. At constant speed $\Sigma F = 0$; The forces acting parallel to the incline are F (up), F_f (down) and $mg\sin\theta$ (down), which gives $F - F_f - mg\sin\theta = 0$, where $F_f = \mu F_N = \mu mg\cos\theta$ and $\cos\theta = 4/5$ B

31. $\Sigma F = ma = F\cos\phi - f$ D

32. $f = \mu F_N$ where $F_N = mg - F\sin\theta$ E

33. The string pulling all three masses (total 6*m*) must have the largest tension. String A is only pulling the block of mass 3*m* and string B is pulling a total mass of 5*m*. C

34. At t = 2 s the force is 4 N. F = ma B

35. The upward component of the slanted cord is 300 N to balance the weight of the object. Since the slanted cord is at an angle of 45º, it has an equal horizontal component. The horizontal component of the slanted cord is equal to the tension in the horizontal cord. D

36. The normal force must point perpendicular to the surface and the weight must point down. In order to accelerate up the ramp, there must be an applied force up the ramp. If the box is accelerating up the ramp, friction acts down the ramp, opposite the motion. E

37. The normal force must point perpendicular to the surface and the weight must point down. If the box is at rest on the ramp, friction acts up the ramp, opposing the tendency to slide down C

38. The normal force must point perpendicular to the surface and the weight must point down. If the box is sliding down at constant speed, friction acts up the ramp, opposing the motion C

39. $\Sigma F_{external} = m_{total}a$ gives $(Mg) - (mg) = (M + m)a$ E

40. To keep the box from slipping, friction up the wall must balance the weight of the block, or $F_f = mg$, where $F_f = \mu F_N$ and F_N = the applied force F. This gives $\mu F = mg$ C

41. $\Sigma F_{external} = m_{total}a$ gives $(mg) - (10\text{ N}) = (m + 1\text{ kg})(5\text{ m/s}^2)$ A

42. Friction opposes the motion of the block and therefore points to the left. The normal force is found from $\Sigma F_y = 0 = F_N - mg - F\sin\theta$ and the force of friction $F_f = \mu F_N$ E

43. When an object exerts a force **on a second object**, the second object exerts an equal and opposite force back **on the first object**. C

44. Since P is at an upward angle, the normal force is decreased as P supports some of the weight. Since a component of P balances the frictional force, P itself must be larger than f. A

45. Newton's 2nd law applied to an object sliding to rest gives $\Sigma F = -F_f = -\mu F_N = ma$. On a horizontal surface, $F_N = mg$ and we have $-\mu mg = ma$, or $a = -\mu g$. Use this acceleration with $v_f^2 = v_i^2 + 2ad$. D

46. $F = ma = m\Delta v/t$ C

47. $\Sigma F = ma$; $F_{cable} - mg = ma = m(-2\ m/s^2)$ C

48. The force of friction = $\mu F_N = 0.2 \times 10\ kg \times 9.8\ m/s^2 = 19.6\ N$, which is greater than the applied force, which means the object is accelerating to the left, or slowing down A

49. F = ma gives 36 N = (24 kg)a or an acceleration of 1.5 m/s^2. The 20 kg block is accelerating due to the contact force from the 4 kg block $F_{contact} = ma = (20\ kg)(1.5\ m/s^2) = 30\ N$. D

50. The upward component of the tension is $T_{up} = T\sin\theta$, where θ is the angle to the horizontal. This gives $T = T_{up}/\sin\theta$. Since the upward components are all equal to one half the weight, the rope at the smallest angle (and the smallest value of sinθ) will have the greatest tension, and most likely break B

51. $\Sigma F_{external} = m_{total}a$ gives $(3.0\ kg \times 10\ m/s^2) - (1.5\ kg \times 10\ m/s^2) = (4.5\ kg)a$ C

52. From the 1 kg block: F = ma giving a = 2 m/s^2. For the system: F = (4 kg)(2 m/s^2) E

53. For three forces in equilibrium, any one of the forces is equal and opposite to the resultant of the other two forces. D

54. Elevator physics: F_N represents the scale reading. $\Sigma F = ma$; $F_N - mg = ma$, or $F_N = m(g + a)$. The velocity of the elevator is irrelevant. B

55. F = ma, if F is doubled, a is doubled. If m is halved, a will be doubled. D

56. Newton's third law C

57. F = ma gives 24 N = (12 kg)a or an acceleration of 2 m/s^2. The 3 kg block is accelerating due to the tension in the rope $F_T = ma = (3\ kg)(2\ m/s^2) = 6\ N$. B

58. Inertia is mass E

59. The normal force is mgcosθ. For a horizontal surface, $F_N = mg$. At any angle $F_N < mg$ and F_f is proportional to F_N. C

60. F = ma, where m = W/g = 10 kg B

61. Newton's 2nd law applied to an object sliding to rest gives $\Sigma F = -F_f = -\mu F_N = ma$. On a horizontal surface, $F_N = mg$ and we have $-\mu mg = ma$, or $a = -\mu g$. Using this acceleration with $v_f^2 = v_i^2 + 2ad$ gives $d = v_i^2/2\mu g$. There is no dependence on mass. B

62. Newton's 2nd law applied to an object sliding to rest gives $\Sigma F = -F_f = -\mu F_N = ma$. On a horizontal surface, $F_N = mg$ and we have $-\mu mg = ma$, or $a = -\mu g$. Using this acceleration with $v_f^2 = v_i^2 + 2ad$ gives $d = v_i^2/2\mu g$. d is proportional to v_i^2 E

63. F = ma and $v_f^2 = 0\ m/s = v_i^2 + 2ad$ B

64. The normal force on an incline is mgcosθ. The component of gravity acting down the incline is mgsinθ. The coefficient of friction is minimized when static friction is at its maximum value, or $\mu_s F_N$. Keeping the block at rest requires $mg\sin\theta = F_f = \mu mg\cos\theta$, or $\mu = \tan\theta$ C

65. The maximum resultant possible from the sum of any two vectors is the sum of the magnitudes. The minimum resultant possible is the difference between the magnitudes. Forces of 6 N and 10 N produce a maximum resultant of 16 N and a minimum of 4 N. — C

66. An apple is approximately 100 g. It is important to have a sense of basic values of measurement. — D

67. $\Sigma F = ma$. The component of gravity acting down the incline is $mg\sin\theta$, which gives $a = g\sin\theta$ — C

68. $\Sigma F = ma$; $mg\sin\theta - F_f = ma$ — B

69. Slope = $\Delta y/\Delta x$ = Weight/mass = acceleration due to gravity — D

70. Newton's second law applied to m_1: $T = m_1a$, or $a = T/m_1$, substitute this into Newton's second law for the hanging mass: $m_2g - T = m_2a$ — E

71. $\Sigma F_y = 0$ gives $F_N + (250\text{ N} \sin 30^\circ) - mg = 0$, or $F_N = 365$ N. To move at constant speed, the force of friction must balance the horizontal component of the applied force $F\cos\theta = 216.5\text{ N} = \mu F_N$ — D

72. String B is pulling both masses so $F_B = (6\text{ kg})(12\text{ m/s}^2)$ — A

73. String A is only pulling the 4 kg mass so $F_A = (4\text{kg})(12\text{ m/s}^2)$ — B

74. $F_{net} = ma$ — C

75. Elevator physics: R represents the scale reading. $\Sigma F = ma$; $R - mg = ma$, or $R = m(g + a)$. This ranks the value of R from largest to smallest as accelerating upward, constant speed, accelerating downward — A

76. $\Sigma F = ma$ for the whole system gives $F - \mu(3m)g = (3m)a$ and solving for a gives $a = (F - 3\mu mg)/3m$. For the top block, $F_m = ma = m[(F - 3\mu mg)/3m]$ — A

77. $m \times a = \text{kg} \times \text{m/s}^2$ — D

78. The normal force comes from the perpendicular component of the applied force which is $F\cos\theta = 50$ N. The maximum value of static friction is then $\mu F_N = 25$ N. The upward component of the applied force is $F\sin\theta = 87$ N. $\Sigma F_y = F_{up} - mg = 87\text{ N} - 60\text{ N} > 25$ N. Since the net force on the block is great than static friction can hold, the block will begin moving up the wall. Since it is in motion, kinetic friction is acting opposite the direction of the block's motion — D

79. Since P is at a downward angle, the normal force is increased. Since a component of P balances the frictional force, P itself must be larger than f. — A

80. Since the force is applied horizontally, the mass has no effect. — C

81. Newton's third law — E

82. If they are not moving, the net force must be zero. While the book and crate are pushing each other apart, there is friction from the table pointing inward against each object on the table to keep them at rest. — C

83. The only force in the direction of the crate's acceleration is the force of friction from the sleigh — B

84. Elevator physics: F_N represents the scale reading. $\Sigma F = ma$; $F_N - mg = ma$, or $F_N = m(g + a)$. When $F_N > mg$, the elevator is accelerating upward (a is positive) — A

85. Changing direction (choices A and C (the astronaut is still orbiting the earth!)) cannot occur with a zero net force. Choices B and D represent accelerated motion. — E

86.	Given that the box accelerates toward Ted, Ted's force must be greater than Mario's force plus the force of friction. Since Mario's force is ½ of Ted's force, the force of frction must be less than half of Ted's force.	A
87.	For a Newton's third law pair, just switch the nouns.	C
88.	The component of gravity acting down the incline (+x) is $mg\sin\theta$ and the component perpendicularly intothe incline (–y) is $mg\cos\theta$. 36.9º indicates a 3-4-5 triangle.	A
89.	$\Sigma F = ma$; $F - mg = m(5g)$ or $F = 6mg$	A
90.	$\Sigma F_y = F\sin\theta + F_N - mg = 0$, which gives $F_N = 170$ N. The force of friction is equal to the horizontal component of the force applied by the student which is $F\cos\theta = 86.6$N. $F_f = \mu F_N$	C
91.	constant speed means $F_{net} = 0$ N	A
92.	As the initial and final velocities and the displacement are given, as well as an indication that the acceleration is constant, this is merely a kinematics problem. $v_f^2 = v_i^2 + 2ad$	E
93.	The maximum value of static friction in this case is $\mu_s F_N = 120$ N. Since the person is pushing with only 60 N of force, the box remains at rest.	C
94.	Between the lower block and the tabletop, there is a force of friction to the left of maximum magnitude $\mu(2W)$ as both blocks are pushing down on the tabletop. There is also a force of friction acting to the left on the upper surface of the lower block due to the upper block of maximum magnitude μW. The total maximum static frictional force holding the lower block in place is therefore $\mu(2W) + \mu W$	E
95.	The normal force on the block can be found from $\Sigma F_y = 0 = F_N - mg\cos\theta - F$. The force of friction necessary to hold the block in place is $mg\sin\theta$. Setting the force of friction equal to $mg\sin\theta$ gives $\mu F_N = mg\sin\theta = \mu(F + mg\cos\theta)$	E
96.	In equilibrium, $mg = kx$ and the equilibrium position $x = mg/k$. In an accelerating elevator, we can just adjust gravity to its effective value $g_{eff} = g + a$, thus making the new equilibrium position mg_{eff}/k	C
97.	This is a tricky one. In order to move the car forward, the rear tires roll back against the ground, the force of friction pushing forward on the rear tires. The front tires, however, are not trying to roll on their own, rather they begin rolling due to the friction acting backward, increasing their rate of rotation	A
98.	Gravity is still the only force acting vertically so we can find the total time in the air from kinematics: $v_y = 0$ at the top $= v_0\sin\theta - gt$ giving t (to the top) $= v_0\sin\theta/g$ and the total time is twice the time to the top, or $2v_0\sin\theta/g$. In this time, the ball is also accelerating horizontally (think of it as a "sideways" gravity) and in this time, should return to its starting location. Using $x = 0 = (v_0\cos\theta)t + ½ at^2$, where $a = F_0/m$ and t is found above, we can solve for θ	B
99.	The external forces acting on the system of masses are the weights of block 1 (pulling the system to the left), the weight of block 3 (pulling the system to the right) and the force of friction on block 2 (pulling the system to the left with a magnitude $\mu F_N = \mu m_2 g$) $\Sigma F_{external} = m_{total}a$ gives $(m_1 g - \mu m_2 g - m_3 g) = (m_1 + m_2 + m_3)a$	E
100.	$F = ma$ gives 30 N = (12 kg)a or an acceleration of 2.5 m/s². The 5 kg block is accelerating due to the tension in the rope $F_T = ma = (5\text{ kg})(2.5\text{ m/s}^2) = 12.5$ N.	C
101.	$\Sigma F_{external} = m_{total}a$ gives $(5.0\text{ kg} \times 10\text{ m/s}^2) - (3\text{ kg} \times 10\text{ m/s}^2) = (8\text{ kg})a$	E
102.	As they are all at the same position after 8 seconds, they all have the same average velocity	D

103. Car A decelerates with the same magnitude that C accelerates. Car B is moving at constant speed, which means $F_B = 0$. B

104. When falling with terminal velocity, the force of air resistance equals your weight, regardless of the speed. E

105. For each case, $\Sigma F_{external} = m_{total}a$ gives $Mg - mg = (M + m)a$, or $a = \frac{M - m}{M + m} g$. A

106. The two ends of the light string must have the same tension, eliminating choices A, C and D. If choice E was correct, both masses would be accelerating downward and T_A must be greater than the weight of block A. B

107. If F = ma, then m = F/a. For the second object m′ = 2F/5a = 2/5(F/a) = (2/5)m A

108. $\Sigma F_{external} = m_{total}a$ gives $(M + m)g - Mg = (2M + m)a$ A

109. As the entire system moves as one, F = (3m)a, or a = F/(3m). The force of friction acting on block 1 is the force moving block 1 and we have $\mu mg = m(F/(3m))$ E

110. $F = ma = m\Delta v/t$ B

111. This is really no different than any other incline problem. The normal force on an incline with no other forces acting into the incline is $mg\cos\theta$ E

112. Since the system is moving at constant velocity, m_1 is pushing m_2 and m_3 with a force equal to the force of friction acting on those two blocks, which is $\mu(F_{N2} + F_{N3})$ A

113. $\Sigma F_{external} = m_{total}a$ gives $(5\text{ kg} \times 9.8\text{ m/s}^2) - F_f = (10\text{ kg})a$, where F_f is the force of friction acting on the 5 kg block on the table: $\mu mg = 0.2 \times 5\text{ kg} \times 9.8\text{ m/s}^2 = 9.8\text{ N}$ D

SECTION B – Circular Motion

1.	Newton's third law	B
2.	F = mv^2/r; $v = \sqrt{\frac{Fr}{m}}$; all other variables being constant, if r is quadrupled, v is doubled	D
3.	With acceleration south the car is at the top (north side) of the track as the acceleration points toward the center of the circular track. Moving east indicates the car is travelling clockwise. The magnitude of the acceleration is found from $a = v^2/r$	A
4.	The frictional force acts as the centripetal force (toward the center)	D
5.	Acceleration occurs when an object is changing speed and/or direction	D
6.	Velocity is tangential, acceleration points toward the center of the circular path	B
7.	To move in a circle, a force directed toward the center of the circle is required. While the package slides to the right in the car, it is actually moving in its original straight line path while the car turns from under it.	E
8.	$a = v^2/r$ and $v = 2\pi r/T$ giving $a = 4\pi^2 r/T^2$	B
9.	Once projected, the ball is no longer subject to a force and will travel in a straight line with a component of its velocity tangent to the circular path and a component outward due to the spring	E
10.	There is a normal force directed upward and a centripetal force directed inward.	D
11.	$a = v^2/r$ where $v = 2\pi rf$ and f = 2.0 rev/sec	D
12.	At Q the ball is in circular motion and the acceleration should point to the center of the circle. At R, the ball comes to rest and is subject to gravity as in free-fall.	C
13.	The net force and the acceleration must point in the same direction. Velocity points tangent to the objects path.	D
14.	The centripetal force is provided by the spring where $F_C = F_s = kx$	B
15.	In the straight sections there is no acceleration, in the circular sections, there is a centripetal acceleration	B
16.	Once the stone is stuck, it is moving in circular motion. At the bottom of the circle, the acceleration points toward the center of the circle at that point.	A
17.	Feeling weightless is when the normal force goes to zero, which in only possible going over the top of the hill where mg (inward) – F_N (outward) = mv^2/R. Setting F_N to zero gives a maximum speed of $\sqrt{gR}$	A
18.	Centripetal force points toward the center of the circle	B
19.	While speed may be constant, the changing direction means velocity cannot be constant as velocity is a vector	B
20.	$F = mv^2/r$. $F_{new} = (2m)(2v)^2/(2r) = 4(mv^2/r) = 4F$	D
21.	Assuming the track is circular at the bottom, the acceleration points toward the center of the circular path	A
22.	Average speed = (total *distance*)/(total time). Lowest average speed is the car that covered the	C

least distance

23. As all the cars are changing direction, there must be a net force to change the direction of their velocity vectors — D

24. $F = mv^2/r$; $v^2 = rF/m$, if r decreases, v will decrease with the same applied force. Also, $v = 2\pi r f$ so $4\pi^2 r^2 f = rF/m$, or $f = F/(4\pi^2 rm)$ and as r decreases, f increases. — D

25. f = 4 rev/sec. $a = v^2/r$ and $v = 2\pi rf$ — D

26. $F = mv^2/r$ — D

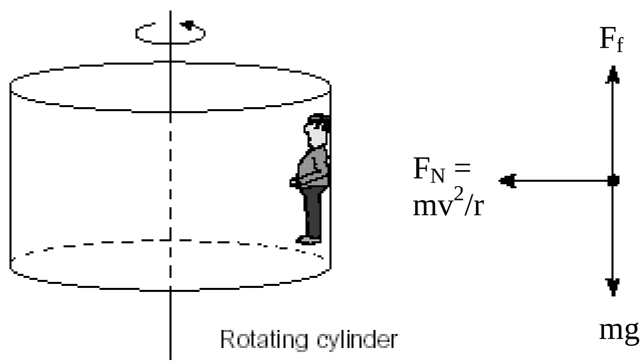
27. There is a force acting downward (gravity) and a centripetal force acting toward the center of the circle (up and to the right). Adding these vectors cannot produce resultants in the directions of B, C, D or E. — A

28. $\Sigma F = ma$; $mg + F_T = mv^2/r$ giving $F_T = mv^2/r - mg$ — B

29. At the top of the circle, $\Sigma F = F_T + mg = mv^2/R$, giving $F_T = mv^2/R - mg$. At the bottom of the circle, $\Sigma F = F_T - mg = mv^2/R$, giving $F_T = mv^2/R + mg$ The difference is $(mv^2/R + mg) - (mv^2/R - mg)$ — B

30. At the bottom of the swing, $\Sigma F = F_T - mg = ma_c$; since the tension is 1.5 times the weight of the object we can write $1.5mg - mg = ma_c$, giving $0.5mg = ma_c$ — B

31. — C



$F_f = mg$ to balance

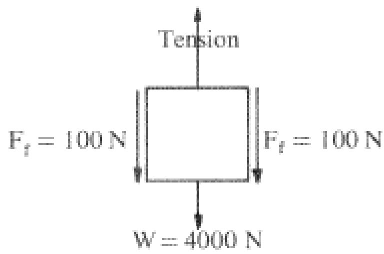
$\mu F_N = \mu mv^2/r = mg$, where $v = 2\pi rf$ which gives $\mu = g/(4\pi^2 rf^2)$

Be careful! f is given in rev/min (45 rev/min = 0.75 rev/sec) and 8.0 m is the ride's diameter

SECTION A – Linear Dynamics

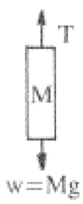
1976B1

a.



b. $\Sigma F = ma$; $T – W – 2F_f = 800$ N; $T = 5000$ N

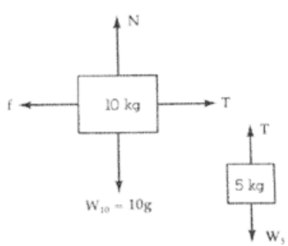
c. Looking at the FBD for the counterweight we have $\Sigma F = ma$; $Mg – T = Ma$
$M = T/(g – a)$ where $T = 5000$ N gives $M = 625$ kg



1979B2

a. $\Sigma F = ma$; 50 N – f = ma where $f = \mu N$ and $N = mg$ gives $50\text{ N} – \mu mg = ma$; $a = 3\text{ m/s}^2$

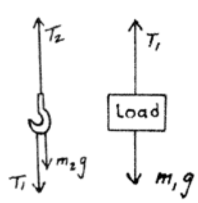
b.



c. $\Sigma F = ma$ for each block gives $W_5 – T = m_5a$ and $T – f = m_{10}a$. Adding the two equations gives $W_5 – f = (m_5 + m_{10})a$, or $a = 2\text{ m/s}^2$

1982B2

a.

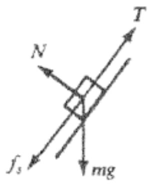


b. T_1 is in internal system force and will cancel in combined equations. Using $\Sigma F_{external} = m_{total}a$ gives $T_2 – m_1g – m_2g = (m_1 + m_2)a$, solving yields $T_2 = 6600$ N. Now using $\Sigma F = ma$ for the load gives $T_1 – m_1g = m_1a$ and $T_1 = 6000$ N

1985B2

a. Note that the system is at rest. The only forces on the hanging block are gravity and the tension in the rope, which means the tension must equal the weight of the hanging block, or 100 N. You cannot use the block on the incline because friction is acting on that block and the amount of friction is unknown.

b.

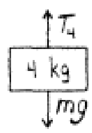


c. $\Sigma F = 0$; $f_s + mg \sin\theta - T = 0$ gives $f_s = 13$ N

1986B1

a. $\Sigma F_{external} = m_{total}a$; $m_4g - m_1g - m_2g = (m_4 + m_2 + m_1)a$ gives $a = 1.4$ m/s^2

b. For the 4 kg block:
$\Sigma F = ma$
$mg - T_4 = ma$

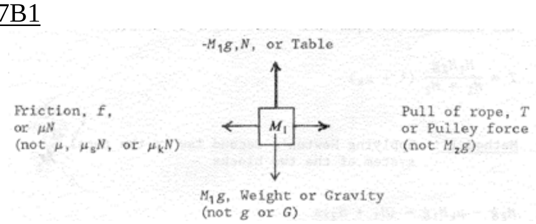


gives
$T_4 = 33.6$ N

c. Similarly for the 1 kg block: $T_1 - mg = ma$ gives $T_1 = 11.2$ N

1987B1

a.

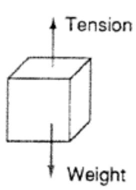


b. $\Sigma F_{ext} = m_{tot}a$; Where the maximum force of static friction on mass M_1 is μ_sN and $N = M_1g$; $M2g - \mu_sM_1g = 0$ gives $\mu_s = M_2/M_1$

c/d. $\Sigma F_{ext} = m_{tot}a$ where we now have kinetic friction acting gives $M_2g - \mu_kM_1g = (M_1 + M_2)a$
so $a = (M_2g - \mu_kM_1g)/(M_1 + M_2)$
$\Sigma F = ma$ for the hanging block gives $M_2g - T = M_2a$ and substituting a from above gives $T = \frac{M_1M_2g}{M_1+M_2}(1 + \mu_k)$

1988B1

a.



b. $\Sigma F = ma$ gives $T - mg = ma$ and $T = 1050$ N

c. The helicopter and the package have the same initial velocity, 30 m/s upward. Use $d = v_it + ½ at^2$
$d_h = (+30 \text{ m/s})t + ½ (+5.2 \text{ m/s}^2)t^2$ and $d_p = (+30 \text{ m/s})t + ½ (-9.8 \text{ m/s}^2)t^2$.
The difference between d_h and d_p is 30 m, but they began 5 m apart so the total distance is 35 m.

1998B1

a. $\Sigma F_{ext} = m_{tot}a$ gives mg = 2ma, or a = g/2

b. $d = v_0t + \frac{1}{2}at^2$; $h = 0 + \frac{1}{2}(g/2)t^2$ gives $t = 2\sqrt{\frac{h}{g}}$

c. Block A accelerates across the table with an acceleration equal to block B (g/2).

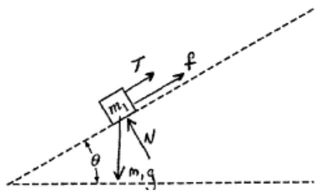
d. Block A is still in motion, but with no more applied force, Block A will move at constant speed across the table.

e. Since block B falls straight to the floor and stops, the distance between the landing points is equal to the horizontal distance block A lands from the edge of the table. The speed with which block A leaves the tabletop is the speed with which block B landed, which is found from $v = v_0 + at = \frac{g}{2}\left(2\sqrt{\frac{h}{g}}\right) = \sqrt{hg}$ and the time for block A to reach the floor is found from $2h = \frac{1}{2}gt^2$, which gives $t = 2\sqrt{\frac{h}{g}}$.

The distance is now $d = vt = \sqrt{hg} \times 2\sqrt{\frac{h}{g}} = 2h$

2000B2

a.



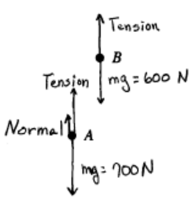
b. $f = \mu N$ where $N = m_1 g \cos\theta$ gives $\mu = \frac{f}{m_1 g \cos\theta}$

c. constant velocity means $\Sigma F = 0$ where $\Sigma F_{external} = m_1 g \sin\theta + m_2 g \sin\theta - f - 2f - Mg = 0$
solving for M gives $M = (m_1 + m_2)\sin\theta - (3f)/g$

d. Applying Newton's second law to block 1 gives $\Sigma F = m_1 g \sin\theta - f = m_1 a$ which gives $a = g\sin\theta - f/m_1$

2003B1

a.



b. The tension in the rope is equal to the weight of student B: $T = m_B g = 600$ N
$\Sigma F_A = T + N - m_A g = 0$ gives N = 100 N

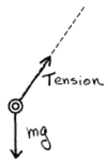
c. For the climbing student $\Sigma F = ma$; $T - m_B g = m_B a$ gives T = 615 N

d. For student A to be pulled off the floor, the tension must exceed the weight of the student, 700 N. No, the student is not pulled off the floor.

e. Applying Newton's second law to student B with a tension of 700 N gives $\Sigma F = T - m_B g = m_B a$ and solving gives $a = 1.67\ m/s^2$

2003Bb1

a.



b. We can find the acceleration from $a = \Delta v/t = 2.17\ m/s^2$ and use $d = ½\ at^2$ to find $d = 975\ m$

c. The x and y components of the tension are $T_x = T \sin\theta$ and $T_y = T \cos\theta$ (this is using the angle to the vertical)
Relating these to the other variables gives $T \sin\theta = ma$ and $T \cos\theta = mg$.
Dividing the two equations gives $\tan\theta = a/g = (2.17\ m/s^2)/(9.8\ m/s^2)$ and $\theta = 12.5º$

1996B2

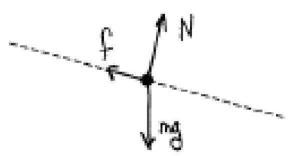
a. There are other methods, but answers are restricted to those taught to this point in the year.
   i. A device to measure distance and a calibrated mass or force scale or sensor
   ii. Hang the mass from the bottom of the spring and measure the spring extension (Δx) or pull on the spring with a known force and measure the resulting extension.
   iii. Use hooke's law with the known force or weight of the known mass $F = k\Delta x$ or $mg = k\Delta x$ and solve for k

b. Many methods are correct, for example, place the object held by the scale on an inclined plane and find the weight using $W\sin\theta = k\Delta x$. One could similarly use a pulley system to reduce the effort applied by the spring scale.

2007B1

a. $x = vt$ gives $t = (21\ m)/(2.4\ m/s) = 8.75\ s$

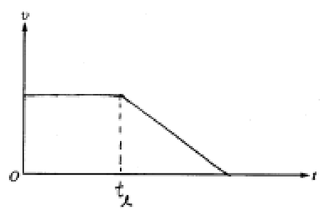
b.



c. $\Sigma F = 0$ if the sled moves at constant speed. This gives $mg \sin\theta - f = 0$, or $f = mg \sin\theta = 63.4\ N$

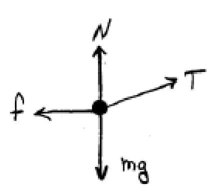
d. $f = \mu N$ where $N = mg \cos\theta$ so $\mu = f/N = (mg \sin\theta)/(mg \cos\theta) = \tan\theta = 0.27$

e. i. The velocity of the sled decreases while its acceleration remains constant
   ii.



2007B1B

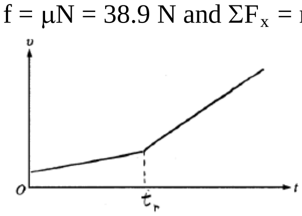
a.



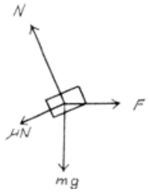
b. $\Sigma F_y = 0$; $N + T \sin\theta - mg = 0$ gives $N = mg - T \sin\theta = 177\ N$

c. $f = \mu N = 38.9\ N$ and $\Sigma F_x = ma$; $T \cos\theta - f = ma$ yields $a = 0.64\ m/s^2$

d.



1981M1

a. 

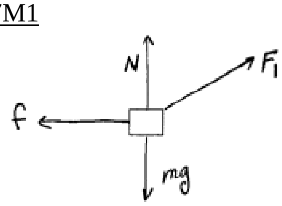
b. F can be resolved into two components: F sin θ acting into the incline and F cos θ acting up the incline. The normal force is then calculated with ΣF = 0; N – F sin θ – mg cos θ = 0 and f = μN
Putting this together gives ΣF = ma; F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = ma, solve for a

c. for constant velocity, a = 0 in the above equation becomes F cos θ – mg sin θ – μ(F sin θ + mg cos θ) = 0 solving for F gives $F = mg\left(\frac{\mu \cos\theta + \sin\theta}{\cos\theta - \mu\sin\theta}\right)$ In order that F remain positive (acting to the right), the denominator must remain positive. That is cos θ > μ sin θ, or tan θ < 1/μ

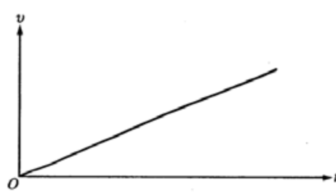
1986M1

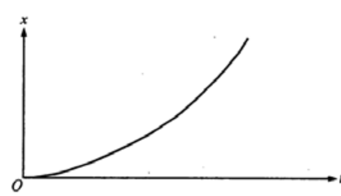
a. Combining the person and the platform into one object, held up by two sides of the rope we have ΣF = ma; 2T = (80 kg + 20 kg)g giving T = 500 N
b. Similarly, ΣF = ma; 2T – 1000 N = (100 kg)(2 m/s^2) giving T = 600 N
c. For the person only: ΣF = ma; N + 600 N – mg = ma gives N = 360 N

2007M1

a. 

b. $\Sigma F_y = 0$; N + F_1 sin θ – mg = 0 gives N = mg – F_1 sin θ
c. ΣF_x = ma; F_1 cos θ – μN = ma_1. Substituting N from above gives μ = (F_1 cos θ – ma_1)/(mg – F_1 sin θ)
d.





e. The condition for the block losing contact is when the normal force goes to zero, which means friction is zero as well. $\Sigma F_x = F_{max}$ cos θ = ma_{max} and $\Sigma F_y = F_{max}$ sin θ – mg = 0 giving F_{max} = mg/(sin θ) and a_{max} = (F_{max} cos θ)/m which results in a_{max} = g cot θ

1996M2

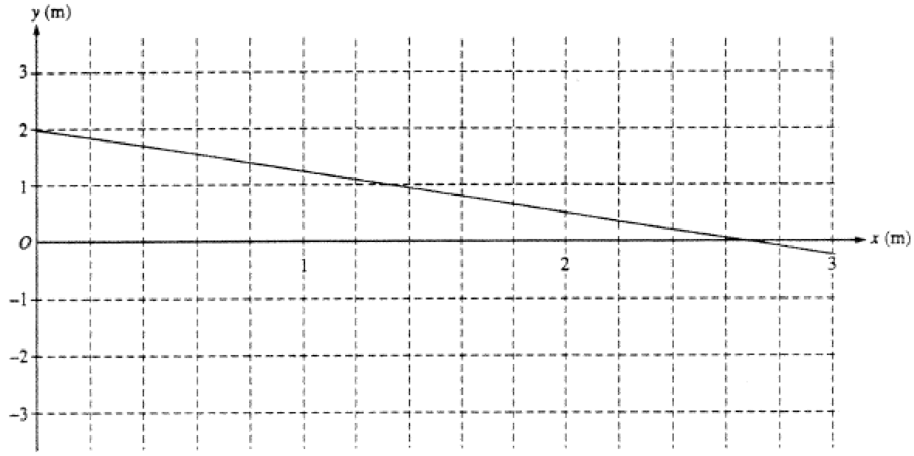
a. $\Sigma F = ma$; using downward as the positive direction, $mg - N = ma_y$ gives $N = m(g - a_y) = 2490$ N

b. Friction is the only horizontal force exerted; $\Sigma F = f = ma_x = 600$ N

c. At the minimum coefficient of friction, static friction will be at its maximum value $f = \mu N$, giving $\mu = f/N = (600\text{ N})/(2490\text{ N}) = 0.24$

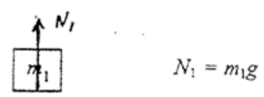
d. $y = y_0 + v_{0y}t + \frac{1}{2} a_y t^2 = 2\text{ m} + \frac{1}{2}(-1.5\text{ m/s}^2)t^2$ and $x = x_0 + v_{0x}t + \frac{1}{2} a_x t^2 = \frac{1}{2}(2\text{ m/s}^2)t^2$, solving for t^2 in the x equation gives $t^2 = x$. Substituting into the y equation gives y as a function of x: $y = 2 - 0.75x$

e.



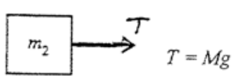
1998M3

a. i.

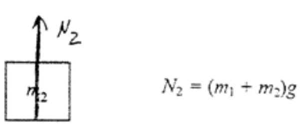


ii.

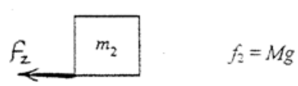
iii.



iv.

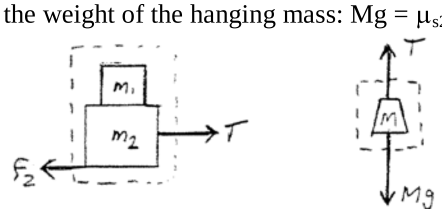


v.



b. The maximum friction force on the blocks on the table is $f_{2max} = \mu_{s2}N_2 = \mu_{s2}(m_1 + m_2)g$ which is balanced by the weight of the hanging mass: $Mg = \mu_{s2}(m_1 + m_2)g$ giving $M = \mu_{s2}(m_1 + m_2)$

c.

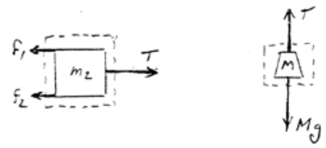


For the hanging block: $Mg - T = Ma$; For the two blocks on the plane: $T - f_2 = (m_1 + m_2)a$

Combining these equations (by adding them to eliminate T) and solving for a gives $a = \left[\frac{M - \mu_{k2}(m_1 + m_2)}{M + m_1 + m_2}\right] g$

d. i. $f_1 = \mu_{k1} m_1 g = m_1 a_1$ giving $a_1 = \mu_{k1} g$

ii.



For the two blocks: $Mg - T = Ma_2$ and $T - f_1 - f_2 = m_2 a_2$. Eliminating T and substituting values for friction gives $a_2 = \left[\frac{M - \mu_{k1} m_1 - \mu_{k2}(m_1 + m_2)}{M + m_2}\right] g$

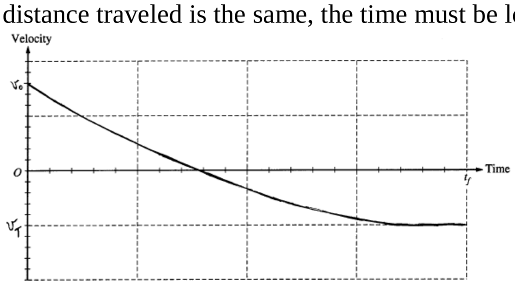
<u>2005M1</u>

a. The magnitude of the acceleration decreases as the ball moves upward. Since the velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

b. At terminal speed $\Sigma F = 0$. $\Sigma F = -Mg + kv_T$ giving $v_T = Mg/k$

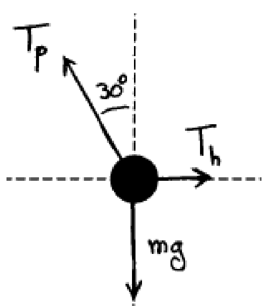
c. It takes longer for the ball to fall. Friction is acting on the ball on the way up and on the way down, where it begins from rest. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.

d.



<u>2005B2.</u>

(a)

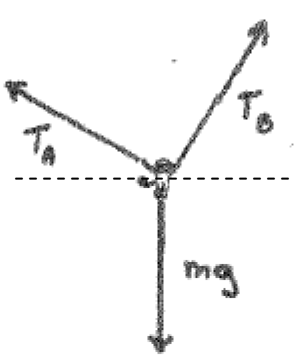


(b) Apply

$F_{net(X)} = 0$	$F_{net(Y)} = 0$
$T_P \cos 30 = mg$	$T_P \sin 30 = T_H$
$T_P = 20.37$ N	$T_H = 10.18$ N

<u>1991B1.</u>

a)



(b) SIMULTANEOUS EQUATIONS

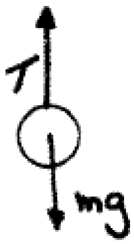
$F_{net(X)} = 0$	$F_{net(Y)} = 0$
$T_a \cos 30 = T_b \cos 60$	$T_a \sin 30 + T_b \sin 60 - mg = 0$

…. Solve above for T_b and plug into Fnet(y) eqn and solve

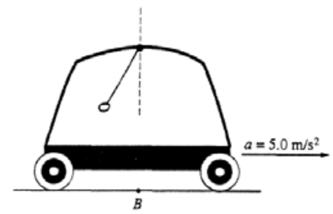
$T_a = 24$ N $\qquad T_b = 42$ N

1995B3

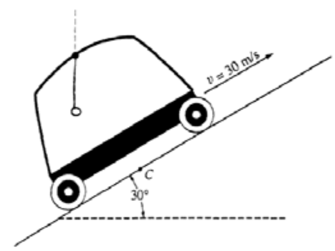
a) i)



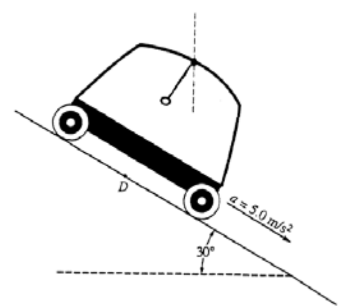
ii) T = mg = 1 N



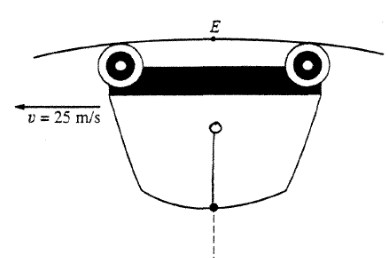
b) The horizontal component of the tension supplies the horizontal acceleration.
$T_h = ma = 0.5$ N
The vertical component of the tension is equal to the weight of the ball, as in (a) ii. $T_v = 1$ N



c) Since there is no acceleration, the sum of the forces must be zero, so the tension is equal and opposite to the weight of the ball. T_h =zero, T_v = 1N



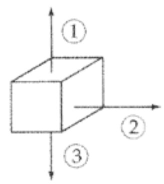
d) The horizontal component of the tension is responsible for the horizontal component of the acceleration. Applying Newton's second law:
$T_h = ma \cos \theta$, where θ is the angle between the acceleration and horizontal
$T_h = (0.10 \text{ kg})(5.0 \text{ m/s}^2) \cos 30°$, $T_h = 0.43$ N
The vertical component of the tension counteracts only part of the gravitational force, resulting in a vertical component of the acceleration. Applying Newton's second law. $Tv = mg – ma \sin \theta$
$Tv = (0.10 \text{ kg})(10 \text{ m/s}^2) – (0.10 \text{ kg})(5.0 \text{ m/s}^2) \sin 30°$, T_v =0.75 N



e) Since there is no horizontal acceleration, there is no horizontal component of the tension. T_h = zero
Assuming for the moment that the string is hanging downward, the centripetal is the difference between the gravitational force and the tension. Applying Newton's second law.
$mv^2/r = mg – T_v$, Solving for the vertical component of tension:
$T_v = – 1.5$ N i.e. the string is actually pulling down on the ball.

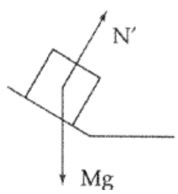
SECTION B – Circular Motion

1977B2

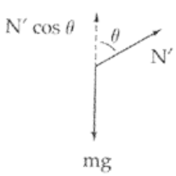
a. 

1 = normal force; 2 = friction; 3 = weight

b. Friction, $f \le \mu N$ where $N = Mg$. Friction provides the necessary centripetal force so we have $f = Mv^2/R$
$Mv^2/R \le \mu Mg$, or $\mu \ge v^2/Rg$

c. 

d. from the diagram below, a component of the normal force N′ balances gravity so N′ must be greater than mg



1984B1

a. At the top of the path, tension and gravity apply forces downward, toward the center of the circle.
$\Sigma F = T + mg = 2Mg + Mg = 3Mg$

b. In the circular path, $F = mv^2/r$ which gives $3Mg = mv_0^2/L$ and $v_0 = \sqrt{3Lg}$

c. The ball is moving horizontally ($v_{0y} = 0$) from a height of 2L so this gives $2L = ½ gt^2$ or $t = 2\sqrt{L/g}$

d. $x = v_0 t = \sqrt{3Lg} \times 2\sqrt{L/g} = 2\sqrt{3}L$

1989B1

a. i. $v_{iy} = 0$ so we have $h = ½ gt^2$ which gives $t = \sqrt{\frac{2h}{g}}$

ii. $x = v_0 t = v_0\sqrt{\frac{2h}{g}}$

iii. $v_x = v_0$; $v_y = v_{iy} + gt = \sqrt{2gh}$
$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{{v_0}^2 + 2gh}$

b. 

c. Horizontal forces: $T \cos\theta = Mv_0^2/R$; Vertical forces: $T \sin\theta = Mg$. Squaring and adding the equations gives
$T = M\sqrt{g^2 + \frac{{v_0}^4}{R^2}}$

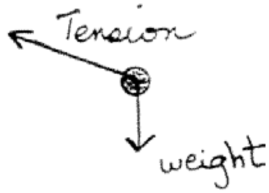
1997B2

a. The circumference of the path, d, can be calculated from the given radius. Use the timer to obtain the period of revolution, t, by timing a number of revolutions and dividing the total time by that number of revolutions. Calculate the speed using $v = d/t$.

b. If the cord is horizontal, $T = mv^2/r = 5.5$ N

c. (5.5 N – 5.8 N)/(5.8 N) × 100 = –5.2%

d. i.

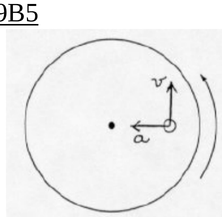


ii. The cord cannot be horizontal because the tension must have a vertical component to balance the weight of the ball.

iii. Resolving tension into components gives $T \sin\theta = mg$ and $T \cos\theta = mv^2/r$ which gives $\theta = 21^\circ$

1999B5

a.



b. v = circumference/period = $2\pi R/T = 2\pi(0.14 \text{ m})/(1.5 \text{ s}) = 0.6$ m/s

c. The coin will slip when static friction has reached its maximum value of $\mu_s N = \mu_s mg = mv^2/r$ which gives $v = \sqrt{\mu_s g r} = 0.83$ m/s

d. It would not affect the answer to part (c) as the mass cancelled out of the equation for the speed of the coin.

2001B1

a.

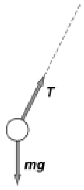
b. The minimum speed occurs when gravity alone supplies the necessary centripetal force at the top of the circle (i.e. tension is zero and is not required). Therefore we have $Mg = Mv_{min}^2/R$ which gives $v_{min} = \sqrt{Rg}$

c. At the bottom of the swing $\Sigma F = ma$ becomes $T - Mg = Mv^2/R$ which gives $T_{max} - Mg = Mv_{max}^2/R$ and solving for v_{max} gives $v_{max} = \sqrt{\frac{R}{M}(T_{max} - Mg)}$

d. At point P the ball is moving straight up. If the string breaks at that point, the ball would continue to move straight up, slowing down until it reaches a maximum height and fall straight back to the ground.

2002B2B

a.



b. $\Sigma F_y = 0$; $T \cos\theta - mg = 0$ gives $m = (T\cos\theta)/g$

c. The centripetal force is supplied by the horizontal component of the tension: $F_C = T\sin\theta = mv^2/r$. Substituting the value of m found in part b. and the radius as ($l \sin\theta$) gives $v = \sqrt{gl\sin\theta\tan\theta}$

d. substituting the answer above into $v = 2\pi rf$ gives $f = \frac{1}{2\pi}\sqrt{\frac{g}{l\cos\theta}}$

e. The initial velocity of the ball is horizontal and the subsequent trajectory is parabolic.

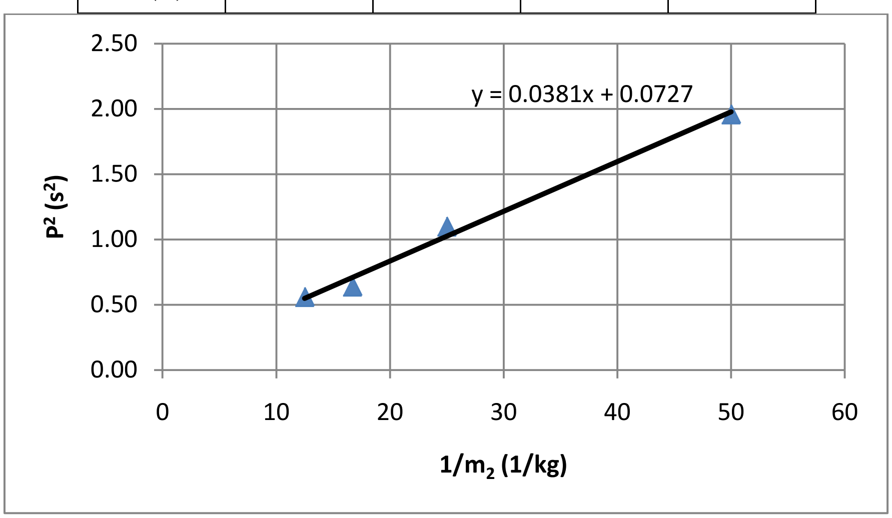
2009B1B

a. The centripetal force is provided by the weight of the hanging mass: $F_C = m_2 g = m_1 v^2/r$ and v is related to the period of the motion $v = 2\pi r/P$. This gives $m_2 g = \frac{m_1 v^2}{r} = \frac{m_1}{r}\frac{4\pi^2 r^2}{P^2}$ and thus $P^2 = 4\pi^2\left(\frac{m_1 r}{m_2 g}\right)$

b. The quantities that may be graphed to give a straight line are P^2 and $1/m_2$, which will yield a straight line with a slope of $4\pi^2\left(\frac{m_1 r}{g}\right)$

c.

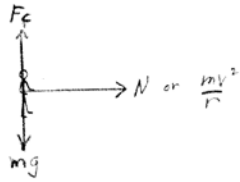
$1/m_2$ (kg^{-1})	50	25	16.7	12.5
m_2 (kg)	0.020	0.040	0.060	0.080
P (s)	1.40	1.05	0.80	0.75
P^2 (s^2)	1.96	1.10	0.64	0.56



d. Using the slope of the line (0.038 kg/s^2) in the equation from part b. gives $g = 9.97\ m/s^2$

1984M1

a.



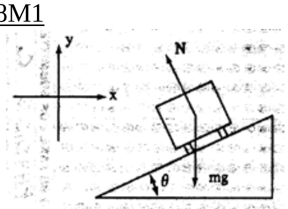
b. $F = mv^2/r$ where $v = 2\pi rf = 2\pi r(1/\pi) = 2r = 10$ m/s giving F = 1000 N provided by the normal force

c. $\Sigma F_y = 0$ so the upward force provided by friction equals the weight of the rider = mg = 490 N

d. Since the frictional force is proportional to the normal force and equal to the weight of the rider, m will cancel from the equation, meaning a rider with twice the mass, or any different mass, will not slide down the wall as mass is irrelevant for this condition.

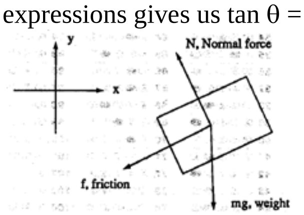
1988M1

a.



Toward the center of the turn we have $\Sigma F = N \sin \theta = mv^2/r$ and vertically $N \cos \theta = mg$. Dividing the two expressions gives us $\tan \theta = v^2/rg$ and v = 16 m/s

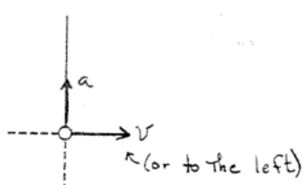
b.



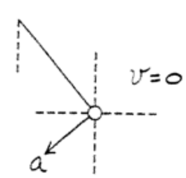
c. $\Sigma F_y = N \cos \theta - f \sin \theta - mg = 0$ and $\Sigma F_x = N \sin \theta + f \cos \theta = mv^2/r$ solve for N and f and substitute into $f = \mu N$ gives $\mu_{min} = 0.32$

1998B6

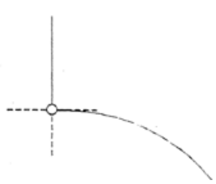
a. i.



ii.

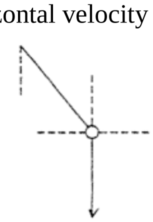


b. i.



The horizontal velocity is constant, the vertical motion is in free fall and the path is parabolic

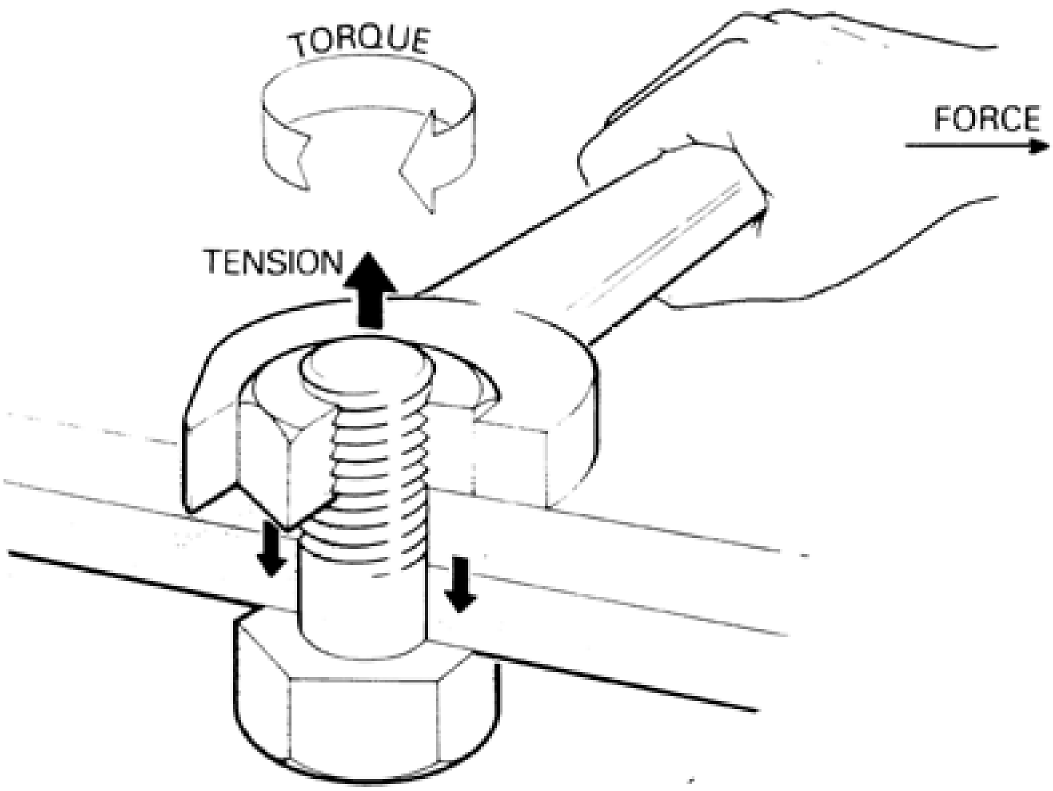
ii.



The ball falls straight down in free fall

Chapter 3

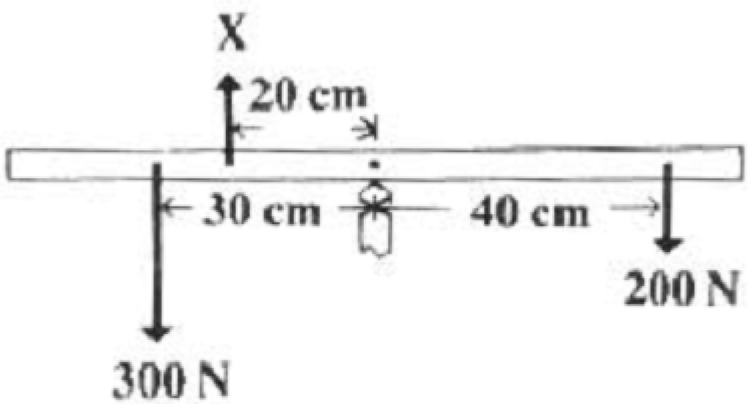
Torque



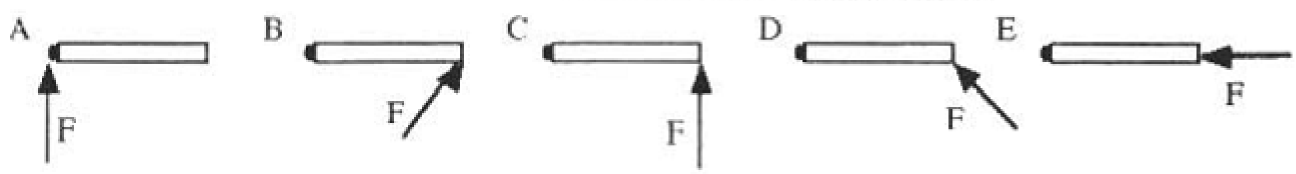
AP Physics Multiple Choice Practice – Torque

1. A uniform meterstick of mass 0.20 kg is pivoted at the 40 cm mark. Where should one hang a mass of 0.50 kg to balance the stick?
(A) 16 cm (B) 36 cm (C) 44 cm (D) 46 cm (E) 54 cm

2. A uniform meterstick is balanced at its midpoint with several forces applied as shown below. If the stick is in equilibrium, the magnitude of the force X in newtons (N) is
(A) 50 N (B) 100 N (C) 200 N (D) 300 N (E) impossible to determine without the weight of the stick

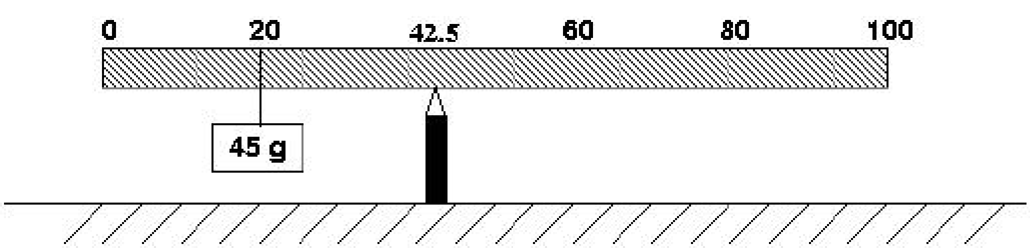


3. A door (seen from above in the figures below) has hinges on the left hand side. Which force produces the largest torque? The magnitudes of all forces are equal.



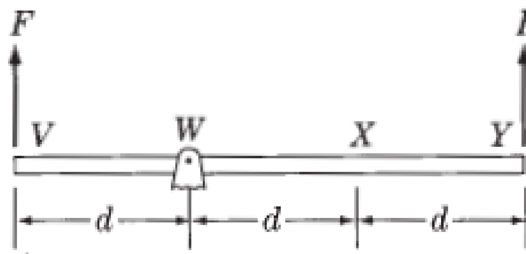
4. A meterstick is supported at each side by a spring scale. A heavy mass is then hung on the meterstick so that the spring scale on the left hand side reads four times the value of the spring scale on the right hand side. If the mass of the meterstick is negligible compared to the hanging mass, how far from the right hand side is the large mass hanging.
(A) 25 cm (B) 50 cm (C) 67 cm (D) 75 cm (E) 80 cm

5. A uniform meter stick has a 45.0 *g* mass placed at the 20 *cm* mark as shown in the figure. If a pivot is placed at the 42.5 *cm* mark and the meter stick remains horizontal in static equilibrium, what is the mass of the meter stick?



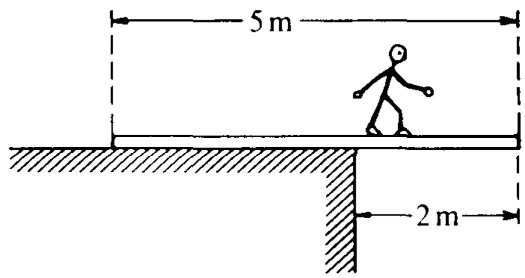
(A) 18.0 *g* (B) 45.0 *g* (C) 72.0 *g* (D) 120.0 *g* (E) 135.0 *g*

6. A massless rigid rod of length 3*d* is pivoted at a fixed point *W*, and two forces each of magnitude *F* are applied vertically upward as shown. A third vertical force of magnitude *F* may be applied, either upward or downward, at one of the labeled points. With the proper choice of direction at each point, the rod can be in equilibrium if the third force of magnitude *F* is applied at point

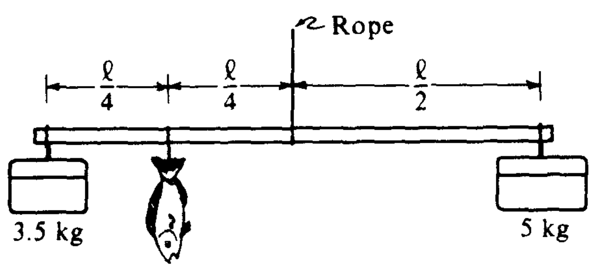


(A) *W* only (B) *Y* only (C) *V* or *X* only (D) *V* or *Y* only (E) *V*, *W*, or *X*

7. A 5-meter uniform plank of mass 100 kilograms rests on the top of a building with 2 meters extended over the edge as shown. How far can a 50-kilogram person venture past the edge of the building on the plank before the plank just begins to tip?
(A) 0.5 m (B) 1 m (C) 1.5 m (D) 2 m
(E) It is impossible to make the plank tip since the person would have to be more than 2 meters from the edge of the building.

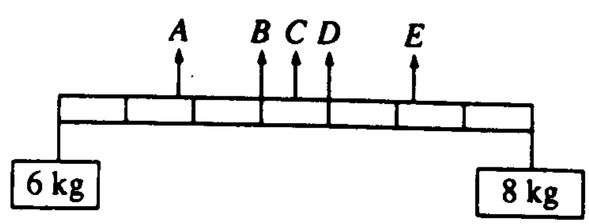


8. To weigh a fish, a person hangs a tackle box of mass 3.5 kilograms and a cooler of mass 5 kilograms from the ends of a uniform rigid pole that is suspended by a rope attached to its center. The system balances when the fish hangs at a point 1/4 of the rod's length from the tackle box. What is the mass of the fish?

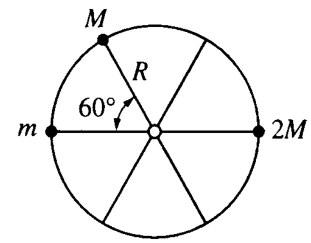


(A) 1.5 kg (B) 2 kg (C) 3 kg (D) 6 kg (E) 6.5 kg

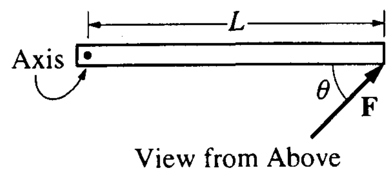
9. Two objects, of masses 6 and 8 kilograms, are hung from the ends of a stick that is 70 cm long and has marks every 10 cm, as shown. If the mass of the stick is negligible, at which of the points indicated should a cord be attached if the stick is to remain horizontal when suspended from the cord?
(A) A (B) B (C) C (D) D (E) E



10. A wheel of radius R and negligible mass is mounted on a horizontal frictionless axle so that the wheel is in a vertical plane. Three small objects having masses m, M, and $2M$, respectively, are mounted on the rim of the wheel, as shown. If the system is in static equilibrium, what is the value of m in terms of M?
(A) $M/2$ (B) M (C) $3M/2$ (D) $2M$ (E) $5M/2$



11. A rod on a horizontal tabletop is pivoted at one end and is free to rotate without friction about a vertical axis, as shown. A force **F** is applied at the other end, at an angle θ to the rod. If **F** were to be applied perpendicular to the rod, at what distance from the axis should it be applied in order to produce the same torque?



(A) L sin θ (B) L cos θ (C) L (D) L tan θ (E) $\sqrt{2}$ L

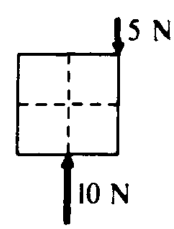
Questions 12-13
A horizontal, uniform board of weight 125 N and length 4 m is supported by vertical chains at each end. A person weighing 500 N is sitting on the board. The tension in the right chain is 250 N.

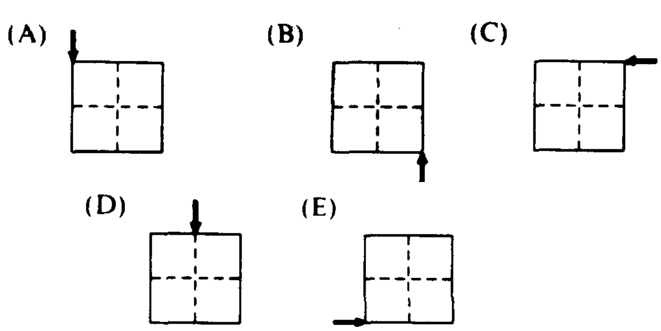
12. What is the tension in the left chain?
(A) 250 N (B) 375 N (C) 500 N (D) 625 N (E) 875 N

13. How far from the left end of the board is the person sitting?
(A) 0.4 m (B) 1.5 m (C) 2 m (D) 2.5 m (E) 3 m

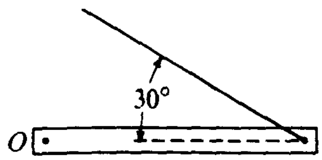
14. Torque is the rotational analogue of
(A) kinetic energy (B) linear momentum (C) acceleration (D) force (E) mass

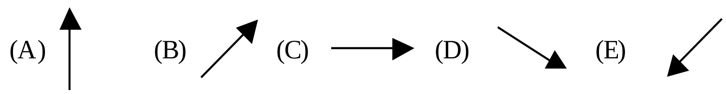
15. A square piece of plywood on a horizontal tabletop is subjected to the two horizontal forces shown. Where should a third force of magnitude 5 newtons be applied to put the piece of plywood into equilibrium?



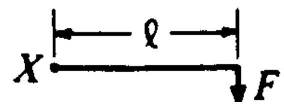


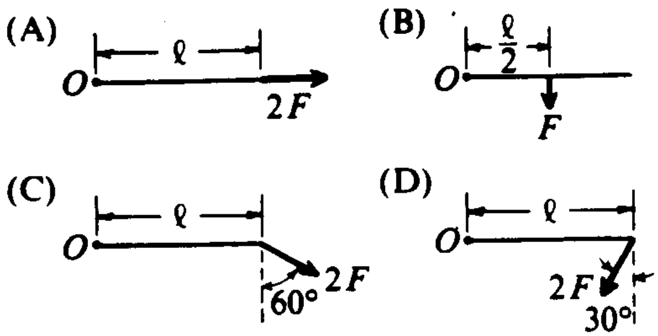
16. A uniform rigid bar of weight W is supported in a horizontal orientation as shown by a rope that makes a 30° angle with the horizontal. The force exerted on the bar at point O, where it is pivoted, is best represented by a vector whose direction is which of the following?





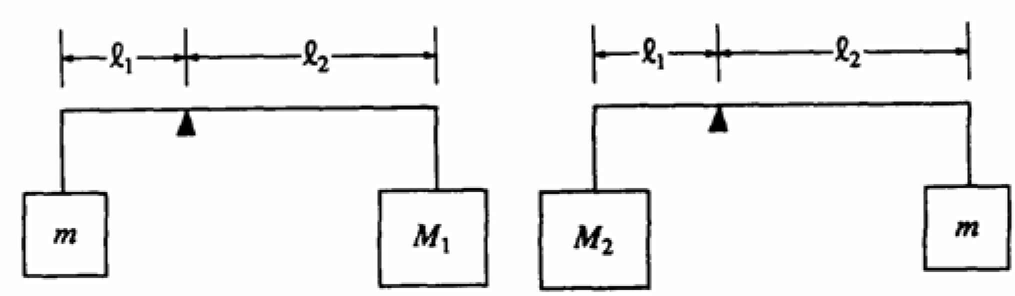
17. In which of the following diagrams is the torque about point O equal in magnitude to the torque about point X in the diagram? (All forces lie in the plane of the paper.)





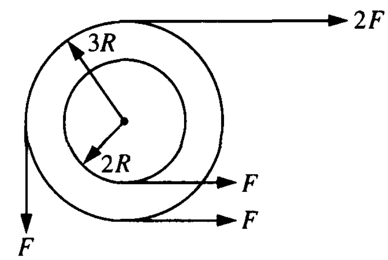
(E) None of the above

18. A rod of length L and of negligible mass is pivoted at a point that is off-center with lengths shown in the figure below. The figures show two cases in which masses are suspended from the ends of the rod. In each case the unknown mass m is balanced by a known mass, M_1 or M_2, so that the rod remains horizontal. What is the value of m in terms of the known masses?

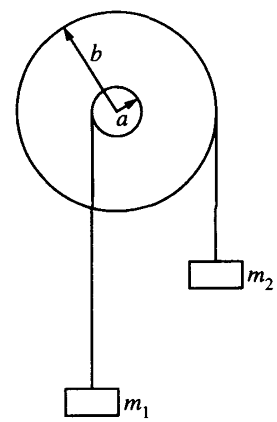


(A) $M_1 + M_2$ (B) $\frac{1}{2}(M_1 + M_2)$ (C) $M_1 M_2$ (D) $\frac{1}{2}M_1M_2$ (E) $\sqrt{M_1M_2}$

19. A system of two wheels fixed to each other is free to rotate about a frictionless axis through the common center of the wheels and perpendicular to the page. Four forces are exerted tangentially to the rims of the wheels, as shown. The magnitude of the net torque on the system about the axis is
(A) zero (B) FR (C) 2FR (D) 5FR (E) 14FR

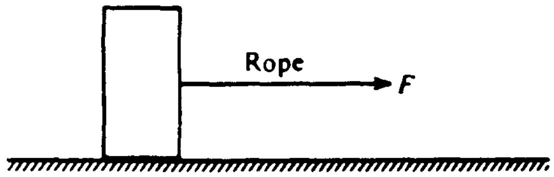


20. For the wheel-and-axle system shown, which of the following expresses the condition required for the system to be in static equilibrium?
(A) $m_1 = m_2$ (B) $am_1 = bm_2$ (C) $am_2 = bm_1$
(D) $a^2m_l = b^2m^2$ (E) $b^2m_1 = a^2m_2$



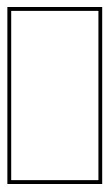
21. A meterstick of negligible mass is placed on a fulcrum at the 0.60 m mark, with a 2.0 kg mass hung at the 0 m mark and a 1.0 kg mass hung at the 1.0 m mark. The meterstick is released from rest in a horizontal position. Immediately after release, the magnitude of the net torque on the meterstick about the fulcrum is most nearly
(A) 2.0 N•m (B) 8.0 N•m (C) 10 N•m (D) 14 N•m (E) 16 N•m

AP Physics Free Response Practice – Torque

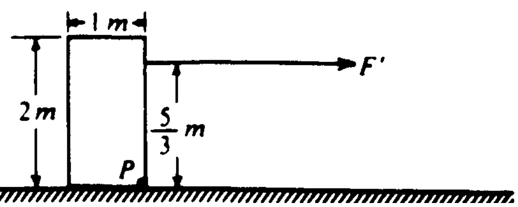


1983B1. A box of uniform density weighing 100 newtons moves in a straight line with constant speed along a horizontal surface. The coefficient of sliding friction is 0.4 and a rope exerts a force F in the direction of motion as shown above.

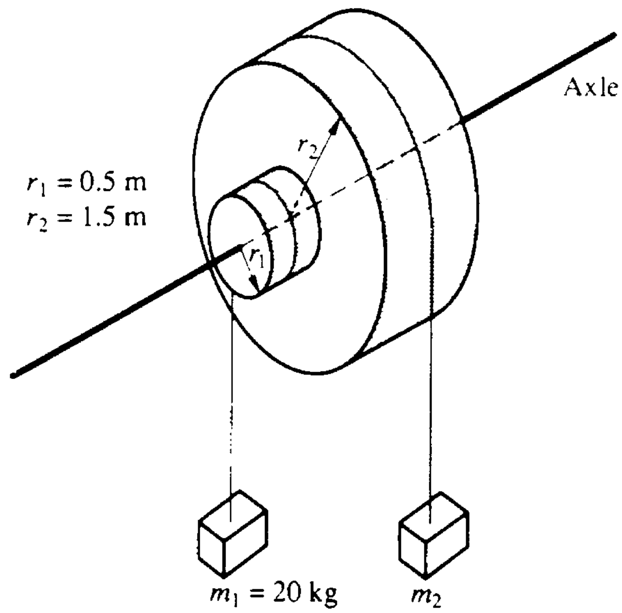
a. On the diagram below, draw and identify all the forces on the box.



b. Calculate the force F exerted by the rope that keeps the box moving with constant speed.

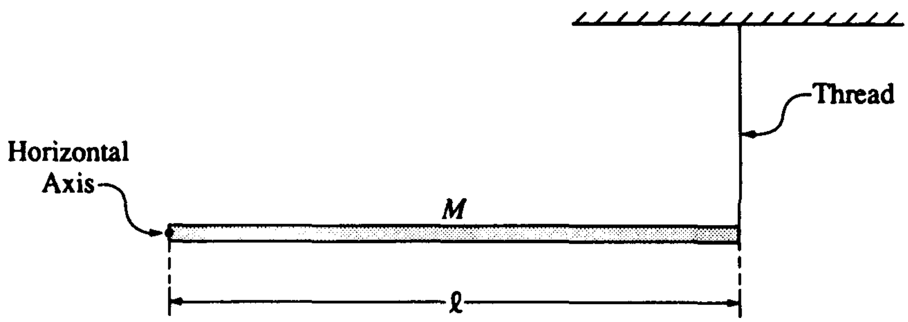


c. A horizontal force F', applied at a height 5/3 meters above the surface as shown in the diagram above, is just sufficient to cause the box to begin to tip forward about an axis through point P. The box is 1 meter wide and 2 meters high. Calculate the force F'.



C1991M2. Two masses. m_1 and m_2, are connected by light cables to the perimeters of two cylinders of radii r_1 and r_2, respectively, as shown in the diagram above with $r_1 = 0.5$ meter, $r_2 = 1.5$ meters, and $m_1 = 20$ kilograms.

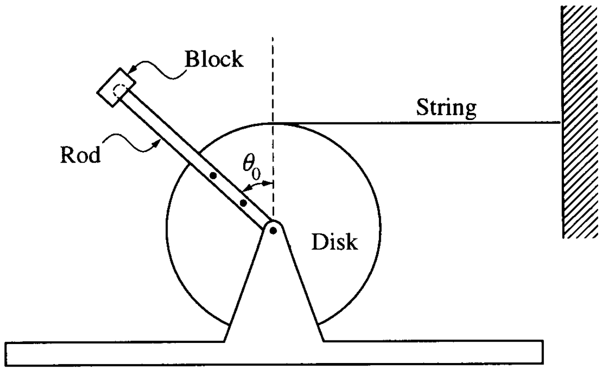
a. Determine m_2 such that the system will remain in equilibrium.



C1993M3. A long, uniform rod of mass M and length *l* is supported at the left end by a horizontal axis into the page and perpendicular to the rod, as shown above. The right end is connected to the ceiling by a thin vertical thread so that the rod is horizontal. Express the answers to all parts of this question in terms of M, L and g.

a. Determine the magnitude and direction of the force exerted on the rod by the axis.

b. If the breaking strength of the thread is 2Mg, determine the maximum distance, r, measured from the hinge axis, that a box of mass 4M could be placed without breaking the thread

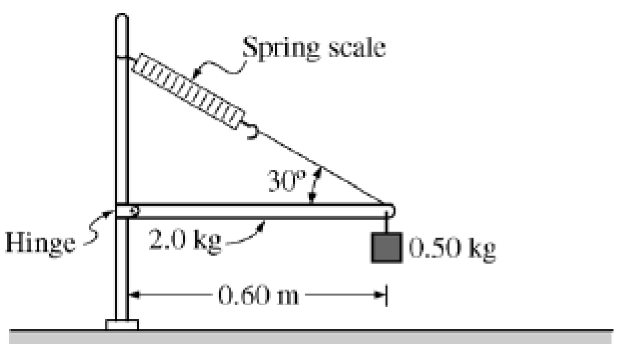


C1999M3. As shown above, a uniform disk is mounted to an axle and is free to rotate without friction. A thin uniform rod is rigidly attached to the disk. A block is attached to the end of the rod. Properties of the rod, and block are as follows.

Rod: mass = m, length = 2R
Block: mass = 2m
Disk: radius = R

The system is held in equilibrium with the rod at an angle θ_o to the vertical, as shown above, by a horizontal string of negligible mass with one end attached to the disk and the other to a wall. Determine the tension in the string in terms of m, θ_o, and g.

C2008M2.

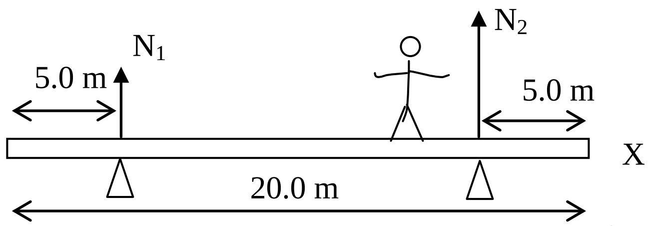


The horizontal uniform rod shown above has length 0.60 m and mass 2.0 kg. The left end of the rod is attached to a vertical support by a frictionless hinge that allows the rod to swing up or down. The right end of the rod is supported by a cord that makes an angle of 30° with the rod. A spring scale of negligible mass measures the tension in the cord. A 0.50 kg block is also attached to the right end of the rod.
(a) On the diagram below, draw and label vectors to represent all the forces acting on the rod. Show each force vector originating at its point of application.

(b) Calculate the reading on the spring scale.
(c) Calculate the magnitude of the force exerted by the hinge on the rod

Supplemental Problem

The diagram below shows a beam of length 20.0 m and mass 40.0 kg resting on two supports placed at 5.0 m from each end.



A person of mass 50.0 kg stands on the beam between the supports. The reaction forces at the supports are shown.

(a) State the value of $N_1 + N_2$

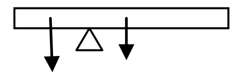
(b) The person now moves toward the X end of the beam to the position where the beam just begins to tip and reaction force N_1 becomes zero as the beam starts to leave the left support. Determine the distance of the girl from the end X when the beam is about to tip.

ANSWERS - AP Physics Multiple Choice Practice – Torque

Solution	Answer

1. Mass of stick m_1=0.20 kg at midpoint, Total length L=1.0 m, Pivot at 0.40 m, attached mass m_2=0.50 kg. **B**

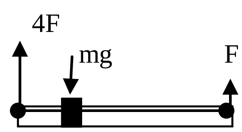
   Applying rotational equilibrium $\tau_{net}=0$
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (0.2) (0.1 m) = (0.5)(x)
   x = 0.04 m (measured away from 40 cm mark)
   → gives a position on the stick of 36 cm



2. As above, apply rotational equilibrium **A**
   + (300) (30cm) – X (20 cm) – (200) (40 cm) = 0

3. Torque = $(Fr)_\perp$ Choices A and E make zero torque, Of the remaining choices, each has moment arm = r but choice C has the full value of F to create torque (perpendicular) while the others would only use a component of F to make less torque **C**

4. Applying rotational equilibrium, using location of unknown mass as pivot … **E**



   4F (1 – x) = (F) (x)
   4F = 5Fx
   x = 4/5 = 0.80 m measured from right side

5. Applying rotational equilibrium ("g" cancels on each side) **E**
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (45) (22.5 cm) = (m) (7.5 cm) → m = 135 g

6. On the left of the pivot τ=Fd, on the right side of the pivot τ = F(2d). So we either have to add 1(Fd) to the left side to balance out the torque or remove 1(Fd) on the right side to balance out torque. Putting an upwards force on the left side at V gives (2Fd) on the left to balance torques, or putting a downwards force on the right side at X give a total of Fd on the right also causing a balance **C**

7. Apply rotational equilibrium using the corner of the building as the pivot point. Weight of plank (acting at midpoint) provides torque on left and weight of man provides torque on right. **B**
   $(m_1g) \bullet r_1 = (m_2g) \bullet r_2$
   (100 kg) (0.5m) = (50 kg) (r) → r = 1m

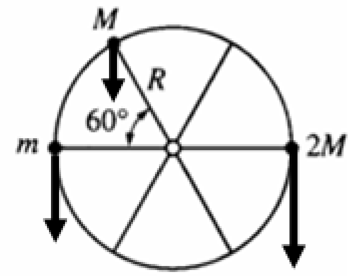
8. Apply rotational equilibrium using the rope as the pivot point. **C**
   (3.5)(9.8)(L/2) + m(9.8)(L/4) – (5)(9.8)(L/2) = 0 → m = 3 kg

9. To balance the torques on each side, we obviously need to be closer to the heavier mass. **D**
   Trying point D as a pivot point we have:
   $(m_1g) \bullet r_1$?=? $(m_2g) \bullet r_2$
   (6kg) (40 cm) ?=? (8kg) (30 cm) and we see it works.

10. Applying rotational equilibrium at the center pivot we get: C

$+mg(R) + Mg(R\cos 60°) – 2Mg(R) = 0$.

Using $\cos 60° = ½$ we arrive at the answer 3M/2



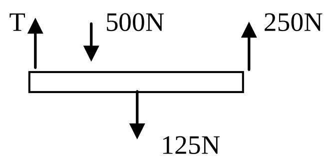
11. Finding the torque in the current configuration we have: A

$(F\sin\theta)(L) = FL \sin \theta$.

To get the same torque with F applied perpendicular we would have to change the L to get F $(L\sin\theta)$

12. Diagram B

T = 375 N



Simple $F_{net(y)} = 0$

$T – 500 + 250 – 125 = 0$

13. Same Diagram B

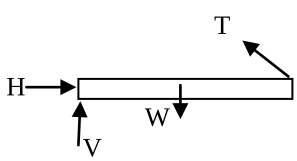
Apply rotational equilibrium using left end as pivot:

$+ (250)(4) – (125)(2) – (500)(r) = 0 \rightarrow r = 1.5m$

14. Definition of Torque D

15. To balance the forces (Fnet=0) the answer must be A or D, to prevent rotation, obviously A would be needed. A

16. FBD B



Since the rope is at an angle it has x and y components of force.

Therefore, H would have to exist to counteract Tx.

Based on $\tau_{net} = 0$ requirement, V also would have to exist to balance W if we were to chose a pivot point at the right end of the bar

17. In the given diagram the torque is = FL. C

Finding the torque of all the choices reveals C as correct.

$(2F\sin 60°)(L) = 2F ½ L = FL$

18. Applying rotational equilibrium to each diagram gives E

DIAGRAM 1: $(mg)(L_1) = (M_1g)(L_2)$

$L_1 = M_1(L_2)/m$

(sub this L_1) into the Diagram 2 eqn, and solve. →

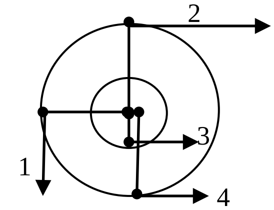
DIAGRAM 2: $(M_2g)(L_1) = mg(L_2)$

$M_2 (L_1) = m (L_2)$

19. Find the torques of each using proper signs and add up. C

$+ (1) - (2) + (3) + (4)$

$+F(3R) - (2F)(3R) + F(2R) + F(3R) = 2FR$



20. Simply apply rotational equilibrium B

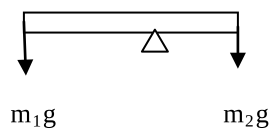
$(m_1 g) \bullet r_1 = (m_2 g) \bullet r_2$

$m_1 a = m_2 b$

21. Question says meterstick has no mass, so ignore that force. Pivot placed at 0.60 m. Based on the applied masses, this meterstick would have a net torque and rotate. Find the net Torque as follows B

$\tau_{net} = + (m_1 g) \bullet r_1 - (m_2 g) \bullet r_2$

$+ (2)(10\ m/s^2)(0.6\ m) - (1)(10\ m/s^2)(0.4\ m)$



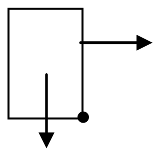
1983B1.

a) FBD. F_n pointing up, F_g pointing down, f_k applied to base of box pointing left

b) Constant speed → a=0.

Fnet = 0 F – fk = 0 F – uFn = 0 F – (0.4)(100)=0 F = 40 N

c) The force F' occurs at the limit point of tipping which is when the torque trying to tip it (caused by F') is equal to the torque trying to stop it from tipping (from the weight) using the tipping pivot point of the bottom right corner of the box.



(F')(5/3 m) = Fg (0.5m)
(F') (5/3) = (100)(0.5)
F' = 30 N

C1991M2.

Apply rotational equilibrium with the center as the pivot

$(m_1g) \cdot r_1 = (m_2g) \cdot r_2$ (20)(9.8)(0.5) = m_2(9.8)(1.5) m_2= 6.67 kg

C1993M3

(a) There is no H support force at the hinge since there are no other horizontal forces acting, so there is only vertical support for V. The tension in the thread T acts upwards and the weight of the rod acts at the midpoint. Apply rotational equilibrium using the hinge axis as the pivot:

+(T)(L) – (Mg)(L/2) = 0 T = Mg/2

Then using Fnet(y) = 0 V + T – Mg = 0 V + Mg/2 – Mg =0 V = Mg / 2

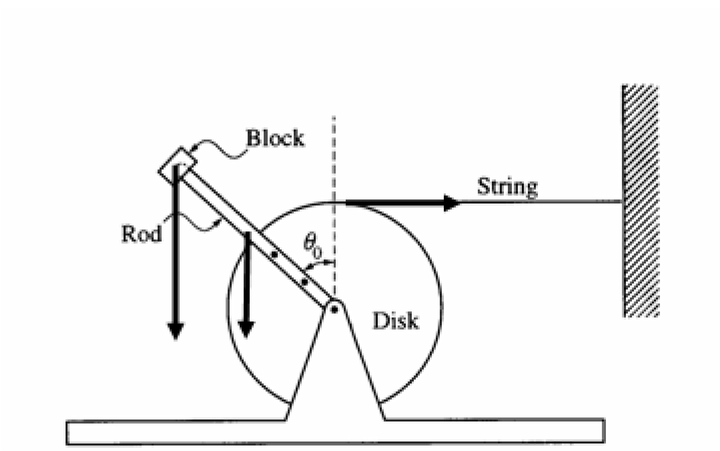
(b) Apply rotational equilibrium using the hinge axis as the pivot and "r" as the unknown distance of the box

Thread torque – Box torque - Rod Torque= 0

(2Mg)(L) – (4Mg)(r) – (Mg)(L/2) = 0

2L – 4r – L/2 = 0 r = 3/8 L

C1999M3



Apply rotational equilibrium using the center of the disk as the pivot

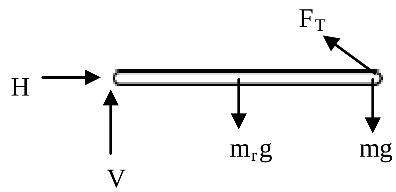
$(m_bg)(2R\sin\theta_o) + (m_rg)(R\sin\theta_o) – T(R) = 0$

$(2mg)(2R\sin\theta_o) + (mg)(R\sin\theta_o) – T(R) = 0$

$T = 5mg(\sin\theta_o)$

C2008M2

a) FBD



b) Apply rotational equilibrium using the hinge as the pivot

$+(F_T \sin30)(0.6) - (mg)(0.6) - (m_r g)(0.3) = 0$

$+(F_T \sin30)(0.6) - (0.5)(9.8)(0.6) - (2)(9.8)(0.3)=0$ $\qquad F_T = 29.4$ N

c) Apply Fnet(x), Fnet(y) = 0 to find H and V $\qquad$ V=9.8N, H=25.46N

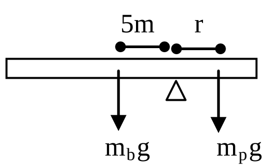
combining H and V $\qquad F_{hinge} = 27.28$ N

Supplemental

(a) Simple application of Fnet(y) = 0 $\qquad N_1 + N_2 - m_b g - m_p g = 0$

$N_1 + N_2 = (40)(9.8) + (50)(9.8) \quad = 882$ N

(b)



Apply rotational equilibrium

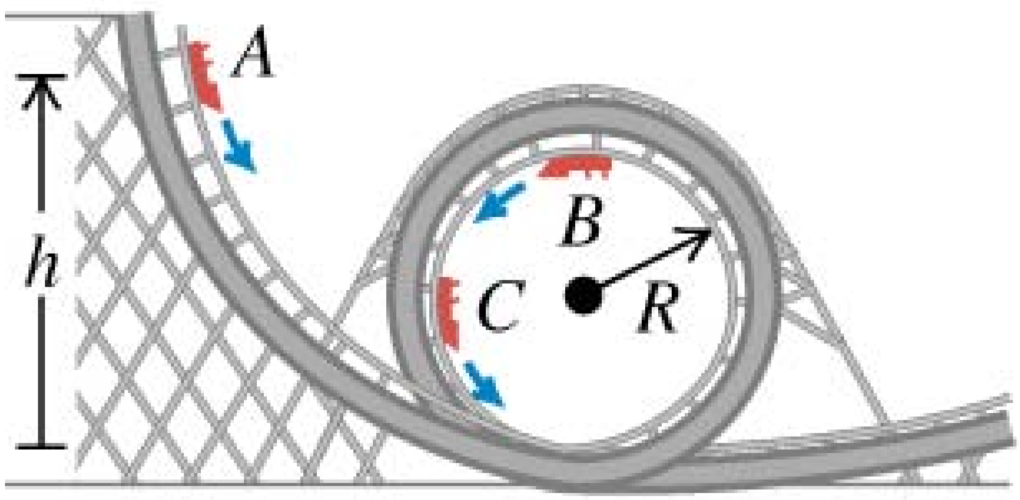
$(m_b g) \bullet r_1 = (m_p g) \bullet r_2$

(40) (5m) = (50) (r) $\qquad$ r = 4m from hinge

➔ 1 m from point X

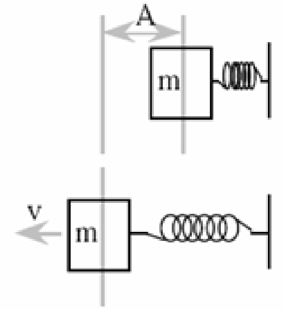
Chapter 4

Work, Power and Energy



AP Physics Multiple Choice Practice – Work-Energy

1. A mass m attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the masses speed as it passes through its equilibrium position?



$(A)\ 0 \quad (B)\ A\sqrt{\frac{k}{m}} \quad (C)\ A\sqrt{\frac{m}{k}} \quad (D)\ \frac{1}{A}\sqrt{\frac{k}{m}} \quad (E)\ \frac{1}{A}\sqrt{\frac{m}{k}}$

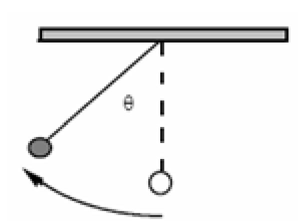
2. A force F at an angle θ above the horizontal is used to pull a heavy suitcase of weight mg a distance d along a level floor at constant velocity. The coefficient of friction between the floor and the suitcase is μ. The work done by the frictional force is:
(A) –Fd cos θ (B) mgh – Fd cos θ (C) – μ Fd cos θ (D) -μmgd (E) -μ mgd cos θ

3. If the unit for force is F, the unit for velocity V, and the unit for time T, then the unit for energy is:
(A) FVT (B) F/T (C) FV/T (D) F/T^2 (E) FV^2/T^2

4. A force of 10 N stretches a spring that has a spring constant of 20 N/m. The potential energy stored in the spring is:
(A) 2.5 J (B) 5.0 J (C) 10 J (D) 40 J (E) 200 J

5. A 2 kg ball is attached to a 0.80 m string and whirled in a horizontal circle at a constant speed of 6 m/s. The work done on the ball during each revolution is:
(A) 450 J (B) 90 J (C) 72 J (D) 16 J (E) zero

6. A pendulum bob of mass m on a cord of length L is pulled sideways until the cord makes an angle θ with the vertical as shown in the figure to the right. The change in potential energy of the bob during the displacement is:
(A) mgL (1–cos θ) (B) mgL (1–sin θ) (C) mgL sin θ
(D) mgL cos θ (E) 2mgL (1–sin θ)



7. A force F directed at an angle θ above the horizontal is used to pull a crate a distance D across a level floor. The work done by the force F is
(A) FD (B) FD cos θ (C) FD sin θ (D) mg sin θ (E) mgD cos θ

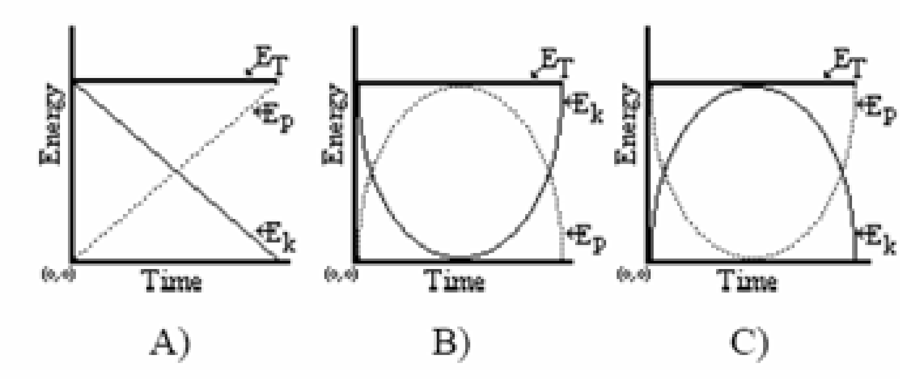
8. A compressed spring has 16 J of potential energy. What is the maximum speed it can impart to a 2 kg object?
(A) 2.8 m/s (B) 4.0 m/s (C) 5.6 m/s (D) 8.0 m/s (E) 16 m/s

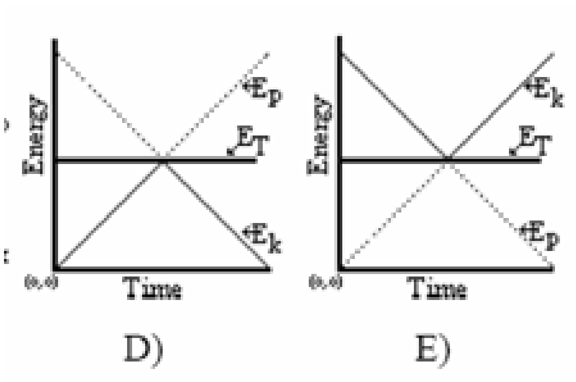
9. A softball player catches a ball of mass m, which is moving towards her with horizontal speed V. While bringing the ball to rest, her hand moved back a distance d. Assuming constant deceleration, the horizontal force exerted on the ball by the hand is
(A) $mV^2/(2d)$ (B) mV^2/d (C) mVd (D) 2mV/d (E) mV/d

10. A 3 kg block with initial speed 4 m/s slides across a rough horizontal floor before coming to rest. The frictional force acting on the block is 3 N. How far does the block slide before coming to rest?
(A) 1.0 m (B) 2.0 m (C) 4.0 m (D) 8.0 m (E) 16 m

11. A construction laborer holds a 20 kg sheet of wallboard 3 m above the floor for 4 seconds. During these 4 seconds how much power was expended on the wallboard?
(A) 2400 W (B) 340 W (C) 27 W (D) 15 W (E) none of these

12. A pendulum is pulled to one side and released. It swings freely to the opposite side and stops. Which of the following might best represent graphs of kinetic energy (E_k), potential energy (E_p) and total mechanical energy (E_T)





Problems 13 and 14 refer to the following situation: A car of mass m slides across a patch of ice at a speed v with its brakes locked. It the hits dry pavement and skids to a stop in a distance d. The coefficient of kinetic friction between the tires and the dry road isμ.

13. If the car has a mass of 2m, it would have skidded a distance of
(A) 0.5 d (B) d (C) 1.41 d (D) 2d (E) 4 d

14. If the car has a speed of 2v, it would have skidded a distance of
(A) 0.5 d (B) d (C) 1.41 d (D) 2d (E) 4 d

15. A ball is thrown vertically upwards with a velocity v and an initial kinetic energy E_k. When half way to the top of its flight, it has a velocity and kinetic energy respectively of
(A) $\frac{v}{2}, \frac{E_k}{2}$ (B) $\frac{v}{\sqrt{2}}, \frac{E_k}{2}$ (C) $\frac{v}{4}, \frac{E_k}{2}$ (D) $\frac{v}{2}, \frac{E_k}{\sqrt{2}}$ (E) $\frac{v}{\sqrt{2}}, \frac{E_k}{\sqrt{2}}$

16. A football is kicked off the ground a distance of 50 yards downfield. Neglecting air resistance, which of the following statements would be INCORRECT when the football reaches the highest point?
(A) all of the balls original kinetic energy has been changed into potential energy
(B) the balls horizontal velocity is the same as when it left the kickers foot
(C) the ball will have been in the air one-half of its total flight time
(D) the ball has an acceleration of g
(E) the vertical component of the velocity is equal to zero

17. A mass m is attached to a spring with a spring constant k. If the mass is set into motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to equilibrium position?
(A) $v = \sqrt{\frac{kd}{m}}$ (B) $v^2 = \frac{kd}{m}$ (C) $v = \frac{kd}{mg}$ (D) $v^2 = \frac{mgd}{k}$ (E) $v = d\sqrt{\frac{k}{m}}$

18. If M represents units of mass, L represents units of length, and T represents units of time, the dimensions of power would be:
(A) $\frac{ML}{T^2}$ (B) $\frac{ML^2}{T^2}$ (C) $\frac{ML^2}{T^3}$ (D) $\frac{ML}{T}$ (E) $\frac{ML^2}{T}$

19. An automobile engine delivers 24000 watts of power to a car's driving wheels. If the car maintains a constant speed of 30 m/s, what is the magnitude of the retarding force acting on the car?
(A) 800 N (B) 960 N (C) 1950 N (D) 720,000 N (E) 1,560,000 N

20. A fan blows the air and gives it kinetic energy. An hour after the fan has been turned off, what has happened to the kinetic energy of the air?
(A) it disappears (B) it turns into potential energy (C) it turns into thermal energy
(D) it turns into sound energy (E) it turns into electrical energy

21. A box of old textbooks is on the middle shelf in the bookroom 1.3 m from the floor. If the janitor relocates the box to a shelf that is 2.6 m from the floor, how much work does he do on the box? The box has a mass of 10 kg.
(A) 13 J (B) 26 J (C) 52 J (D) 130 J (E) 260 J

22. A mass, M, is at rest on a frictionless surface, connected to an ideal horizontal spring that is unstretched. A person extends the spring 30 cm from equilibrium and holds it by applying a 10 N force. The spring is brought back to equilibrium and the mass connected to it is now doubled to 2M. If the spring is extended back 30 cm from equilibrium, what is the necessary force applied by the person to hold the mass stationary there?
(A) 20 N (B) 14.1 N (C) 10 N (D) 7.07 N (E) 5 N

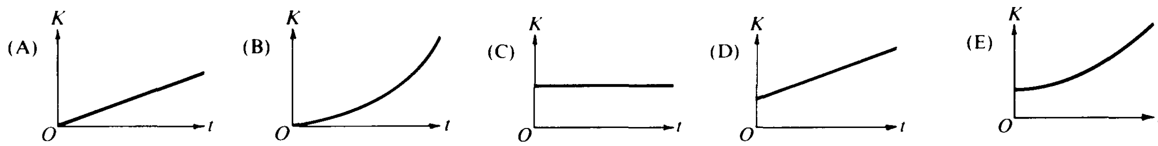
23. A deliveryman moves 10 cartons from the sidewalk, along a 10-meter ramp to a loading dock, which is 1.5 meters above the sidewalk. If each carton has a mass of 25 kg, what is the total work done by the deliveryman on the cartons to move them to the loading dock?
(A) 2500 J (B) 3750 J (C) 10000 J (D) 25000 J (E) 37500 J

24. A rock is dropped from the top of a tall tower. Half a second later another rock, twice as massive as the first, is dropped. Ignoring air resistance,
(A) the distance between the rocks increases while both are falling.
(B) the acceleration is greater for the more massive rock.
(C) the speed of both rocks is constant while they fall.
(D) they strike the ground more than half a second apart.
(E) they strike the ground with the same kinetic energy.

25. A 60.0-kg ball of clay is tossed vertically in the air with an initial speed of 4.60 m/s. Ignoring air resistance, what is the change in its potential energy when it reaches its highest point?
(A) 0 J (B) 45 J (C) 280 J (D) 635 J (E) 2700 J

26. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The kinetic and potential energies are both constant.
(C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
(E) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position

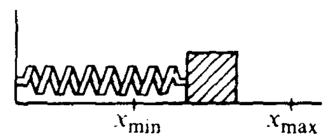
27. From the top of a high cliff, a ball is thrown horizontally with initial speed v_o. Which of the following graphs best represents the ball's kinetic energy K as a function of time t ?

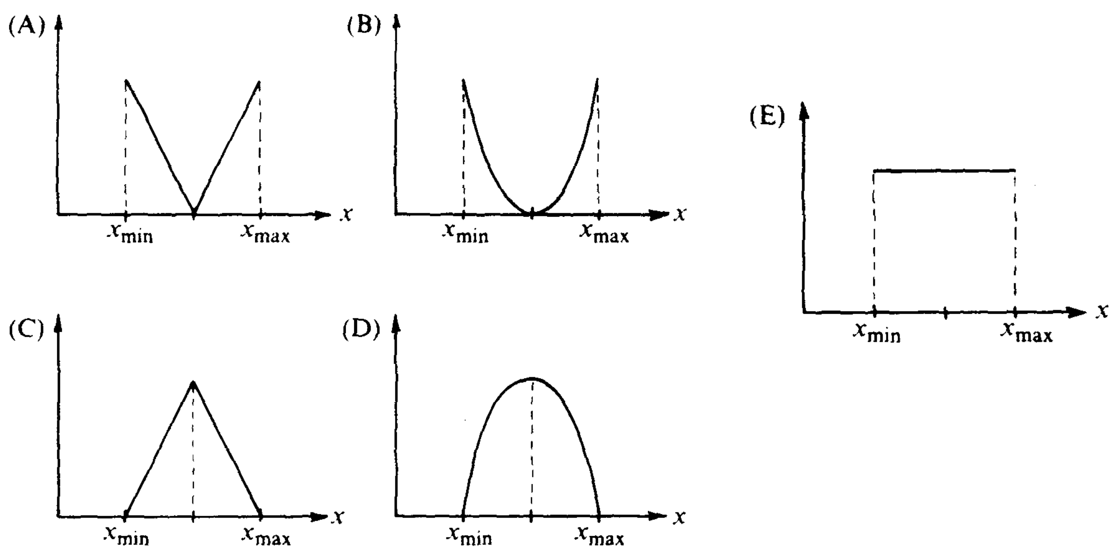


28. A person pushes a box across a horizontal surface at a constant speed of 0.5 meter per second. The box has a mass of 40 kilograms, and the coefficient of sliding friction is 0.25. The power supplied to the box by the person is (A) 0.2 W (B) 5 W (C) 50 W (D) 100 W (E) 200 W

29. A horizontal force F is used to pull a 5-kilogram block across a floor at a constant speed of 3 meters per second. The frictional force between the block and the floor is 10 newtons. The work done by the force F in 1 minute is most nearly
(A) 0 J (B) 30 J (C) 600 J (D) 1,350 J (E) 1,800 J

Questions 30-31: A block oscillates without friction on the end of a spring as shown. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



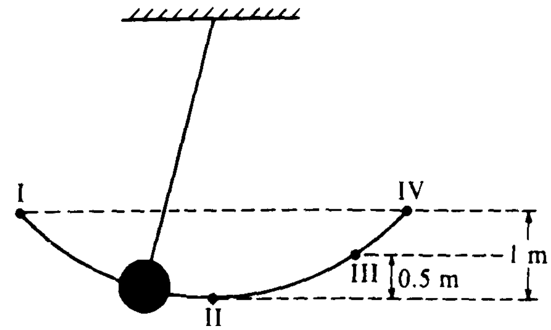


30. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
(A) A (B) B (C) C (D) D (E) E

31. Which graph can represent the kinetic energy of the block as a function of x ?
(A) A (B) B (C) C (D) D (E) E

Questions 32-33

A ball swings freely back and forth in an arc from point I to point IV, as shown. Point II is the lowest point in the path, III is located 0.5 meter above II, and IV is I meter above II. Air resistance is negligible.

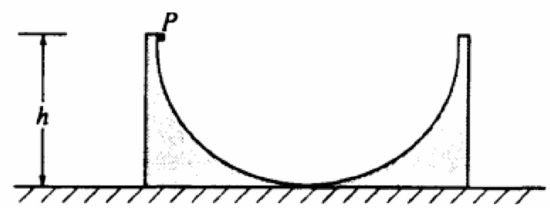


32. If the potential energy is zero at point II, where will the kinetic and potential energies of the ball be equal?
(A) At point II (B) At some point between II and III
(C) At point III (D) At some point between III and IV
(E) At point IV

33. The speed of the ball at point II is most nearly
(A) 3.0 m/s (B) 4.5 m/s (C) 9.8 m/s (D) 14 m/s (E) 20 m/s

34. An ideal spring obeys Hooke's law, F = -kx. A mass of 0.50 kilogram hung vertically from this spring stretches the spring 0.075 meter. The value of the force constant for the spring is most nearly
(A) 0.33 N/m (B) 0.66 N/m (C) 6.6 N/m (D) 33 N/m (E) 66 N/m

35. The figure shows a rough semicircular track whose ends are at a vertical height h. A block placed at point P at one end of the track is released from rest and slides past the bottom of the track. Which of the following is true of the height to which the block rises on the other side of the track?



(A) It is equal to h/(2π) (B) It is equal to h/4
(C) It is equal to h/2 (D) It is equal to h
(E) It is between zero and h; the exact height depends on how much energy is lost to friction.

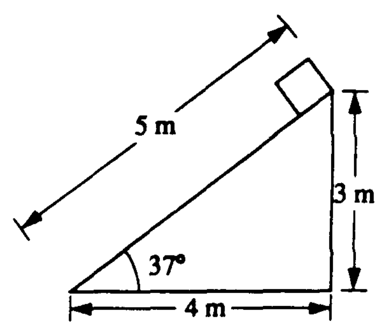
36. A weight lifter lifts a mass m at constant speed to a height h in time t. What is the average power output of the weight lifter? (A) mg (B) mh (C) mgh (D) mght (E) mgh /t

37. A block of mass m slides on a horizontal frictionless table with an initial speed v_o. It then compresses a spring of force constant k and is brought to rest. How much is the spring compressed from its natural length?

(A) $\frac{v_0^2}{2g}$ (B) $\frac{mg}{k}v_0$ (C) $\frac{m}{k}v_0$ (D) $\sqrt{\frac{m}{k}}v_0$ (E) $\sqrt{\frac{k}{m}}v_0$

Questions 38-40

A plane 5 meters in length is inclined at an angle of 37°, as shown. A block of weight 20 newtons is placed at the top of the plane and allowed to slide down.



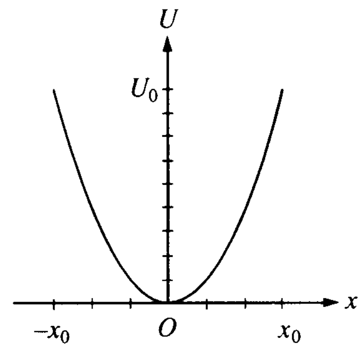
38. The mass of the block is most nearly
(A) 1.0 kg (B) 1.2 kg (C) 1.6 kg (D) 2.0 kg (E) 2.5 kg

39. The magnitude of the normal force exerted on the block by the plane is most nearly
(A) 10 N (B) 12 N (C) 16 N (D) 20 N (E) 33 N

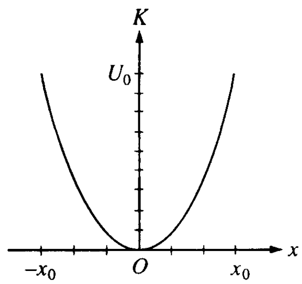
40. The work done on the block by the gravitational force during the 5-meter slide down the plane is most nearly
(A) 20 J (B) 60 J (C) 80 J (D) 100 J (E) 130 J

41. A student weighing 700 N climbs at constant speed to the top of an 8 m vertical rope in 10 s. The average power expended by the student to overcome gravity is most nearly
(A) 1.1 W (B) 87.5 W (C) 560 W (D) 875 W (E) 5,600 W

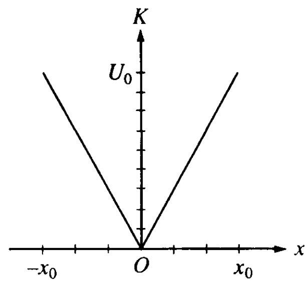
42. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring moving back and forth with amplitude x_0. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?



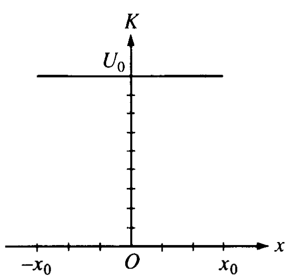
(A)



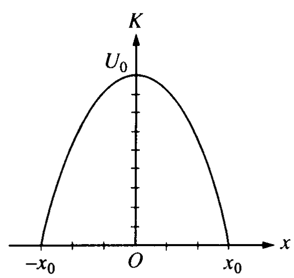
(B)



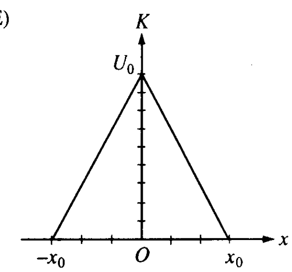
(C)



(D)



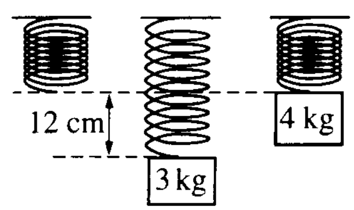
(E)



43. A child pushes horizontally on a box of mass m which moves with constant speed v across a horizontal floor. The coefficient of friction between the box and the floor is μ. At what rate does the child do work on the box?

(A) μmgv (B) mgv (C) $\mu mg/v$ (D) $\mu mg/v$ (E) μmv^2

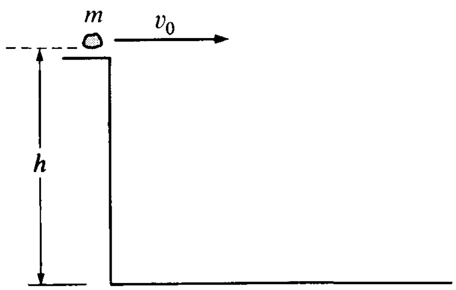
44. A block of mass 3.0 kg is hung from a spring, causing it to stretch 12 cm at equilibrium, as shown. The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?
(A) 9 cm (B) 18 cm (C) 24 cm
(D) 32 cm (E) 48 cm



45. What is the kinetic energy of a satellite of mass m that orbits the Earth, of mass M, in a circular orbit of radius R?

(A) Zero (B) $\frac{1}{2}\frac{GMm}{R}$ (C) $\frac{1}{4}\frac{GMm}{R}$ (D) $\frac{1}{2}\frac{GMm}{R^2}$ (E) $\frac{GMm}{R^2}$

Questions 46-47



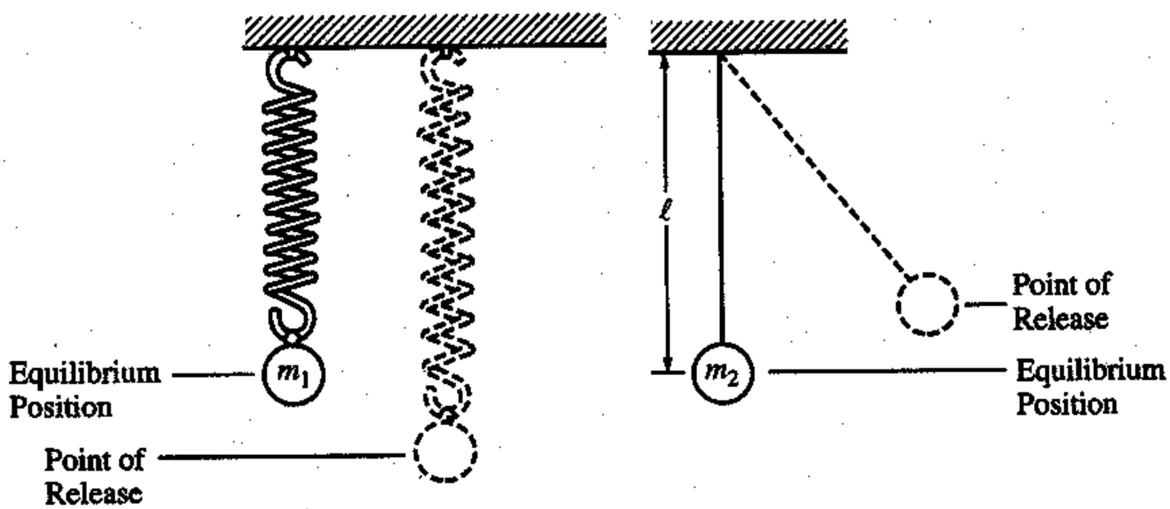
A rock of mass *m* is thrown horizontally off a building from a height *h*, as shown above. The speed of the rock as it leaves the thrower's hand at the edge of the building is v_0.

46. How much time does it take the rock to travel from the edge of the building to the ground?

(A) $\sqrt{hv_o}$ (B) $\frac{h}{v_0}$ (C) $\frac{hv_0}{g}$ (D) $\frac{2h}{g}$ (E) $\sqrt{2h/g}$

47. What is the kinetic energy of the rock just before it hits the ground?

(A) mgh (B) $\frac{1}{2}mv_0^2$ (C) $\frac{1}{2}mv_0^2 - mgh$ (D) $\frac{1}{2}mv_0^2 + mgh$ (E) $mgh - \frac{1}{2}mv_0^2$



A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion

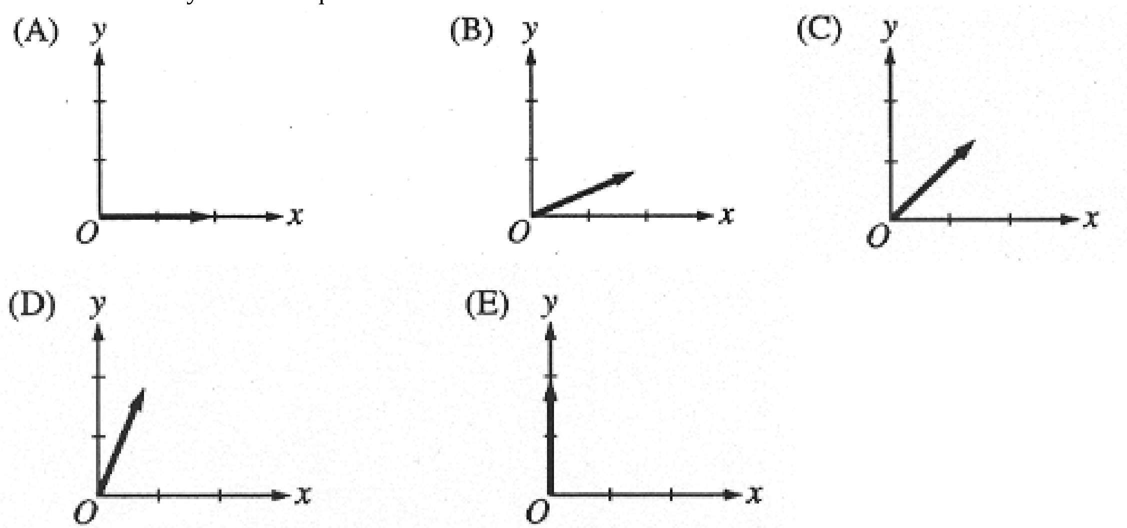
48. Which of the following is true for both spheres?

(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position.
(B) The maximum kinetic energy is attained as the sphere reaches its point of release.
(C) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(D) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(E) The maximum total energy is attained only as the sphere passes through its equilibrium position.

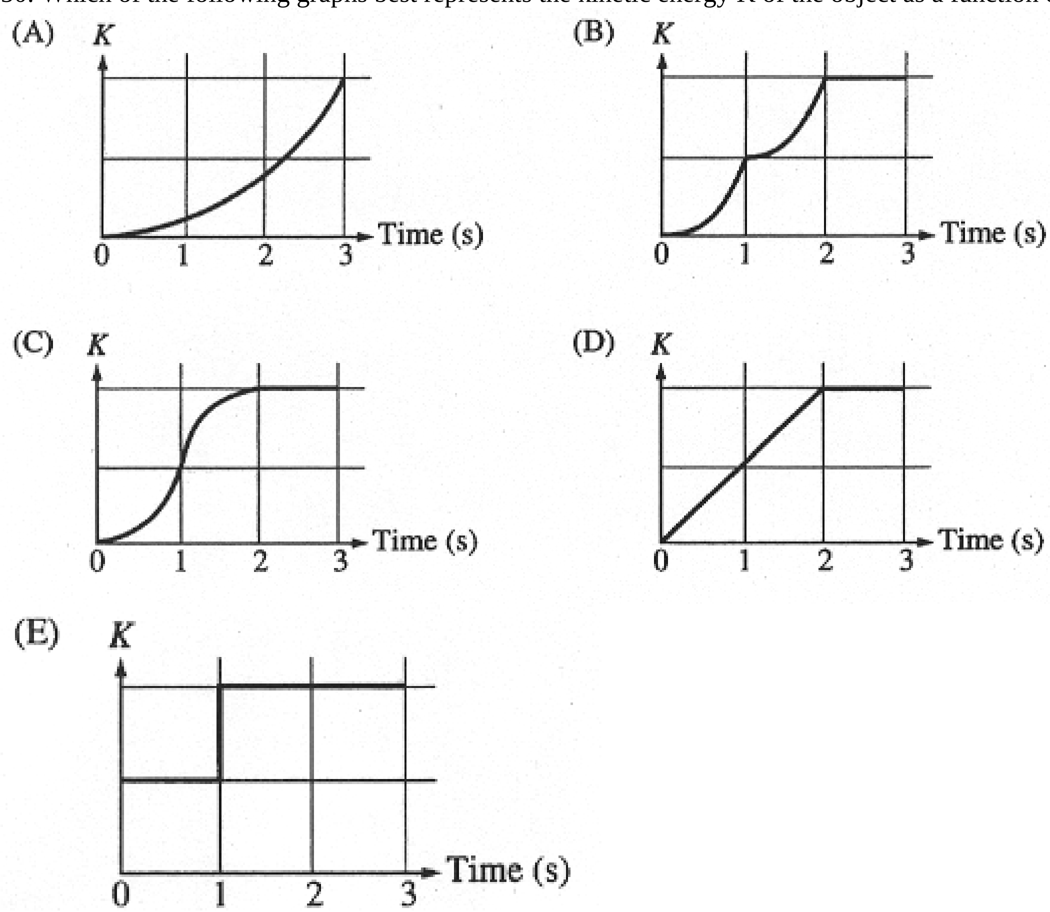
Questions 49-50

An object of mass m is initially at rest and free to move without friction in any direction in the xy-plane. A constant net force of magnitude F directed in the +x direction acts on the object for 1 s. Immediately thereafter a constant net force of the same magnitude F directed in the +y direction acts on the object for 1 s. After this, no forces act on the object.

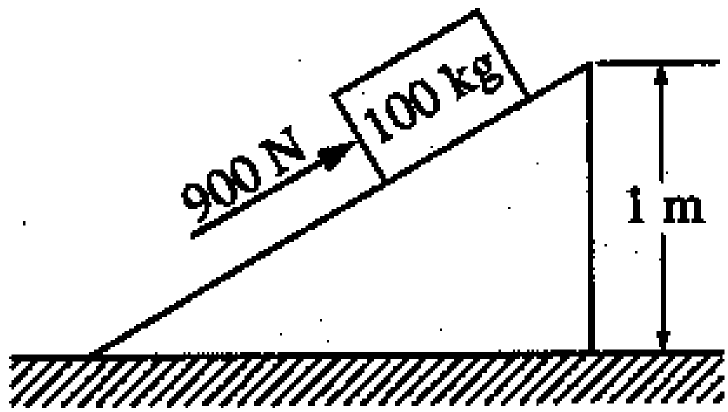
49. Which of the following vectors could represent the velocity of the object at the end of 3 s, assuming the scales on the x and y axes are equal?



50. Which of the following graphs best represents the kinetic energy K of the object as a function of time?

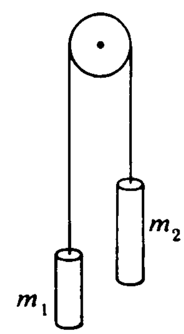


51. A constant force of 900 N pushes a 100 kg mass up the inclined plane shown at a uniform speed of 4 m/s. The power developed by the 900 N force is most nearly
(A) 400 W
(B) 800 W
(C) 900 W
(D) 1000W
(E) 3600 W

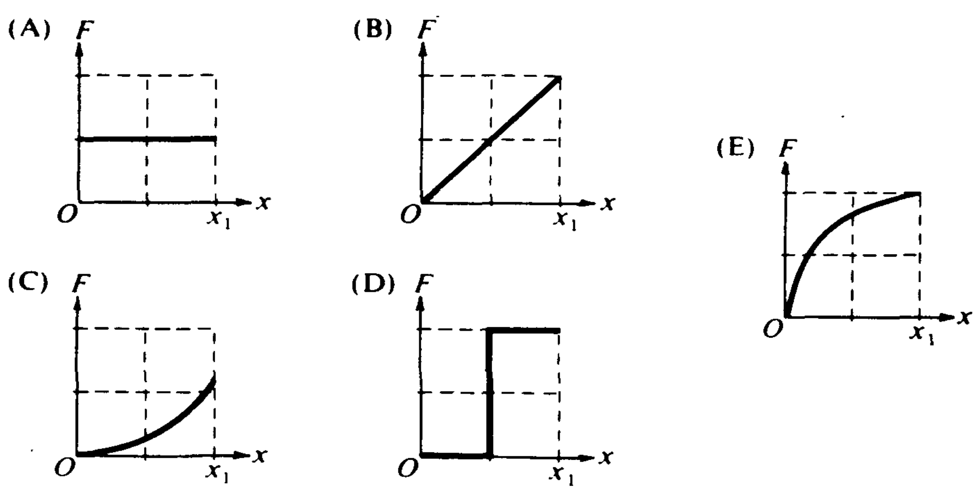


52. An object of mass m is lifted at constant velocity a vertical distance H in time T. The power supplied by the lifting force is (A) mgHT (B) mgH/T (C) mg/HT (D) mgT/H (E) zero

53. A system consists of two objects having masses m_1 and m_2 ($m_1 < m_2$). The objects are connected by a massless string, hung over a pulley as shown, and then released. When the object of mass m_2 has descended a distance h, the potential energy of the system has decreased by
(A) $(m_2 - m_1)gh$ (B) m_2gh (C) $(m_1 + m_2)gh$ (D) $\frac{1}{2}(m_1 + m_2)gh$ (E) 0

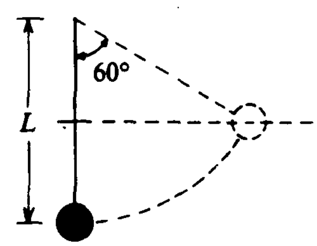


54. The following graphs, all drawn to the same scale, represent the net force F as a function of displacement x for an object that moves along a straight line. Which graph represents the force that will cause the greatest change in the kinetic energy of the object from x = 0 to x = x_1?



55. From the top of a 70-meter-high building, a l-kilogram ball is thrown directly downward with an initial speed of 10 meters per second. If the ball reaches the ground with a speed of 30 meters per second, the energy lost to friction is most nearly (A) 0J (B) 100 J (C) 300 J (D) 400 J (E) 700 J

56. A pendulum consists of a ball of mass m suspended at the end of a massless cord of length L as shown. The pendulum is drawn aside through an angle of 60° with the vertical and released. At the low point of its swing, the speed of the pendulum ball is

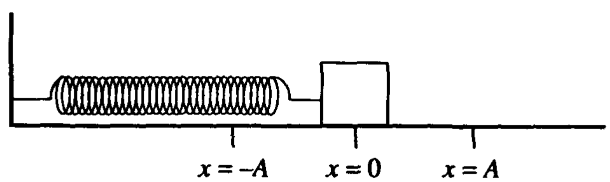


(A) $\sqrt{gL}$ (B) $\sqrt{2gL}$ (C) ½gL (D) gL (E) 2gL

57. A rock is lifted for a certain time by a force F that is greater in magnitude than the rock's weight W. The change in kinetic energy of the rock during this time is equal to the
(A) work done by the net force (F - W)
(B) work done by F alone
(C) work done by W alone
(D) difference in the momentum of the rock before and after this time
(E) difference in the potential energy of the rock before and after this time.

58. A ball is thrown upward. At a height of 10 meters above the ground, the ball has a potential energy of 50 joules (with the potential energy equal to zero at ground level) and is moving upward with a kinetic energy of 50 joules. Air friction is negligible. The maximum height reached by the ball is most nearly
(A) 10 m (B) 20 m (C) 30 m (D) 40 m (E) 50 m

59. A block on a horizontal frictionless plane is attached to a spring, as shown. The block oscillates along the x-axis with amplitude A. Which of the following statements about energy is correct?



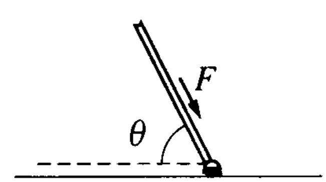
(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.
(E) The kinetic energy of the block is always equal to the potential energy of the spring.

60. During a certain time interval, a constant force delivers an average power of 4 watts to an object. If the object has an average speed of 2 meters per second and the force acts in the direction of motion of the object, the magnitude of the force is
(A) 16 N (B) 8 N (C) 6 N (D) 4N (E) 2N

61. A spring-loaded gun can fire a projectile to a height h if it is fired straight up. If the same gun is pointed at an angle of 45° from the vertical, what maximum height can now be reached by the projectile?

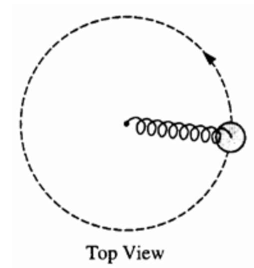
(A) h/4 (B) $\frac{h}{2\sqrt{2}}$ (C) h/2 (D) $\frac{h}{\sqrt{2}}$ (E) h

62. A force F is exerted by a broom handle on the head of the broom, which has a mass m. The handle is at an angle θ to the horizontal, as shown. The work done by the force on the head of the broom as it moves a distance d across a horizontal floor is
(A) Fd sin θ (B) Fd cosθ (C) Fm cosθ (D) Fm tanθ (E) Fmd sinθ



Question 63-64

A spring has a force constant of 100 N/m and an unstretched length of 0.07 m. One end is attached to a post that is free to rotate in the center of a smooth table, as shown in the top view. The other end is attached to a 1 kg disc moving in uniform circular motion on the table, which stretches the spring by 0.03 m. Friction is negligible.

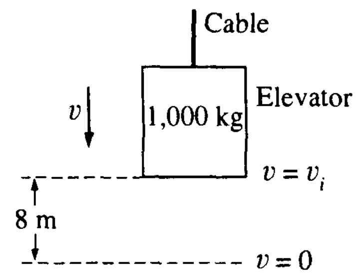


63. What is the centripetal force on the disc?
    (A) 0.3 N (B) 3N (C) 10 N (D) 300 N (E) 1,000 N

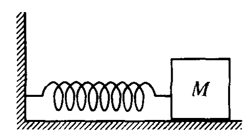
64. What is the work done on the disc by the spring during one full circle?
    (A) 0 J (B) 94 J (C) 186 J (D) 314 J (E) 628 J

65. A frictionless pendulum of length 3 m swings with an amplitude of 10°. At its maximum displacement, the potential energy of the pendulum is 10 J. What is the kinetic energy of the pendulum when its potential energy is 5 J ?
    (A) 3.3 J (B) 5 J (C) 6.7 J (D) 10 J (E) 15 J

66. A descending elevator of mass 1,000 kg is uniformly decelerated to rest over a distance of 8 m by a cable in which the tension is 11,000 N. The speed v_i of the elevator at the beginning of the 8 m descent is most nearly
    (A) 4 m/s (B) 10 m/s (C) 13 m/s
    (D) 16 m/s (E) 21 m/s



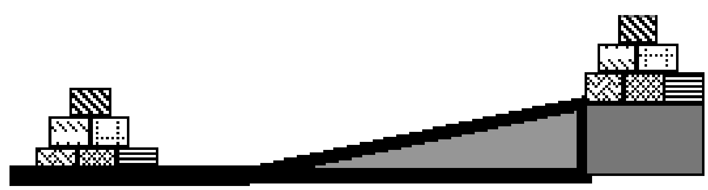
67. An ideal massless spring is fixed to the wall at one end, as shown. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is



(A) $\dfrac{Mg}{A}$ (B) $\dfrac{Mgv_m}{2A}$ (C) $\dfrac{Mv_m^2}{2A}$ (D) $\dfrac{Mv_m^2}{A^2}$ (E) $\dfrac{Mv_m^2}{2A^2}$

68. A 1000 W electric motor lifts a 100 kg safe at constant velocity. The vertical distance through which the motor can raise the safe in 10 s is most nearly
    (A) 1 m (B) 3 m (C) 10 m (D) 32 m (E) 100 m

69. A deliveryman moves 10 cartons from the sidewalk, along a 10-meter ramp to a loading dock, which is 1.5 meters above the sidewalk. If each carton has a mass of 25 kg, what is the total work done by the deliveryman on the cartons to move them to the loading dock?
    (A) 2500 J (|B) 3750 J (C) 10000 J (D) 25000 J (D) 37500 J

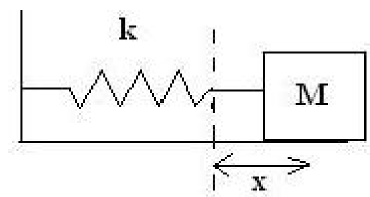


70. A 60.0-kg ball of clay is tossed vertically in the air with an initial speed of 4.60 m/s. Ignoring air resistance, what is the change in its potential energy when it reaches its highest point?
    (A) 0 J (B) 45 J (C) 280 J (D) 635 J (E) 2700

71. A 500-kg car is moving at 28 m/s. The driver sees a barrier ahead. If the car takes 95 meters to come to rest, what is the magnitude of the minimum average net force necessary to stop?
(A) 47.5 N (B) 1400 N (C) 2060 N (D) 19600 N (E) 133000 N

72. A person pushes a block of mass M = 6.0 kg with a constant speed of 5.0 m/s straight up a flat surface inclined 30.0° above the horizontal. The coefficient of kinetic friction between the block and the surface is $\mu = 0.40$. What is the net force acting on the block?
(A) 0*N* (B) 21*N* (C) 30*N* (D) 51*N* (E) 76*N*

73. A block of mass *M* on a horizontal surface is connected to the end of a massless spring of spring constant *k* . The block is pulled a distance *x* from equilibrium and when released from rest, the block moves toward equilibrium. What coefficient of kinetic friction between the surface and the block would allow the block to return to equilibrium and stop?

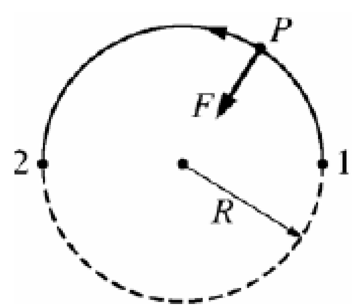


$(A)\ \frac{kx^2}{2Mg}$ $(B)\ \frac{kx}{Mg}$ $(C)\ \frac{kx}{2Mg}$ $(D)\ \frac{Mg}{2kx}$ $(E)\ \frac{k}{4Mgx}$

74. An object is dropped from rest from a certain height. Air resistance is negligible. After falling a distance d, the object's kinetic energy is proportional to which of the following?
(A) $1 / d^2$ (B) 1 / d (C) $\sqrt{d}$ (D) d (E) d^2

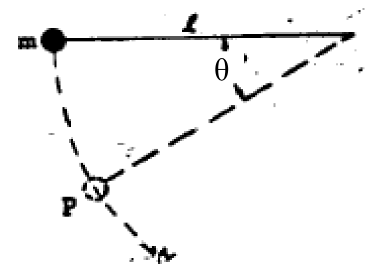
75. An object is projected vertically upward from ground level. It rises to a maximum height H. If air resistance is negligible, which of the following must be true for the object when it is at a height H/2 ?
(A) Its speed is half of its initial speed.
(B) Its kinetic energy is half of its initial kinetic energy.
(C) Its potential energy is half of its initial potential energy.
(D) Its total mechanical energy is half of its initial value.
(E) Its total mechanical energy is half of its value at the highest point.

76. A particle P moves around the circle of radius R under the influence of a radial force of magnitude F as shown. What is the work done by the radial force as the particle moves from position 1 to position 2 halfway around the circle?
(A) Zero (B) RF (C) 2RF (D) πRF (E) 2πRF



AP Physics Free Response Practice – Work Power Energy

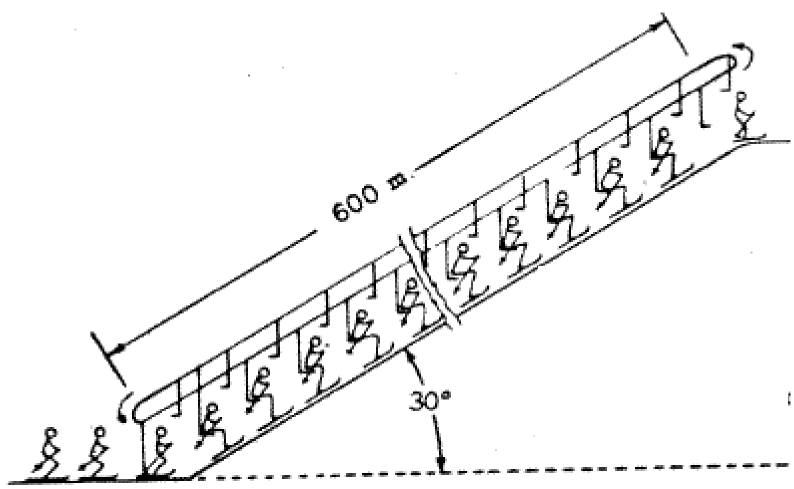
1974B1. A pendulum consisting of a small heavy ball of mass m at the end of a string of length L is released from a horizontal position. When the ball is at point P, the string forms an angle of θ with the horizontal as shown.

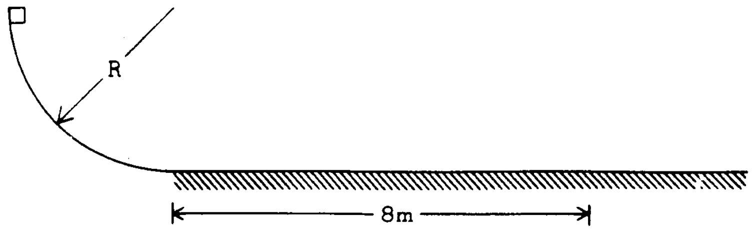


(a) In the space below, draw a force diagram showing all of the forces acting on the ball at P. Identify each force clearly.

(b) Determine the speed of the ball at P.
(c) Determine the tension in the string when the ball is at P.

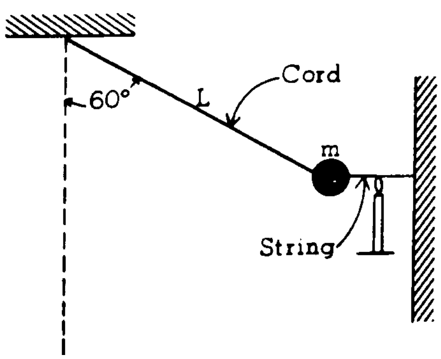
1974B7. A ski lift carries skiers along a 600 meter slope inclined at 30°. To lift a single rider, it is necessary to move 70 kg of mass to the top of the lift. Under maximum load conditions, six riders per minute arrive at the top. If 60 percent of the energy supplied by the motor goes to overcoming friction, what average power must the motor supply?





1975B1. A 2-kilogram block is released from rest at the top of a curved incline in the shape of a quarter of a circle of radius R. The block then slides onto a horizontal plane where it finally comes to rest 8 meters from the beginning of the plane. The curved incline is frictionless, but there is an 8-newton force of friction on the block while it slides horizontally. Assume g = 10 meters per second^2.

a. Determine the magnitude of the acceleration of the block while it slides along the horizontal plane.
b. How much time elapses while the block is sliding horizontally?
c. Calculate the radius of the incline in meters.



1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.
c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

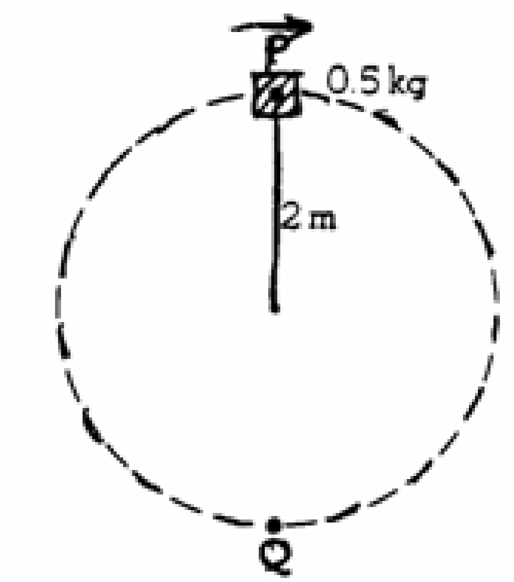
1977 B1. A block of mass 4 kilograms, which has an initial speed of 6 meters per second at time t = 0, slides on a horizontal surface.

a. Calculate the work W that must be done on the block to bring it to rest.

If a constant friction force of magnitude 8 newtons is exerted on the block by the surface, determine the following:

b. The speed v of the block as a function of the time t.

c. The distance x that the block slides as it comes to rest



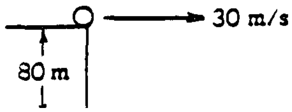
1978B1. A 0.5 kilogram object rotates freely in a vertical circle at the end of a string of length 2 meters as shown above. As the object passes through point P at the top of the circular path, the tension in the string is 20 newtons. Assume g = 10 meters per second squared.

(a) On the following diagram of the object, draw and clearly label all significant forces on the object when it is at the point P.

(b) Calculate the speed of the object at point P.

(c) Calculate the increase in kinetic energy of the object as it moves from point P to point Q.

(d) Calculate the tension in the string as the object passes through point Q.

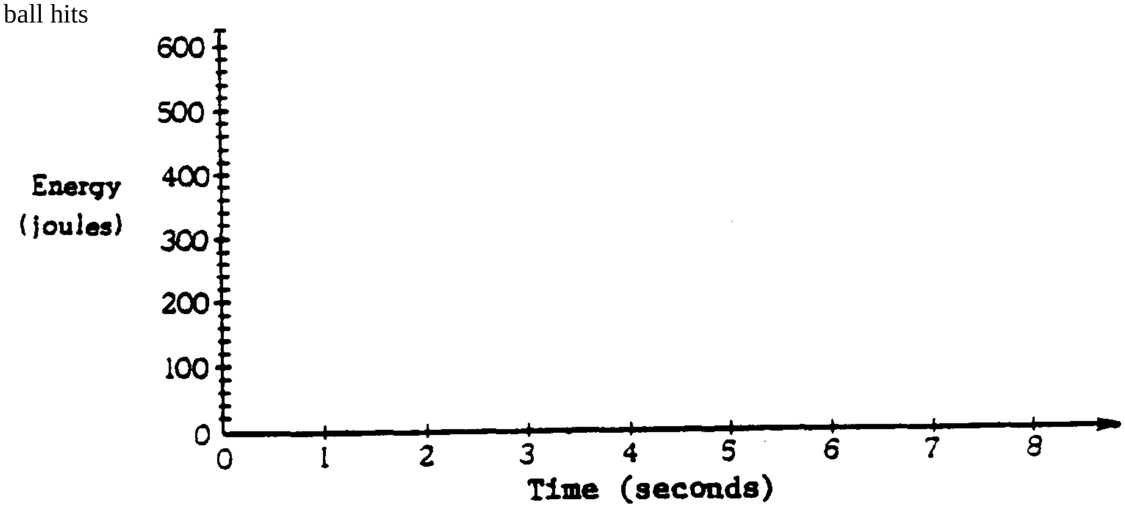


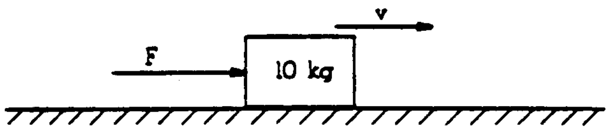
1979B1. From the top of a cliff 80 meters high, a ball of mass 0.4 kilogram is launched horizontally with a velocity of 30 meters per second at time t = 0 as shown above. The potential energy of the ball is zero at the bottom of the cliff. Use g = 10 meters per second squared.

a. Calculate the potential, kinetic, and total energies of the ball at time t = 0.

b. On the axes below, sketch and label graphs of the potential, kinetic, and total energies of the ball as functions of the distance fallen from the top of the cliff

c. On the axes below sketch and label the kinetic and potential energies of the ball as functions of time until the ball hits





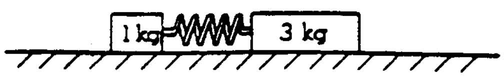
1981B1. A 10-kilogram block is pushed along a rough horizontal surface by a constant horizontal force F as shown above. At time t = 0, the velocity v of the block is 6.0 meters per second in the same direction as the force. The coefficient of sliding friction is 0.2. Assume g = 10 meters per second squared.

a. Calculate the force F necessary to keep the velocity constant.

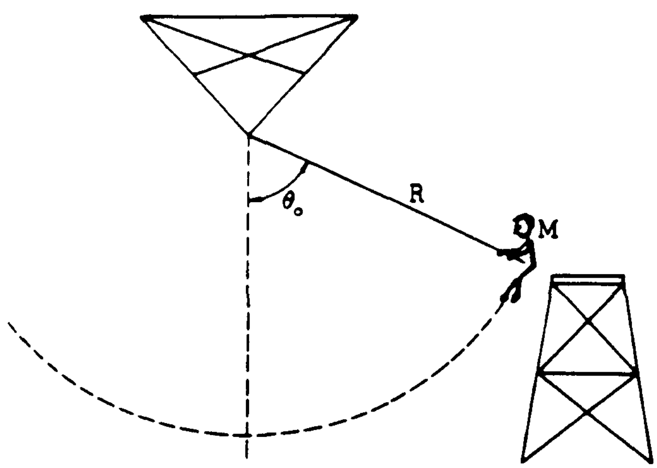
The force is now changed to a larger constant value F'. The block accelerates so that its kinetic energy increases by 60 joules while it slides a distance of 4.0 meters.

b. Calculate the force F'.

c. Calculate the acceleration of the block.



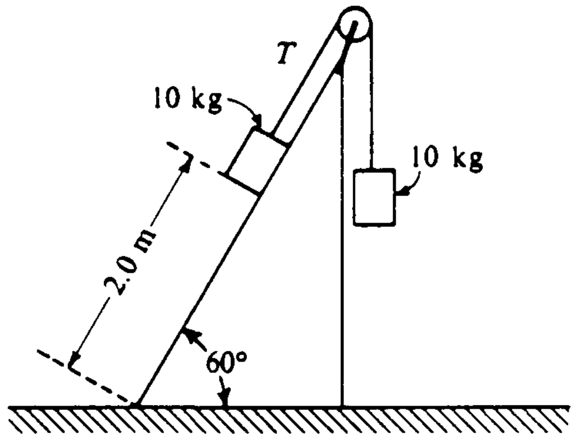
1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table. In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second. Determine the minimum work needed to compress the spring in this experiment.



1982B3. A child of mass M holds onto a rope and steps off a platform. Assume that the initial speed of the child is zero. The rope has length R and negligible mass. The initial angle of the rope with the vertical is θ_o, as shown in the drawing above.

a. Using the principle of conservation of energy, develop an expression for the speed of the child at the lowest point in the swing in terms of g, R, and cos θ_o

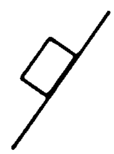
b. The tension in the rope at the lowest point is 1.5 times the weight of the child. Determine the value of cos θ_o.



1985B2. Two 10-kilogram boxes are connected by a massless string that passes over a massless frictionless pulley as shown above. The boxes remain at rest, with the one on the right hanging vertically and the one on the left 2.0 meters from the bottom of an inclined plane that makes an angle of 60° with the horizontal. The coefficients of kinetic friction and static friction between the left-hand box and the plane are 0.15 and 0.30, respectively. You may use $g = 10\ m/s^2$, sin 60° = 0.87, and cos 60° = 0.50.

a. What is the tension T in the string?

b. On the diagram below, draw and label all the forces acting on the box that is on the plane.

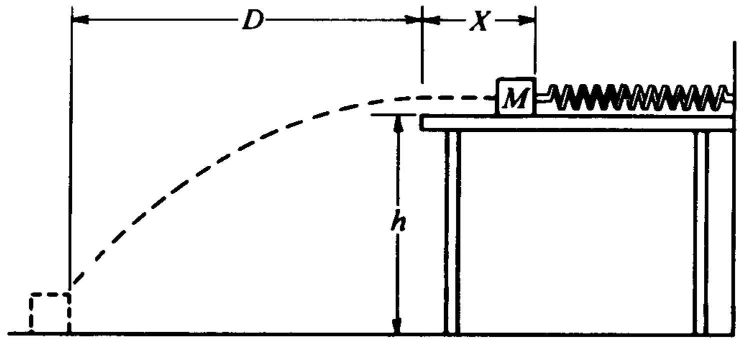


c. Determine the magnitude of the frictional force acting on the box on the plane.

The string is then cut and the left-hand box slides down the inclined plane.

d. Determine the amount of mechanical energy that is converted into thermal energy during the slide to the bottom.

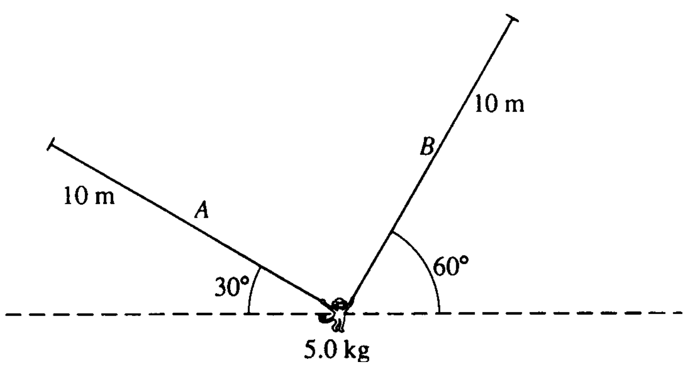
e. Determine the kinetic energy of the left-hand box when it reaches the bottom of the plane.



1986B2. One end of a spring is attached to a solid wall while the other end just reaches to the edge of a horizontal, frictionless tabletop, which is a distance *h* above the floor. A block of mass M is placed against the end of the spring and pushed toward the wall until the spring has been compressed a distance X, as shown above. The block is released, follows the trajectory shown, and strikes the floor a horizontal distance D from the edge of the table. Air resistance is negligible.

Determine expressions for the following quantities in terms of M, X, D, h, and g. Note that these symbols do not include the spring constant.

a. The time elapsed from the instant the block leaves the table to the instant it strikes the floor
b. The horizontal component of the velocity of the block just before it hits the floor
c. The work done on the block by the spring
d. The spring constant



sin 30° = 0.50	sin 60° = 0.87
cos 30° = 0.87	cos 60° = 0.50
tan 30° = 0.58	tan 60° = 1.73

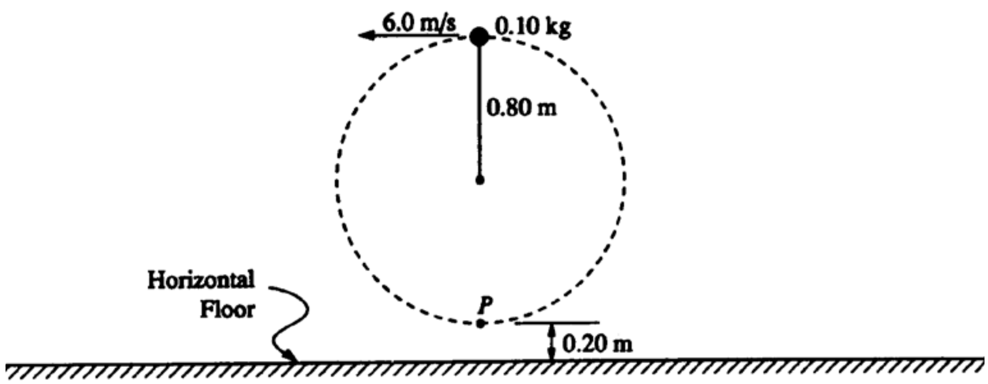
1991B1. A 5.0-kilogram monkey hangs initially at rest from two vines, A and B. as shown above. Each of the vines has length 10 meters and negligible mass.

a. On the figure below, draw and label all of the forces acting on the monkey. (Do not resolve the forces into components, but do indicate their directions.)

b. Determine the tension in vine B while the monkey is at rest.

The monkey releases vine A and swings on vine B. Neglect air resistance.

c. Determine the speed of the monkey as it passes through the lowest point of its first swing.
d. Determine the tension in vine B as the monkey passes through the lowest point of its first swing.

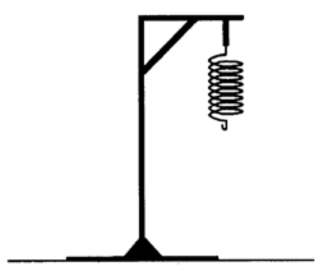


1992B1. A 0.10-kilogram solid rubber ball is attached to the end of an 0.80 meter length of light thread. The ball is swung in a vertical circle, as shown in the diagram above. Point P, the lowest point of the circle, is 0.20 meter above the floor. The speed of the ball at the top of the circle is 6.0 meters per second, and the total energy of the ball is kept constant.

a. Determine the total energy of the ball, using the floor as the zero point for gravitational potential energy.
b. Determine the speed of the ball at point P, the lowest point of the circle.
c. Determine the tension in the thread at
   i. the top of the circle;
   ii. the bottom of the circle.

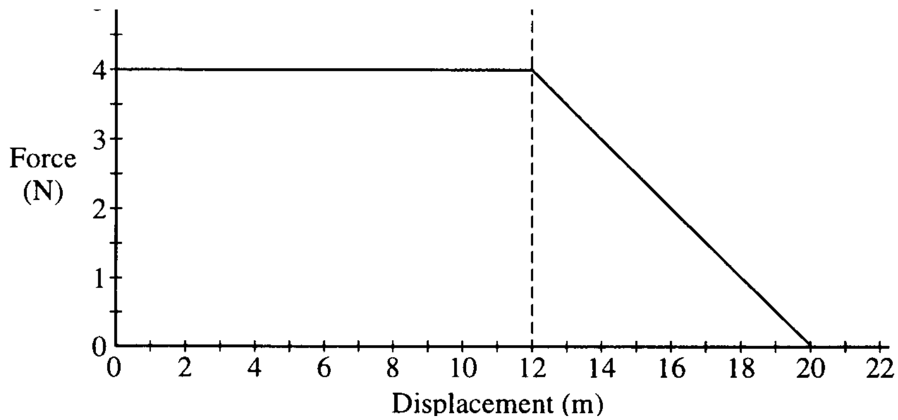
The ball only reaches the top of the circle once before the thread breaks when the ball is at the lowest point of the circle.

d. Determine the horizontal distance that the ball travels before hitting the floor.



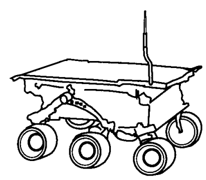
1996B2 (15 points) A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?
b. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

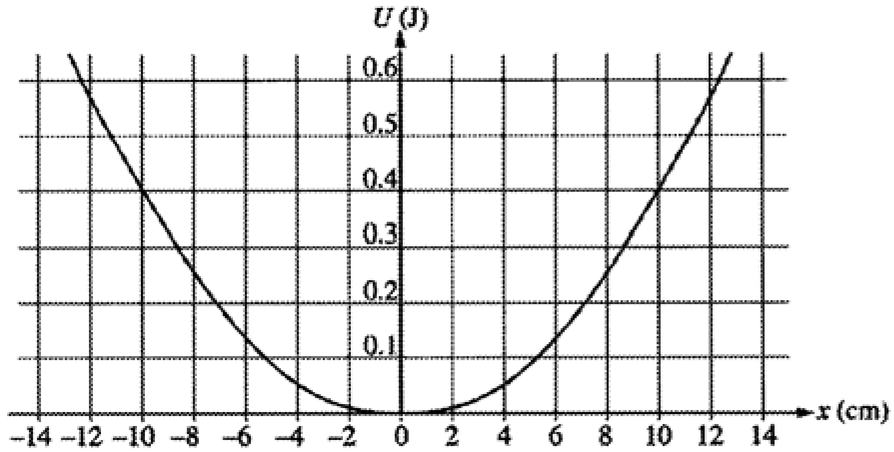
a. The acceleration of the particle when its displacement x is 6 m.
b. The time taken for the object to be displaced the first 12 m.
c. The amount of work done by the net force in displacing the object the first 12 m.
d. The speed of the object at displacement x = 12 m.
e. The final speed of the object at displacement x = 20 m.



1999B1. The Sojourner rover vehicle shown in the sketch above was used to explore the surface of Mars as part of the Pathfinder mission in 1997. Use the data in the tables below to answer the questions that follow.

Mars Data	Sojourner Data	
Radius: 0.53 x Earth's radius	Mass of Sojourner vehicle:	11.5 kg
Mass: 0.11 x Earth's mass	Wheel diameter: 0.13 m	
	Stored energy available:	5.4×10^5 J
	Power required for driving under average conditions:	10 W
	Land speed:	6.7×10^{-3} m/s

a. Determine the acceleration due to gravity at the surface of Mars in terms of g, the acceleration due to gravity at the surface of Earth.
b. Calculate Sojourner's weight on the surface of Mars.
c. Assume that when leaving the Pathfinder spacecraft Sojourner rolls down a ramp inclined at 20° to the horizontal. The ramp must be lightweight but strong enough to support Sojourner. Calculate the minimum normal force that must be supplied by the ramp.
d. What is the net force on Sojourner as it travels across the Martian surface at constant velocity? Justify your answer.
e. Determine the maximum distance that Sojourner can travel on a horizontal Martian surface using its stored energy.
f. Suppose that 0.010% of the power for driving is expended against atmospheric drag as Sojourner travels on the Martian surface. Calculate the magnitude of the drag force.



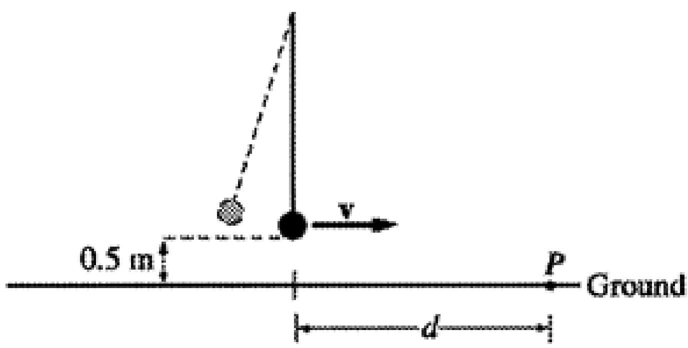
2002B2. A 3.0 kg object subject to a restoring force *F* is undergoing simple harmonic motion with a small amplitude. The potential energy *U* of the object as a function of distance x from its equilibrium position is shown above. This particular object has a total energy *E*: of 0.4 J.

(a) What is the object's potential energy when its displacement is +4 cm from its equilibrium position?

(b) What is the farthest the object moves along the x axis in the positive direction? Explain your reasoning.

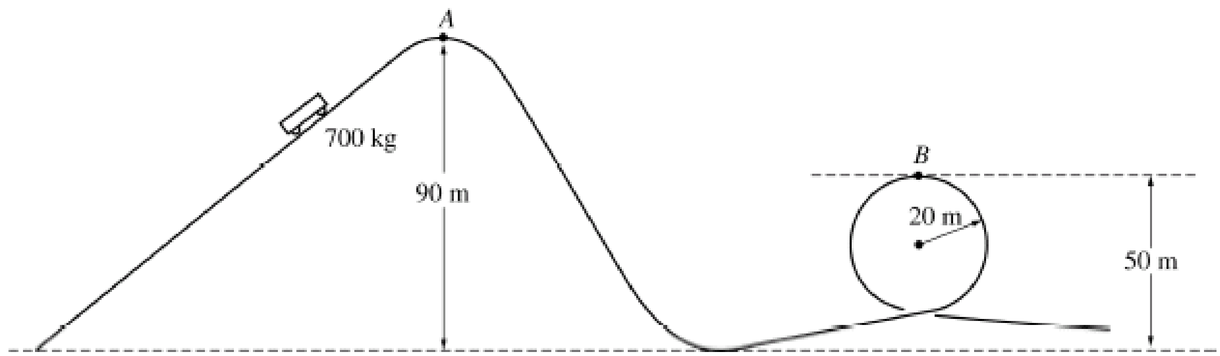
(c) Determine the object's kinetic energy when its displacement is – 7 cm.

(d) What is the object's speed at x = 0 ?



Note: Figure not drawn to scale.

(e) Suppose the object undergoes this motion because it is the bob of a simple pendulum as shown above. If the object breaks loose from the string at the instant the pendulum reaches its lowest point and hits the ground at point *P* shown, what is the horizontal distance *d* that it travels?



2004B1.
A roller coaster ride at an amusement park lifts a car of mass 700 kg to point *A* at a height of 90 m above the lowest point on the track, as shown above. The car starts from rest at point *A*, rolls with negligible friction down the incline and follows the track around a loop of radius 20 m. Point *B*, the highest point on the loop, is at a height of 50 m above the lowest point on the track.

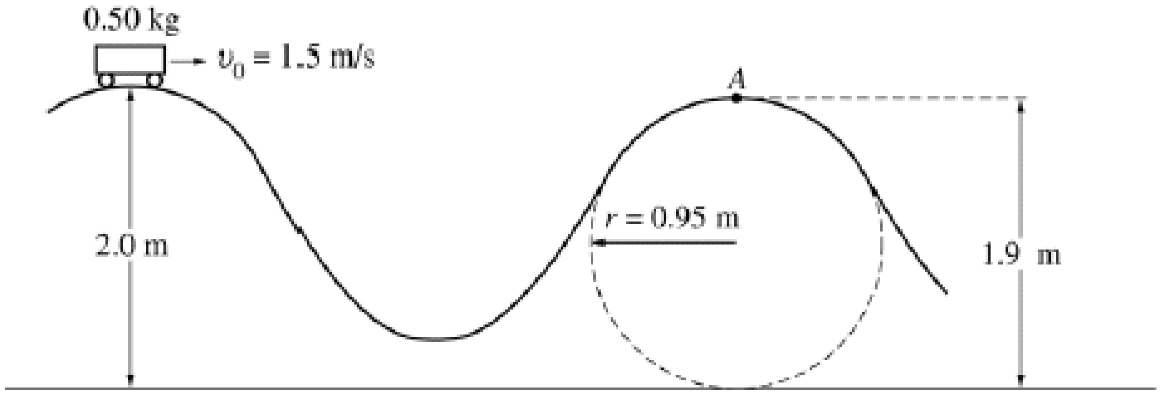
(a)
i. Indicate on the figure the point *P* at which the maximum speed of the car is attained.
ii. Calculate the value v_{msx} of this maximum speed.

(b) Calculate the speed v_B of the car at point *B*.

(c)
i. On the figure of the car below, draw and label vectors to represent the forces acting on the car when it is upside down at point *B*.

ii. Calculate the magnitude of all the forces identified in (c)

(d) Now suppose that friction is not negligible. How could the loop be modified to maintain the same speed at the top of the loop as found in (b)? Justify your answer.



B2004B1.
A designer is working on a new roller coaster, and she begins by making a scale model. On this model, a car of total mass 0.50 kg moves with negligible friction along the track shown in the figure above. The car is given an initial speed v_o = 1.5 m/s at the top of the first hill of height 2.0 m. Point *A* is located at a height of 1.9 m at the top of the second hill, the upper part of which is a circular arc of radius 0.95 m.

(a) Calculate the speed of the car at point *A*.

(b) On the figure of the car below, draw and label vectors to represent the forces on the car at point *A*.

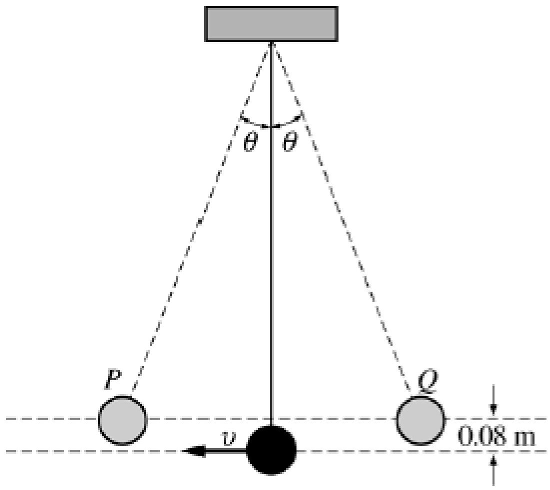
(c) Calculate the magnitude of the force of the track on the car at point *A*.

(d) In order to stop the car at point *A*, some friction must be introduced. Calculate the work that must be done by the friction force in order to stop the car at point *A*.

(e) Explain how to modify the track design to cause the car to lose contact with the track at point *A* before descending down the track. Justify your answer.

B2005B2

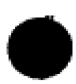
A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



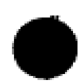
Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

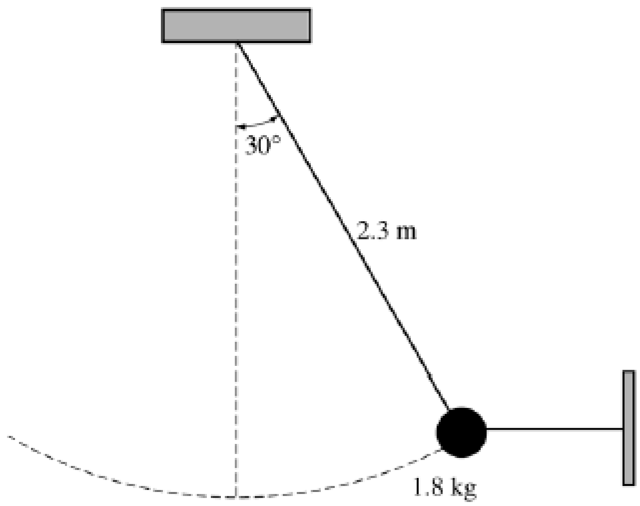


ii. When it is in motion at its lowest position

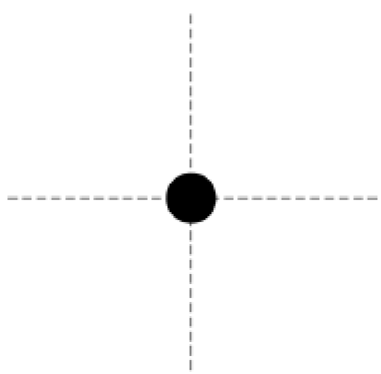


(b) Calculate the speed v of the bob at its lowest position.
(c) Calculate the tension in the string when the bob is passing through its lowest position.

2005B2.

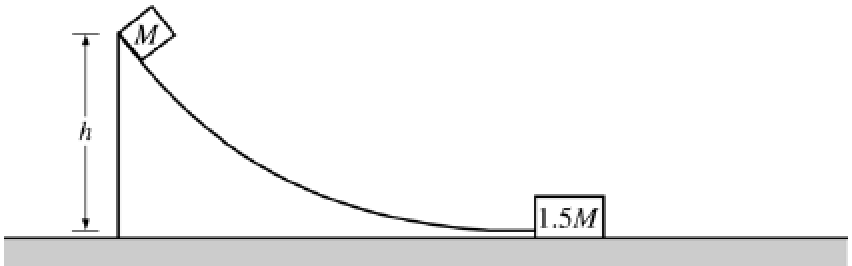


2. (10 points)
A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.



(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

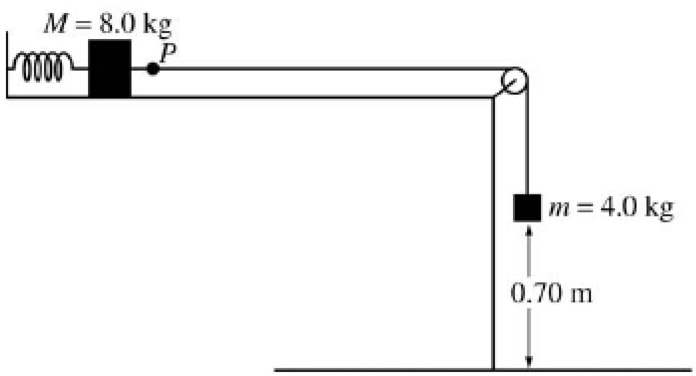
B2006B2



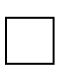
A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2v_o$ immediately after the collision.

Express your answers to the following questions in terms of the given quantities and fundamental constants.
(a) Determine the height h of the ramp from which the small block was released.
(b) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.

2006B1



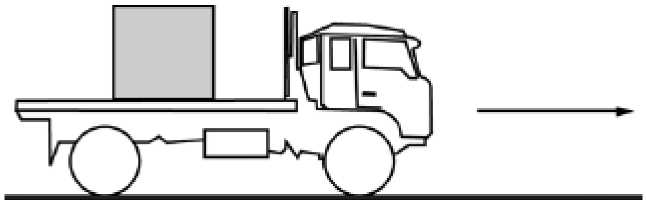
An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.
(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.
M = 8.0 kg $\qquad\qquad$ m = 4.0 kg




(b) Calculate the tension in the string.
(c) Calculate the force constant of the spring.

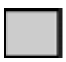
The string is now cut at point P.
(d) Calculate the time taken by the 4.0 kg block to hit the floor.
(e) Calculate the maximum speed attained by the 8.0 kg block as it oscillates back and forth

B2008B2



A 4700 kg truck carrying a 900 kg crate is traveling at 25 m/s to the right along a straight, level highway, as shown above. The truck driver then applies the brakes, and as it slows down, the truck travels 55 m in the next 3.0 s. The crate does not slide on the back of the truck.

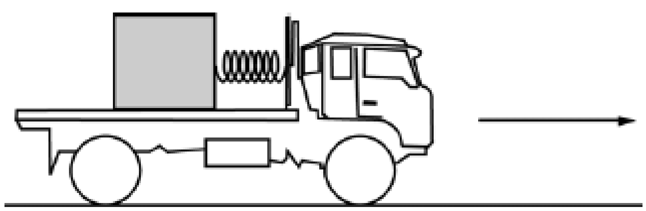
(a) Calculate the magnitude of the acceleration of the truck, assuming it is constant.
(b) On the diagram below, draw and label all the forces acting on the crate during braking.



(c)
i. Calculate the minimum coefficient of friction between the crate and truck that prevents the crate from sliding.

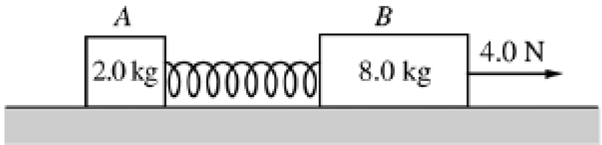
ii. Indicate whether this friction is static or kinetic.
____ Static ____Kinetic

Now assume the bed of the truck is frictionless, but there is a spring of spring constant 9200 N/m attaching the crate to the truck, as shown below. The truck is initially at rest.



(d) If the truck and crate have the same acceleration, calculate the extension of the spring as the truck accelerates from rest to 25 m s in 10 s.
(e) At some later time, the truck is moving at a constant speed of 25 m/s and the crate is in equilibrium. Indicate whether the extension of the spring is greater than, less than, or the same as in part (d) when the truck was accelerating.
___ Greater ___ Less ___ The same
Explain your reasoning.

2008B2

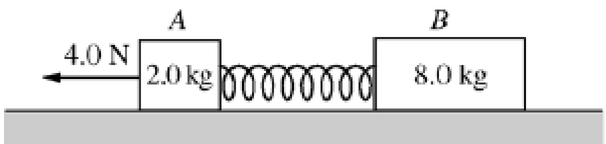


Block *A* of mass 2.0 kg and block *B* of mass 8.0 kg are connected as shown above by a spring of spring constant 80 N/m and negligible mass. The system is being pulled to the right across a horizontal frictionless surface by a horizontal force of 4.0 N, as shown, with both blocks experiencing equal constant acceleration.

(a) Calculate the force that the spring exerts on the 2.0 kg block.

(b**)** Calculate the extension of the spring.

The system is now pulled to the left, as shown below, with both blocks again experiencing equal constant acceleration.



(c**)** Is the magnitude of the acceleration greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

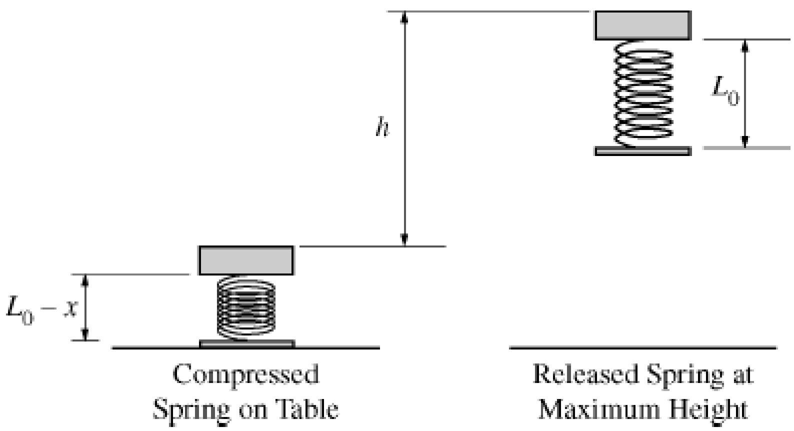
(d) Is the amount the spring has stretched greater than, less than, or the same as before?

____ Greater ____ Less ____ The same

Justify your answer.

(e) In a new situation, the blocks and spring are moving together at a constant speed of 0.50 m s to the left. Block *A* then hits and sticks to a wall. Calculate the maximum compression of the spring.

2009B1



In an experiment, students are to calculate the spring constant *k* of a vertical spring in a small jumping toy that initially rests on a table. When the spring in the toy is compressed a distance *x* from its uncompressed length L_0 and the toy is released, the top of the toy rises to a maximum height *h* above the point of maximum compression. The students repeat the experiment several times, measuring *h* with objects of various masses taped to the top of the toy so that the combined mass of the toy and added objects is *m*. The bottom of the toy and the spring each have negligible mass compared to the top of the toy and the objects taped to it.

(a) Derive an expression for the height *h* in terms of *m*, *x*, *k*, and fundamental constants.

With the spring compressed a distance *x* = 0.020 m in each trial, the students obtained the following data for different values of *m*.

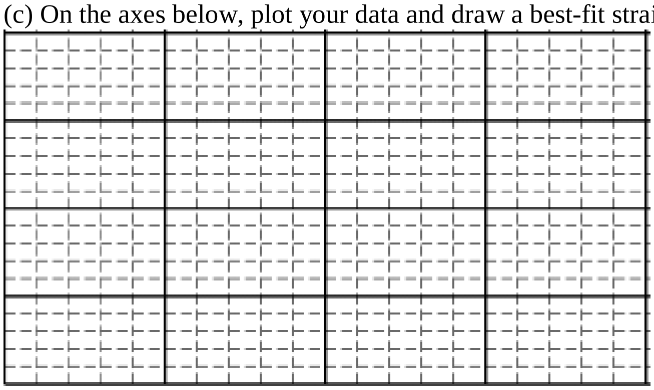
	m (kg)	*h* (m)	
	0.020	0.49	
	0.030	0.34	
	0.040	0.28	
	0.050	0.19	
	0.060	0.18	

(b)

i. What quantities should be graphed so that the slope of a best-fit straight line through the data points can be used to calculate the spring constant *k* ?

ii. Fill in one or both of the blank columns in the table with calculated values of your quantities, including units.

(c) On the axes below, plot your data and draw a best-fit straight line. Label the axes and indicate the scale.



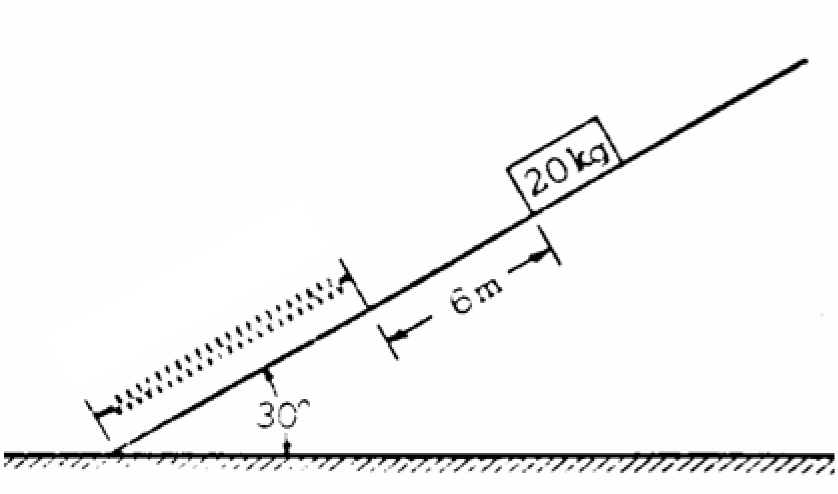
(d) Using your best-fit line, calculate the numerical value of the spring constant.

(e) Describe a procedure for measuring the height *h* in the experiment, given that the toy is only momentarily at that maximum height.

C1973M2. A 30-gram bullet is fired with a speed of 500 meters per second into a wall.

a. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, calculate the force on the bullet while it is stopping.

b. If the deceleration of the bullet is constant and it penetrates 12 centimeters into the wall, how much time is required for the bullet to stop?

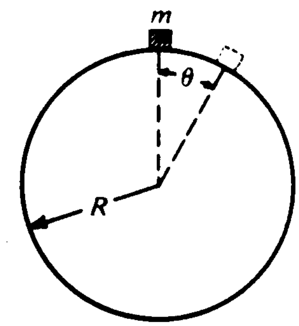


C1982M1. A 20 kg mass, released from rest, slides 6 meters down a frictionless plane inclined at an angle of 30º with the horizontal and strikes a spring of unknown spring constant as shown in the diagram above. Assume that the spring is ideal, that the mass of the spring is negligible, and that mechanical energy is conserved.

a. Determine the speed of the block just before it hits the spring.

b. Determine the spring constant given that the distance the spring compresses along the incline is 3m when the block comes to rest.

c. Is the speed of the block a maximum at the instant the block strikes the spring? Justify your answer.

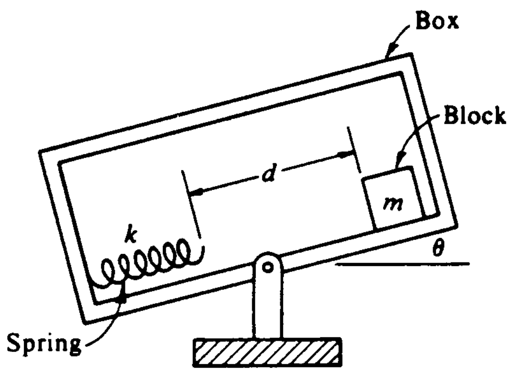


C1983M3. A particle of mass m slides down a fixed, frictionless sphere of radius *R*. starting from rest at the top.

a. In terms of m, g, *R*. and θ, determine each of the following for the particle while it is sliding on the sphere.

i. The kinetic energy of the particle

ii. The centripetal acceleration of the mass



C1985M2. An apparatus to determine coefficients of friction is shown above. At the angle θ shown with the horizontal, the block of mass m just starts to slide. The box then continues to slide a distance d at which point it hits the spring of force constant k, and compresses the spring a distance x before coming to rest. In terms of the given quantities and fundamental constants, derive an expression for each of the following.

a. μ_s the coefficient of static friction.

b. ΔE, the loss in total mechanical energy of the block-spring system from the start of the block down the incline to the moment at which it comes to rest on the compressed spring.

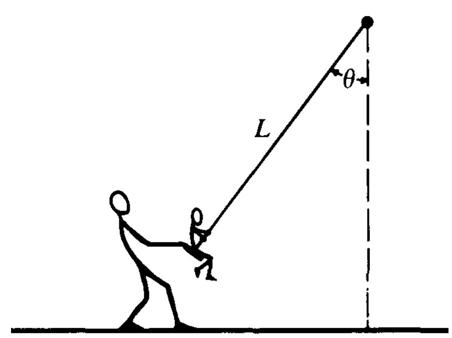
c. μ_k, the coefficient of kinetic friction.

C1987M1. An adult exerts a horizontal force on a swing that is suspended by a rope of length L, holding it at an angle θ with the vertical. The child in the swing has a weight W and dimensions that are negligible compared to L. The weights of the rope and of the seat are negligible. In terms of W and θ, determine

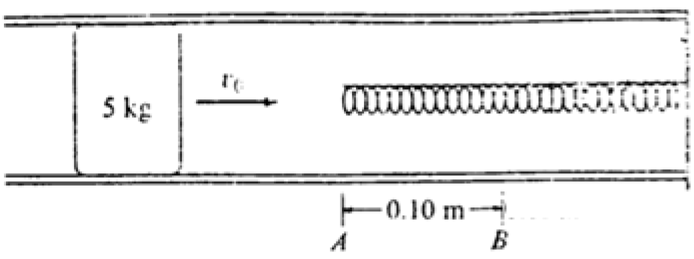
a) The tension in the rope

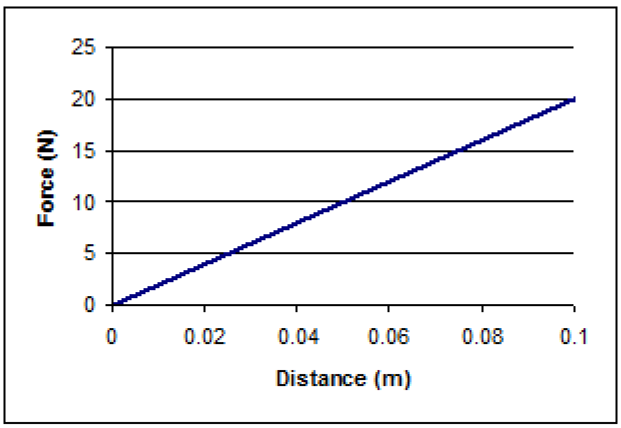
b) The horizontal force exerted by the adult.

c) The adult releases the swing from rest. In terms of W and θ determine the tension in the rope as the swing passes through its lowest point



C1988M2.



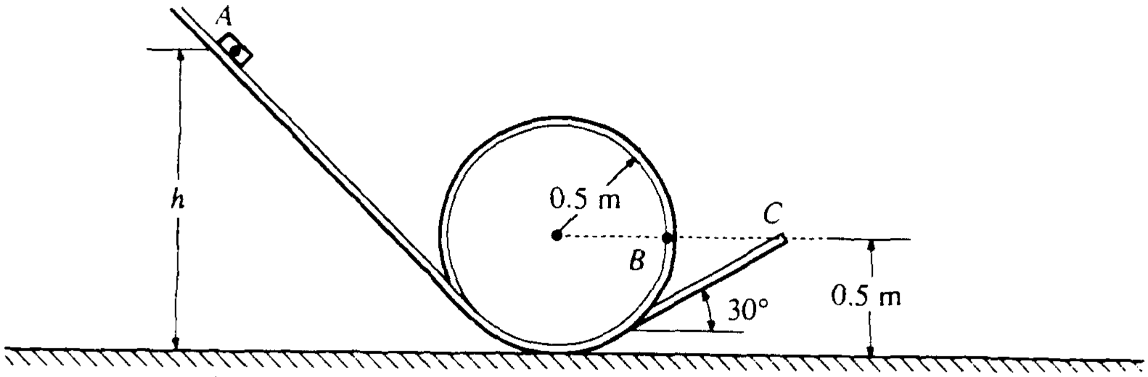


A 5-kilogram object initially slides with speed v_o in a hollow frictionless pipe. The end of the pipe contains a spring as shown. The object makes contact with the spring at point A and moves 0.1 meter before coming to rest at point B. The graph shows the magnitude of the force exerted on the object by the spring as a function of the objects distance from point A.

a. Calculate the spring constant for the spring.

b. Calculate the decrease in kinetic energy of the object as it moves from point A to point B.

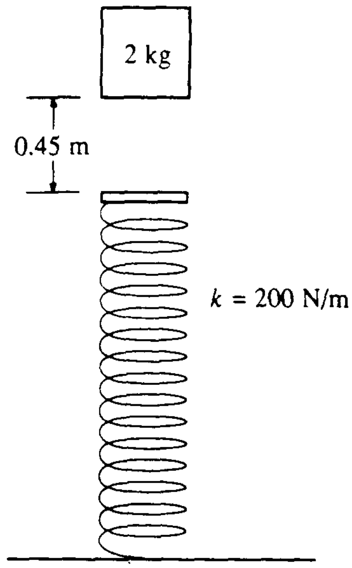
c. Calculate the initial speed v_o of the object



C1989M1. A 0.1 kilogram block is released from rest at point A as shown above, a vertical distance h above the ground. It slides down an inclined track, around a circular loop of radius 0.5 meter, then up another incline that forms an angle of 30° with the horizontal. The block slides off the track with a speed of 4 m/s at point C, which is a height of 0.5 meter above the ground. Assume the entire track to be frictionless and air resistance to be negligible.

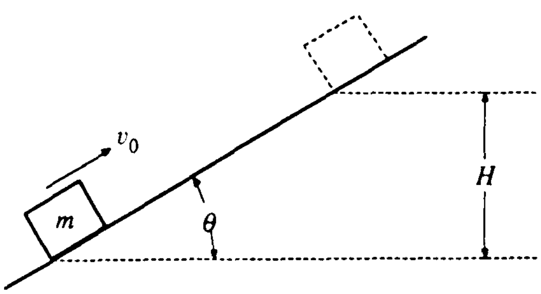
a. Determine the height h .
b. On the figure below, draw and label all the forces acting on the block when it is at point B. which is 0.5 meter above the ground.

c. Determine the magnitude of the velocity of the block when it is at point B.
d. Determine the magnitude of the force exerted by the track on the block when it is at point B.
e. Determine the maximum height above the ground attained by the block after it leaves the track.
f. Another track that has the same configuration, but is **NOT** frictionless, is used. With this track it is found that if the block is to reach point C with a speed of 4 m/s, the height h must be 2 meters. Determine the work done by the frictional force.



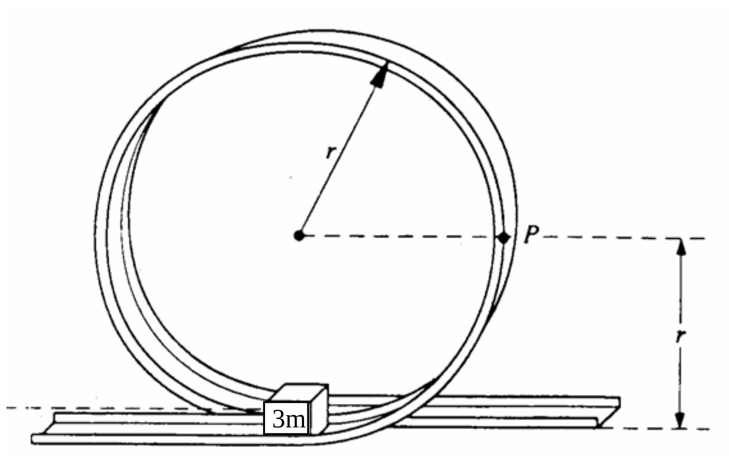
C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.

a. Determine the speed of the block at the instant it hits the end of the spring.
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Is the speed of the block a maximum at the equilibrium position, explain.



C1990M2. A block of mass m slides up the incline shown above with an initial speed v_o in the position shown.

a. If the incline is frictionless, determine the maximum height H to which the block will rise, in terms of the given quantities and appropriate constants.
b. If the incline is rough with coefficient of sliding friction μ, the box slides a distance $d = h_2 / \sin\theta$ along the length of the ramp as it reaches a new maximum height h_2. Determine the new maximum height h_2 in terms of the given quantities.

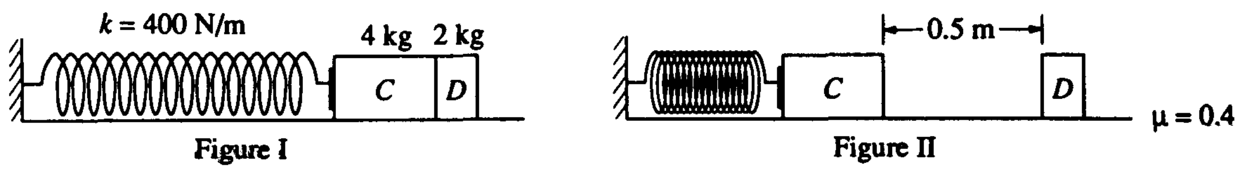


C1991M1. A small block of mass 3m moving at speed v_0 / 3 enters the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. Determine each of the following in terms of m, v_o , r, and g.

a. The kinetic energy of the block and bullet when they reach point P on the loop

b. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times

c The new required entry speed v_o' at the bottom of the loop such that the conditions in part b apply.



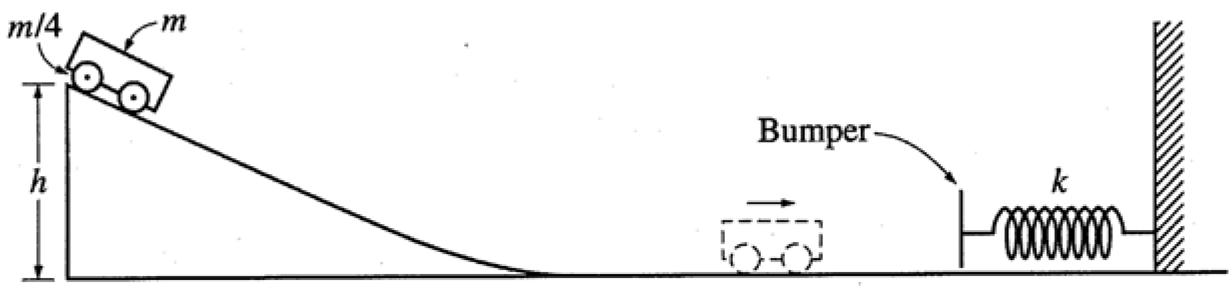
C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a rough horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

Block C is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right at 3 m/s. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

c. The horizontal distance the combined blocks move after leaving the spring before coming to rest

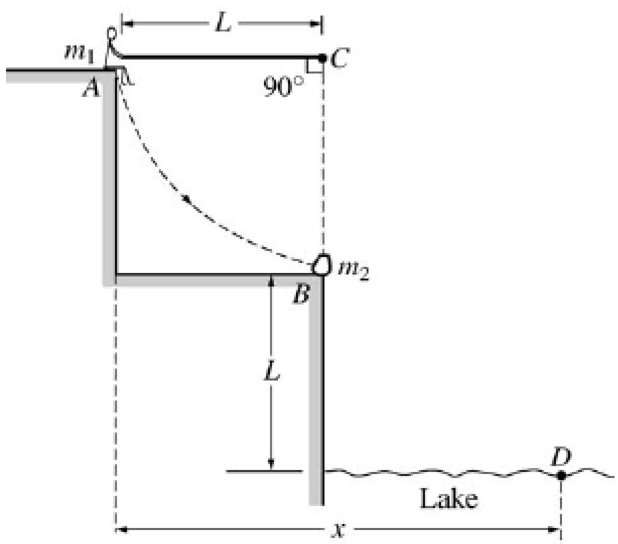


C2002M2. The cart shown above has a mass 2m. The cart is released from rest and slides from the top of an inclined frictionless plane of height h. Express all algebraic answers in terms of the given quantities and fundamental constants.

a. Determine the speed of the cart when it reaches the bottom of the incline.

b. After sliding down the incline and across the frictionless horizontal surface, the cart collides with a bumper of negligible mass attached to an ideal spring, which has a spring constant k. Determine the distance x_m the spring is compressed before the cart and bumper come to rest.

C2004M1



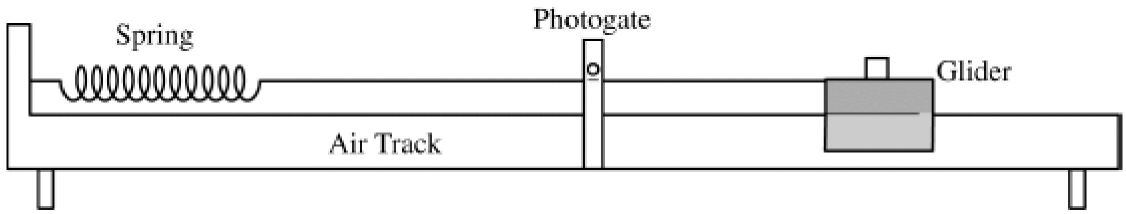
A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object

(b) The tension in the rope just before the collision with the object

(c) After the person hits and grabs the rock, the speed of the combined masses is determined to be v'. In terms of v' and the given quantities, determine the total horizontal displacement *x* of the person from position *A* until the person and object land in the water at point *D*.

C2007M3.

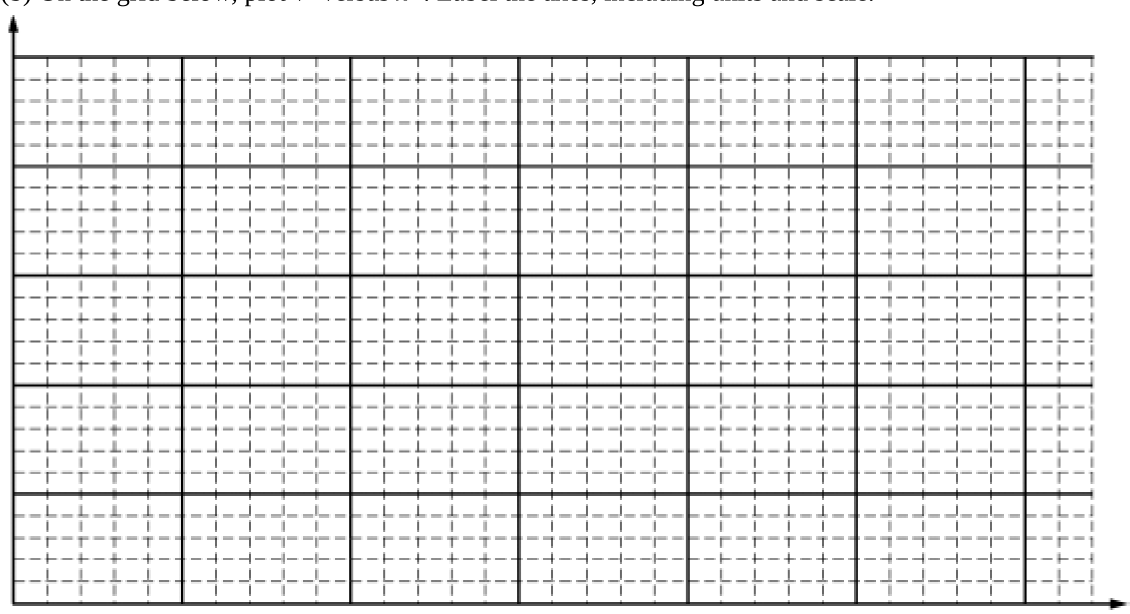


The apparatus above is used to study conservation of mechanical energy. A spring of force constant 40 N/m is held horizontal over a horizontal air track, with one end attached to the air track. A light string is attached to the other end of the spring and connects it to a glider of mass *m*. The glider is pulled to stretch the spring an amount *x* from equilibrium and then released. Before reaching the photogate, the glider attains its maximum speed and the string becomes slack. The photogate measures the time *t* that it takes the small block on top of the glider to pass through. Information about the distance *x* and the speed *v* of the glider as it passes through the photogate are given below.

Trial #	Extension of the Spring x (m)	Speed of Glider v (m/s)	Extension Squared x^2 (m^2)	Speed Squared v^2 (m^2/s^2)
1	0.30×10^{-1}	0.47	0.09×10^{-2}	0.22
2	0.60×10^{-1}	0.87	0.36×10^{-2}	0.76
3	0.90×10^{-1}	1.3	0.81×10^{-2}	1.7
4	1.2×10^{-1}	1.6	1.4×10^{-2}	2.6
5	1.5×10^{-1}	2.2	2.3×10^{-2}	4.8

(a) Assuming no energy is lost, write the equation for conservation of mechanical energy that would apply to this situation.

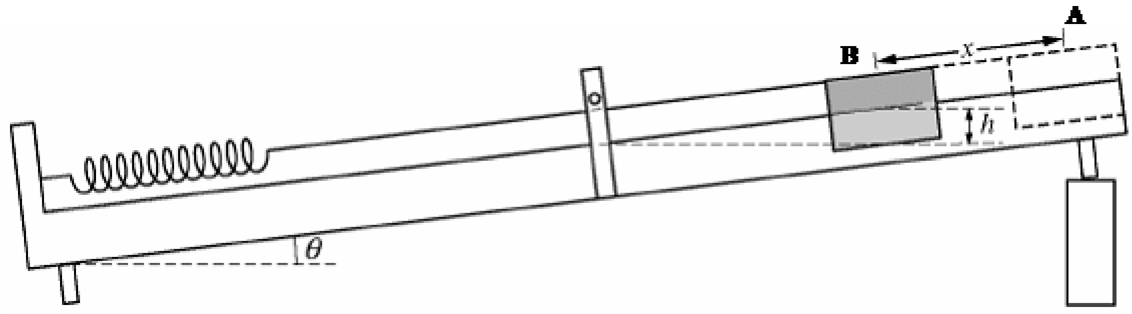
(b) On the grid below, plot v^2 versus x^2. Label the axes, including units and scale.



(c)

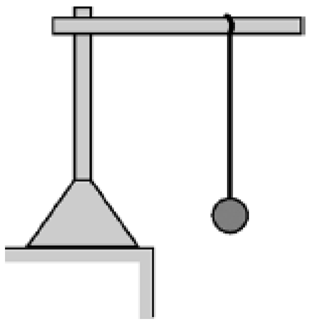
i. Draw a best-fit straight line through the data.

ii. Use the best-fit line to obtain the mass *m* of the glider.
(d) The track is now tilted at an angle θ as shown below. When the spring is unstretched, the center of the glider is a height *h* above the photogate. The experiment is repeated with a variety of values of *x*.



Assuming no energy is lost, write the new equation for conservation of mechanical energy that would apply to this situation starting from position A and ending at position B

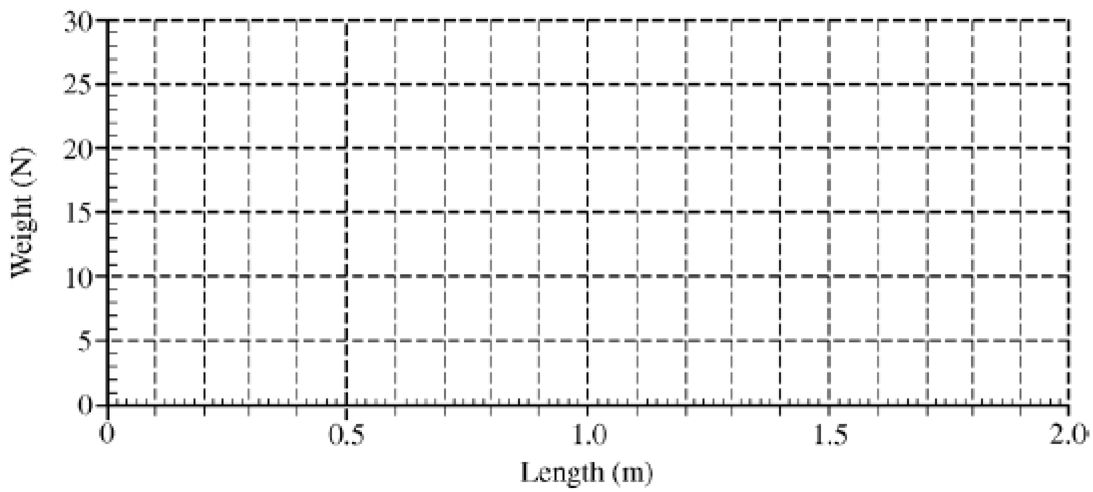
C2008M3



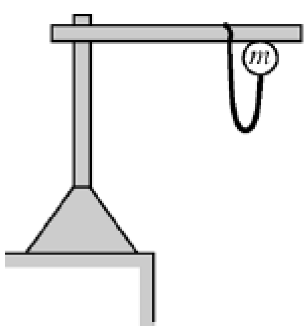
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant *k* of the cord.



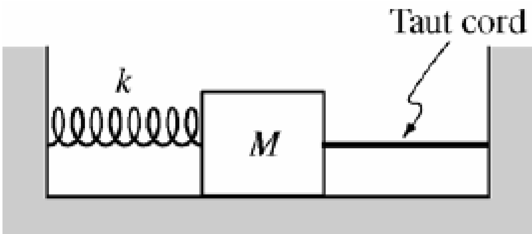
The student now attaches an object of unknown mass *m* to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass *m* of the object.

(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

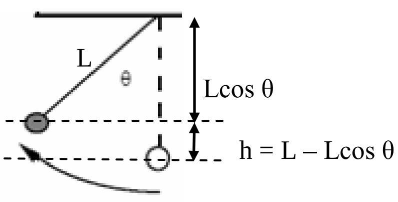
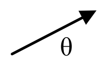
(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Work-Energy

	Solution	Answer
1.	Conservation of Energy, $U_{sp} = K$, $\frac{1}{2} kA^2 = \frac{1}{2} mv^2$ solve for v	B
2.	Constant velocity → $F_{net}=0$, $f_k = Fx = F\cos\theta$ $W_{fk} = -f_k d = -F\cos\theta\, d$	A
3.	Try out the choices with the proper units for each quantity. Choice A … FVT = (N) (m/s) (s) = Nm which is work in joules same as energy.	A
4.	Two step problem. Do $F = k\Delta x$, solve for Δx then sub in the $U_{sp} = \frac{1}{2} k\Delta x^2$	A
5.	In a circle moving at a constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle	E
6.	 The potential energy at the first position will be the amount "lost" as the ball falls and this will be the change in potential. $U=mgh = mg(L-L\cos\theta)$	A
7.	A force directed above the horizontal looks like this  θ To find the work done by this force we use the parallel component of the force (Fx) x distance. = $(F\cos\theta)\, d$	B
8.	The maximum speed would occur if all of the potential energy was converted to kinetic $U = K$ $16 = \frac{1}{2} mv^2$ $16 = \frac{1}{2} (2) v^2$	B
9.	The work done by the stopping force equals the loss of kinetic energy. $-W=\Delta K$ $-Fd = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$ $v_f = 0$ $F = mv^2/2d$	A
10.	The work done by friction equals the loss of kinetic energy $-f_k d = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2$ $v_f = 0$, plug in values to get answer	D
11.	$P = Fd / t$. Since there is no distance moved, the power is zero	E
12.	This is a conservative situation so the total energy should stay same the whole time. It should also start with max potential and min kinetic, which only occurs in choice C	C
13.	Stopping distance is a work-energy relationship. Work done by friction to stop = loss of kinetic $-f_k d = -\frac{1}{2} mv_i^2$ $\mu_k mg = \frac{1}{2} mv_i^2$ The mass cancels in the relationship above so changing mass doesn't change the distance	B
14.	Same relationship as above … double the v gives 4x the distance	E
15.	Half way up you have gained half of the height so you gained ½ of potential energy. Therefore you must have lost ½ of the initial kinetic energy so $E_2 = (E_k/2)$. Subbing into this relationship $E_2 = (E_k/2)$ $\frac{1}{2} mv_2^2 = \frac{1}{2} m v^2 / 2$ $v_2^2 = v^2 / 2$ …. Sqrt both sides gives answer	B
16.	At the top, the ball is still moving (v_x) so would still possess some kinetic energy	A
17.	Same as question #1 with different variables used	E

18. $P = F d / t$ $= (ma)d / t$ $= (kg)(m/s^2)(m) / (s)$ $= kg\ m^2 / s^3$ C

19. $P = Fv$, plug in to get the pushing force F and since its constant speed, $F_{push} = f_k$ A

20. Total energy is always conserved so as the air molecules slow and lose their kinetic energy, there is a heat flow which increases internal (or thermal) energy C

21. The work done must equal the increase in the potential energy $mgh = (10)(10)(1.3)$ D

22. Based on $F = k\ \Delta x$. The mass attached to the spring does not change the spring constant so the same resistive spring force will exist, so the same stretching force would be required C

23. The work done must equal the total gain in potential energy B
10 boxes * mgh (25)(10)(1.5) of each

24. Eliminating obviously wrong choices only leaves A as an option. The answer is A because since the first ball has a head start on the second ball it is moving at a faster rate of speed at all times. When both are moving in the air together for equal time periods the first faster rock will gain more distance than the slower one which will widen the gap between them. A

25. All of the $K = ½\ m\ v^2$ is converted to U. Simply plug in the values D

26. For a mass on a spring, the max U occurs when the mass stops and has no K while the max K occurs when the mass is moving fast and has no U. Since energy is conserved it is transferred from one to the other so both maximums are equal D

27. Since the ball is thrown with initial velocity it must start with some initial K. As the mass falls it gains velocity directly proportional to the time (V=Vi+at) but the K at any time is equal to $1/2\ mv^2$ which gives a parabolic relationship to how the K changes over time. E

28. Since the speed is constant, the pushing force F must equal the friction force $f_k = \mu F_n = \mu mg$. The power is then given by the formula $P = Fv = \mu mgv$ C

29. Since the speed is constant, the pushing force F must equal the friction force (10 N). The distance traveled is found by using d = vt = (3)(60 sec), and then the work is simply found using W=Fd E

30. Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates E

31. The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the K gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph. D

32. Point IV is the endpoint where the ball would stop and have all U and no K. Point II is the minimum height where the ball has all K and no U. Since point III is halfway to the max U point half the energy would be U and half would be K C

33. Apply energy conservation using points IV and II. $U_4 = K_2$ $mgh = ½\ mv^2$ B

34. Force is provided by the weight of the mass (mg). Simply plug into $F=k\Delta x$, $mg=k\Delta x$ and solve E

35. Since the track is rough there is friction and some mechanical energy will be lost as the block slides which means it cannot reach the same height on the other side. The extent of energy lost depends on the surface factors and cannot be determined without more information — E

36. The force needed to lift something at a constant speed is equal to the object weight F=mg. The power is then found by P = Fd / t = mgh / t — E

37. Simple energy conservation $K=U_{sp}$ $\frac{1}{2} mv_o^2 = \frac{1}{2} k \Delta x^2$ solve for Δx — D

38. Simple application of $F_g=mg$ — D

39. $F_n = F_{gy} = mg \cos \theta$. Since you are given the incline with sides listed, $\cos \theta$ can be found by using the dimensions of the incline … $\cos \theta$ = adj / hyp = 4/5 to make math simple. This is a good trick to learn for physics problems — C

40. As the box slides down the incline, the gravity force is parallel to the height of the incline the whole time so when finding the work or gravity you use the gravity force for F and the height of the incline as the parallel distance. Work = $(F_g)(d) = (20)(3)$ — B

41. The student must exert an average force equal to their weight (F_g) in order to lift themselves so the lifting force F=mg. The power is then found with $P = Fd / t = (F_g)d / t$ — C

42. As the object oscillates its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D — D

43. To push the box at a constant speed, the child would need to use a force equal to friction so $F=f_k=\mu mg$. The rate of work (W/t) is the power. Power is given by P=Fv → μmgv — A

44. Two steps. I) use hookes law in the first situation with the 3 kg mass to find the spring constant (k). $F_{sp}=k\Delta x$, $mg=k\Delta x$, k = 30/.12 = 250. II) Now do energy conservation with the second scenario (note that the initial height of drop will be the same as the stretch Δx).
U top = U_{sp} bottom, $mgh = \frac{1}{2} k \Delta x^2$, $(4)(10)(\Delta x) = \frac{1}{2} (250) (\Delta x^2)$ — D

45. In a circular orbit, the velocity of a satellite is given by $v = \sqrt{\frac{Gm_e}{r}}$ with m_e = M. Kinetic energy of the satellite is given by $K = \frac{1}{2} m v^2$. Plug in v from above to get answer — B

46. Projectile. V_x doesn't matter $V_{iy} = 0$. Using $d = v_{iy}t + \frac{1}{2} at^2$ we get the answer — E

47. Energy conservation $E_{top} = E_{bot}$, $K_t+U_t = K_b$. Plug in for K top and U top to get answer — D

48. A is true; both will be moving the fastest when they move through equilibrium. — A

49. X and Y directions are independent and both start with the same velocity of zero in each direction. The same force is applied in each direction for the same amount of time so each should gain the same velocity in each respective direction. — C

50. Kinetic energy is not a vector and the total resultant velocity should be used to determine the KE. For the 1st second the object gains speed at a uniform rate in the x direction and since KE is proportional to v^2 we should get a parabola. However, when the 2nd second starts the new gains in velocity occur only in the y direction and are at smaller values so the gains essentially start over their parabolic trend as shown in graph B — B

51. Simple P = Fv — E

52. The force needed to lift something at a constant speed is equal to the object weight F=mg. The power is then found by P = Fd / t = mgh / t — B

53. As the system moves, m_2 loses energy over distance h and m_1 gains energy over the same distance h but some of this energy is converted to KE so there is a net loss of U. Simply subtract the $U_2 - U_1$ to find this loss — A

54. In a force vs. displacement graph, the area under the line gives the work done by the force and the work done will be the change in the K so the largest area is the most K change — E

55. Compare the U+K (mgh + ½ mv^2) at the top, to the K (½ mv^2) at the bottom and subtract them to get the loss. — C

56. Use energy conservation, U top = K bottom. As in problem #6 (in this document), the initial height is given by L – Lcos θ, with cos 60 = .5 so the initial height is ½ L. — A

57. Use application of the net work energy theorem which says … $W_{net} = \Delta K$. The net work is the work done by the net force which gives you the answer — A

58. First use the given location (h=10m) and the U there (50J) to find the mass. U=mgh, 50=m(10)(10), so m = 0.5 kg. The total mechanical energy is given in the problem as U+K = 100 J. The max height is achieved when all of this energy is potential. So set 100J = mgh and solve for h — B

59. There is no U_{sp} at position x=0 since there is no Δx here so this is the minimum U location — A

60. Simple P = Fv to solve — E

61. Using energy conservation in the first situation presented K=U gives the initial velocity as $v = \sqrt{2gh}$. The gun will fire at this velocity regardless of the angle. In the second scenario, the ball starts with the same initial energy but at the top will have both KE and PE so will be at a lower height. The velocity at the top will be equal to the v_x at the beginning $v_x = v\cos\theta = \left(\sqrt{2gh}\right)\cos 45 = \left(\sqrt{2gh}\right)\left(\frac{\sqrt{2}}{2}\right) = \sqrt{gh}$. Now sub into the full energy conservation problem for situation 2 and solve for h_2. $K_{bottom} = U_{top} + K_{top}$ — C

$$\frac{1}{2}m\left(\sqrt{2gh}\right)^2 = mgh_2 + \frac{1}{2}m\left(\sqrt{gh}\right)^2$$

62. To find work we use the parallel component of the force to the distance, this gives Fcos θ d — B

63 The centripetal force is the force allowing the circular motion which in this case is the spring force $F_{sp} = k\Delta x = (100)(.03)$ — B

64. In a circle at constant speed, the work done is zero since the Force is always perpendicular to the distance moved as you move incrementally around the circle — A

65. At the maximum displacement the K=0 so the 10J of potential energy at this spot is equal to the total amount of mechanical energy for the problem. Since energy is conserved in this situation, the situation listed must have U+K add up to 10J. — B

66. Using the work-energy theorem. $W_{nc} = \Delta$ ME, A

$W_{Ft} = \Delta U + \Delta K$,

$-\,Fd = (mgh_f - mgh_i) + (\tfrac{1}{2}mv_f^{\,2} - \tfrac{1}{2}mv_i^{\,2})$,

$-\,(11000)(8) = (0 - (1000)(10)(8)) + (0 - \tfrac{1}{2}(1000)(v_i^2))$ … solve for v_i

67. Use energy conservation $K=U_{sp}$ $\tfrac{1}{2}\, mv_m^{\,2} = \tfrac{1}{2}\, k\, \Delta x^2$, with Δx=A, solve for k D

68. To lift the mass at a constant velocity a lifting force equal to the objects weight would be needed, so F=(mg). Simply plug into P = Fd / t and solve for d. C

69. Using the vertical distance with the vertical force ($Fd_{\|}$) B

W = 10 cartons * $(mg)(d_y)$ = 10 * (25)(10m/s^2)(1.5m) = 3750J

70. The ΔU will equal the amount of initial K based on energy conservation, U = K = 1/2 m v^2 D

71. Using work-energy. $W_{nc} = \Delta K = K_f - K_i$ $-\,Fd = 0 - \tfrac{1}{2}\, m\, v_f^{\,2}$ $-\,F(95) = -\tfrac{1}{2}\,(500)28^2$ C

72. Based on net work version of work energy theorem. $W_{net} = \Delta K$, we see that since there is a constant speed, the ΔK would be zero, so the net work would be zero requiring the net force to also be zero. A

73. As the block slides back to equilibrium, we want all of the initial spring energy to be dissipated by work of friction so there in no kinetic energy at equilibrium where all of the spring energy is now gone. So set work of friction = initial spring energy and solve for μ. The distance traveled while it comes to rest is the same as the initial spring stretch, d = x. C

$\tfrac{1}{2}\, kx^2 = \mu mg(x)$

74. V at any given time is given by $v = v_i + at$, with $v_i = 0$ gives $v = at$, D

V at any given distance is found by $v^2 = v_i^2 + 2\, ad$, with $v_i = 0$ gives $v^2 = 2ad$

This question asks for the relationship to distance.

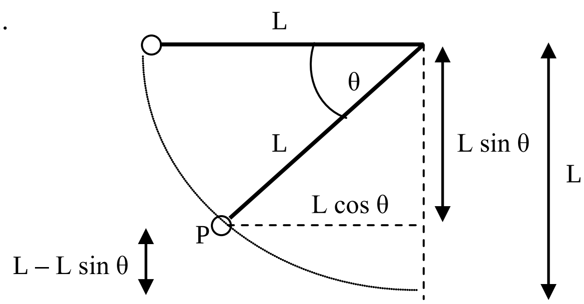
The kinetic energy is given by $K = \tfrac{1}{2}\, m\, v^2$ and since $v^2 = 2ad$ we see a linear direct relationship of kinetic energy to distance (2*d → 2*K)

Another way of thinking about this is in relation to energy conservation. The total of $mgh+1/2mv^2$ must remain constant so for a given change in (h) the $\tfrac{1}{2}\, mv^2$ term would have to increase or decrease directly proportionally in order to maintain energy conservation.

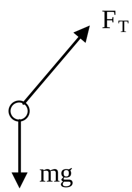
75. Similar to the discussion above. Energy is conserved so the term $mgh + \tfrac{1}{2}\, mv^2$ must remain constant. As the object rises it loses K and gains U. Since the height is H/2 it has gained half of the total potential energy it will end up with which means it must have lost half of its kinetic energy, so its K is half of what it was when it was first shot. B

76. Since the force is always perpendicular to the incremental distances traveled as the particle travels the loop, there is zero work done over each increment and zero total work as well. A

1974B1.



(a) FBD



(b) Apply conservation of energy from top to point P

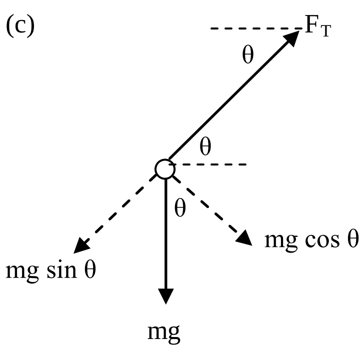
$U_{top} = U_p + K_p$

$mgh = mgh_p + ½ m v_p^2$

$gL = g(L – L\sin θ) + ½ v_p^2$

$$v = \sqrt{2gL\sin\theta}$$

(c)



$F_{NET(C)} = m v^2 / r$

$F_T – mg \sin θ = m v^2 / r$

$F_T – mg \sin θ = m (2gL\sin θ) / L$

$F_T = 2mg \sin θ + mg \sin θ$

$F_T = 3mg \sin θ$

1974B7.

6 riders per minute is equivalent to 6x(70kg)*9.8 = 4116 N of lifting force in 60 seconds

Work to lift riders = work to overcome gravity over the vertical displacement (600 sin 30)
Work lift = Fd = 4116N (300m) = $1.23x10^6$ J

P lift = W / t = $1.23x10^6$ J / 60 sec = 20580 W

But this is only 40% of the necessary power. ➔ 0.40 (total power) = 20580 W

Total power needed = 51450 W

1975B1.

(a) $F_{net} = ma$ $\quad -f_k = ma$ $\quad -8 = 2a$ $\quad a = -4\ m/s^2$

(b) $v_f^2 = v_i^2 + 2ad$ $\quad (0)^2 = v_i^2 + 2(-4)(8)$ $\quad v_i = 8\ m/s$

$v_f = v_i + at$ $\quad t = 2$ sec

(c) Apply energy conservation top to bottom

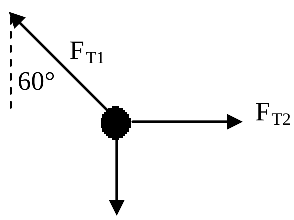
$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$(10)(R) = ½ (8)^2$ $\quad R = 3.2$ m

1975 B7

(a)

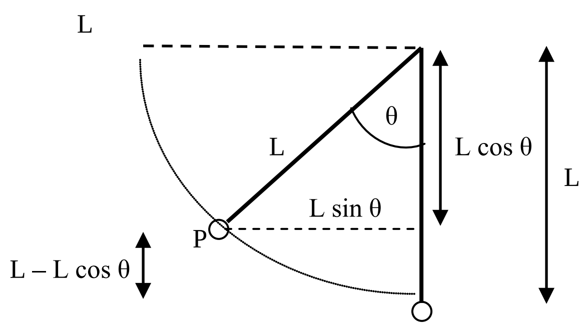


(b) $F_{NET(Y)} = 0$

$F_{T1} \cos\theta = mg$

$F_{T1} = mg / \cos(60) = 2mg$

(c) When string is cut it swing from top to bottom, similar to diagram for 1974B1 with θ moved as shown below



$U_{top} = K_{bot}$

$mgh = ½ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

1977B1.

(a) Apply work–energy theorem

$W_{NC} = \Delta\ ME$

$W_{fk} = \Delta K \quad (K_f - K_i)$

$W = -K_i$

$W = -½\ mv_i2 \qquad -½\ (4)(6)^2 \qquad = -72\ J$

(b) $F_{net} = ma$

$-f_k = m\ a \qquad v = v_i + at$

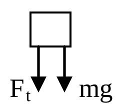
$a = -(8)/4 = -2\ m/s^2 \qquad v = (6) + (-2)\ t$

(c) $W_{fk} = -f_k\ d$

$-72J = -(8)\ d \qquad d = 9\ m$

1978B1,

(a)



(b) Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t + mg) = mv^2/r \qquad (20+0.5(10))=(0.5)v^2 / 2 \qquad v = 10\ m/s$

(c) As the object moves from P to Q, it loses U and gains K. The gain in K is equal to the loss in U.

$\Delta U = mg\Delta h = (0.5)(10)(4) = 20\ J$

(d) First determine the speed at the bottom using energy.

$K_{top} + K_{gain} = K_{bottom} \qquad ½\ mv_{top}^2 + 20\ J = ½\ mv_{bot}^2 \qquad v_{bot} = 13.42\ m/s$

At the bottom, Ft acts up (towards center) and mg acts down (away from center)

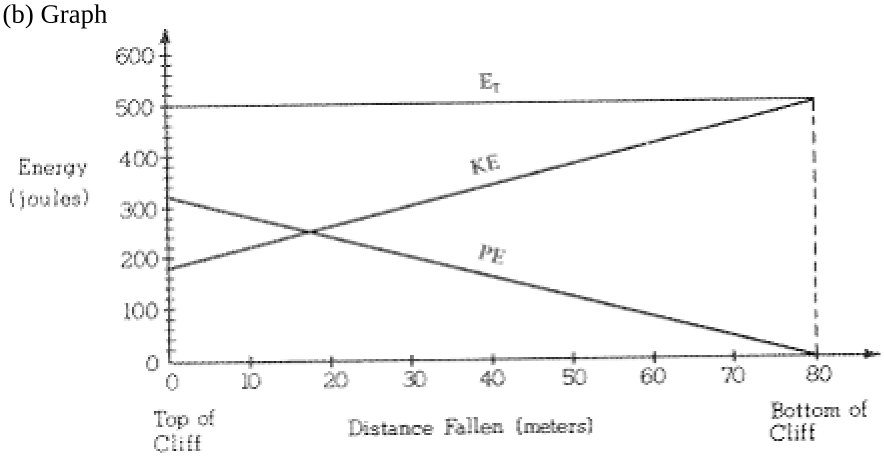
Apply $F_{net(C)} = mv^2 / r$ … towards center as + direction

$(F_t - mg) = mv^2/r \qquad (F_t - 0.5(10)) = (0.5)(13.42)^2 / 2 \qquad F_t = 50\ N$

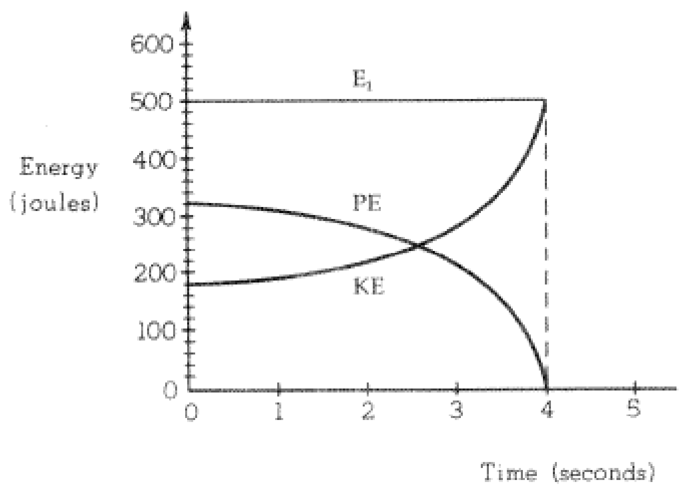
1979B1.

(a) $U = mgh = 320$ J
$K = ½ m v^2 = 180$ J
Total = U + K = 500 J

(b) Graph



(c) First determine the time at which the ball hits the ground, using $d_y = 0 + ½ g t^2$, to find it hits at 4 seconds.



1981B1.

(a) constant velocity means $F_{net} = 0$, $F - f_k = ma$ $F - \mu_k mg = 0$ $F - (0.2)(10)(10) = 0$
$F = 20$ N

(b) A change in K would require net work to be done. By the work–energy theorem:

$W_{net} = \Delta K$
$F_{net}\, d = 60$ J
F_{net} (4m) = 60 $F_{net} = 15$ N
$F' - f_k = 15$
$F' - 20 = 15$ $F = 35$ N

(c) $F_{net} = ma$
(15) = (10) a a= 1.5 m/s^2

1981B2.

The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring

$W = ½\, m\, v^2 = ½\, (3)\, (10)^2 = 150$ J

1982B3.

Same geometry as in problem 1975B7.

(a) Apply energy conservation top to bottom

$U_{top} = K_{bot}$
$mgh = ½\, m\, v^2$
$mg\, (R - R\cos\theta) = ½\, m\, v^2$

$$v = \sqrt{2g(R - R\cos\theta)}$$

(b) Use $F_{NET(C)} = mv^2 / r$

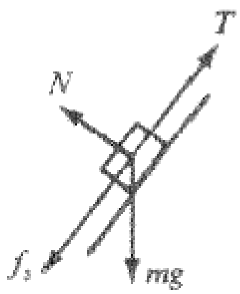
$F_t - mg = m\,(2g(R\text{-}R\cos\theta)) / R$
$1.5\, mg - mg = 2mg(1 - 1\cos\theta)$
$.5 = 2(1 - \cos\theta)$

$2\cos\theta = 1.5$ → $\cos\theta = ¾$

1985B2.

(a) The tension in the string can be found easily by isolating the 10 kg mass. Only two forces act on this mass, the Tension upwards and the weight down (mg) …. Since the systems is at rest, T = mg = 100 N

(b) FBD



(c) Apply $F_{net} = 0$ along the plane. $T - f_s - mg \sin\theta = 0$ $(100\text{ N}) - f_s - (10)(10)(\sin 60)$

$f_s = 13$ N

(d) Loss of mechanical energy = Work done by friction while sliding

First find kinetic friction force Perpendicular to plane $F_{net} = 0$ $F_n = mg\cos\theta$

$F_k = \mu_k F_n = \mu_k mg\cos\theta$

$W_{fk} = f_k d = \mu_k mg\cos\theta\,(d) = (0.15)(10)(10)(\cos(60)) = 15$J converted to thermal energy

(e) Using work-energy theorem … The U at the start – loss of energy from friction = K left over

$U - W_{fk} = K$

$mgh - W_{fk} = K$

$mg(d\sin 60) - 15 = K$

$(10)(10)(2)\sin 60 - 15 = K$ K = 158 J

1986B2.

(a) Use projectile methods to find the time. $d_y = v_{iy}t + ½ a t^2$ $h = 0 + g t_2 / 2$

$$t = \sqrt{\frac{2h}{g}}$$

(b) v_x at ground is the same as v_x top $V_x = d_x /t$ $$v_x = \frac{D}{\sqrt{\frac{2h}{g}}}$$

multiply top and bottom by reciprocal to rationalize

$$v_x = D\sqrt{\frac{g}{2h}}$$

(c) The work done by the spring to move the block is equal to the amount of K gained by it = K_f

$W = K_f = ½ m v^2 = (½ M (D^2 / (2h/g)) = MD^2 g / 4h$

(d) Apply energy conservation $U_{sp} = K$

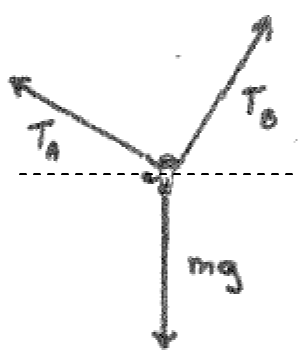
$½ k\Delta x^2 = ½ mv^2$ (plug in V from part b) $$v_x = \frac{MD^2 g}{2hX^2}$$

If using $F=k\Delta x$ you have to plug use F_{avg} for the force

---

1991B1.

(a) FBD

30°  60°

(b) SIMULTANEOUS EQUATIONS

$F_{net(X)} = 0$ $F_{net(Y)} = 0$

$T_a \cos 30 = T_b \cos 60$ $T_a \sin 30 + T_b \sin 60 – mg = 0$

…. Solve above for T_b and plug into Fnet(y) eqn and solve

$T_a = 24$ N $T_b = 42$ N

(c) Using energy conservation with similar diagram as 1974B1 geometry

$U_{top} = U_p + K_p$

$mgh = ½ m v^2$

$g(L – L\sin \theta) = ½ v^2$

$(10)(10 – 10\sin 60) = ½ v^2$ v = 5.2 m/s

(d) $F_{net(C)}= mv^2/r$

$F_t – mg = mv^2 / r$ $F_t = m(g + v^2/r)$ $F_t = (5)(9.8+(5.2)^2/10) = 62$ N

1992B1.

(a) K + U $\quad$ ½ m v^2 + mgh $\quad$ ½ $(0.1)(6)^2 + (0.1)(9.8)(1.8) = 3.6$ J

(b) Apply energy conservation using ground as h=0

$E_{top} = E_p$

3.6 J = K + U

3.6 = ½ m v^2 + mgh

3.6 = ½ $(0.1)(v^2) + (0.1)(9.8)(.2)$ $\quad$ v = 8.2 m/s

(c) Apply net centripetal force with direction towards center as +

i) Top of circle = Ft points down and Fg points down

$F_{net(c)} = mv^2/r$

$Ft + mg = mv^2/r$ $\quad$ $F_t = mv^2/r - mg$ $\quad$ $(0.1)(6)^2/(0.8) - (.1)(9.8)$ $\quad$ $F_t = 3.5$ N

ii) Bottom of circle = Ft points up and Fg points down

$F_{net(c)} = mv^2/r$

$F_t - mg = mv^2/r$ $\quad$ $F_t = mv^2/r + mg$ $\quad$ $(0.1)(8.2)^2/(0.8) + (0.1)(9.8)$ $\quad$ $F_t = 9.5$ N

(d) Ball moves as a projectile.

First find time of fall in y direction

$d_y = v_{iy}t + ½ a t^2$

$(-0.2) = 0 + ½ (-9.8) t^2$

t = .2 sec

Then find range in x direction

$d_x = v_x t$

$d_x = (8.2)(0.2)$

$d_x = 1.6$ m

1996B2.

(a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distance Δx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k \Delta x$ and solve for k

(b) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan \theta$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} - \mu_s$ mg, Giving $\quad$ $mg = F_{spring} / \mu$.
Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid dynamics unit.

1997B1.

(a) The force is constant, so simple $F_{net} = ma$ is sufficient. $(4) = (0.2)\,a$ $a = 20\ m/s^2$

(b) Use $d = v_i t + \frac{1}{2} a t^2$ $12 = (0) + \frac{1}{2}(20)\,t^2$ $t = 1.1$ sec

(c) $W = Fd$ $W = (4\ N)(12\ m) = 48\ J$

(d) Using work energy theorem $W = \Delta K$ $(K_i = 0)$ $W = Kf - Ki$
$W = \frac{1}{2} m v_f^2$

Alternatively, use $v_f^2 = v_i^2 + 2ad$ $48J = \frac{1}{2}(0.2)(v_f^2)$ $v_f = 22\ m/s$

(e) The area under the triangle will give the extra work for the last 8 m
$\frac{1}{2}(8)(4) = 16J$ + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem $W = \frac{1}{2} m v_f^2$ $64\ J = \frac{1}{2}(0.2)\,v_f^2$ $v_f = 25.3\ m/s$

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

1999B1.

(a) Plug into $g = GM_{planet} / r_{planet}^2$ lookup earth mass and radius
$g_{mars} = 3.822\ m/s^2$ to get it in terms of g_{earth} divide by 9.8 $g_{mars} = 0.39\ g_{earth}$

(b) Since on the surface, simply plug into $F_g = mg = (11.5)(3.8) = 44$ N

(c) On the incline, $F_n = mg\cos\theta = (44)\cos(20) = 41$ N

(d) moving at constant velocity → $F_{net} = 0$

(e) $W = Pt$ $(5.4\times10^5\ J) = (10\ W)\,t$ $t = 54000$ sec
$d = vt$ $(6.7\times10^{-3})(54000\ s)$ $d = 362$ m

(f) $P = Fv$ $(10) = F\,(6.7\times10^{-3})$ $F_{push} = 1492.54$ N total pushing force used
* (.0001) use for drag
→ $F_{drag} = 0.15$ N

2002B2.

(a) From graph U = 0.05 J

(b) Since the total energy is 0.4 J, the farthest position would be when all of that energy was potential spring energy. From the graph, when all of the spring potential is 0.4 J, the displacement is 10 cm

(c) At –7 cm we read the potential energy off the graph as 0.18 J. Now we use energy conservation.
$ME = U_{sp} + K$ $0.4J = 0.18\ J + K$ → $K = 0.22$ J

(d) At x=0 all of the energy is kinetic energy $K = \frac{1}{2} m v^2$ $0.4 = \frac{1}{2}(3)\,v^2$ $v = 0.5\ m/s$

(e) The object moves as a horizontally launched projectile when it leaves.

First find time of fall in y direction
$d_y = v_{iy} t + \frac{1}{2} a t^2$
$(-0.5) = 0 + \frac{1}{2}(-9.8)\,t^2$
$t = 0.3$ sec

Then find range in x direction
$d_x = v_x t$
$d_x = (0.5)(0.3)$
$d_x = 0.15$ m

2004B1.

(a) i) fastest speed would be the lowest position which is the bottom of the first hill where you get all sick and puke your brains out.

ii) Applying energy conservation from the top of the hill where we assume the velocity is approximately zero we have

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$ $(9.8)(90) = ½ v^2$ $v = 42$ m/s

(b) Again applying energy conservation from the top to position B

$U_{top} = K_b + U_b$

$mgh = ½ m v_B{}^2 + mgh_B$

$(9.8)(90) = ½ v_B{}^2 + (9.8)(50)$ $v_B = 28$ m/s

(c) i) FBD

ii) $mg = (700)(9.8) = 6860$ N

$F_{net(C)} = mv^2 /r$

$F_n + mg = mv^2 / r$

$F_n = mv^2/r – mg = m (v^2/r – g) = (700) (28^2/20 – 9.8) = 20580$ N

(d) The friction will remove some of the energy so there will not be as much Kinetic energy at the top of the loop. In order to bring the KE back up to its original value to maintain the original speed, we would need less PE at that location. A lower height of the loop would reduce the PE and compensate to allow the same KE as before. To actually modify the track, you could flatten the inclines on either side of the loop to lower the height at B.

B2004B1.

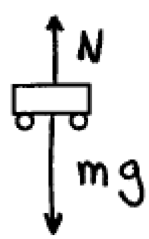
(a) set position A as the h=0 location so that the PE=0 there.

Applying energy conservation with have

$U_{top} + K_{top} = K_A$ $mgh + ½ m v^2 = ½ m v_A{}^2$

$(9.8)(0.1) + ½ (1.5)^2 = ½ v_A{}^2$ $v_A = 2.05$ m/s

(b) FBD



(c) $F_{net(C)} = mv^2 /r$

$mg – F_N = mv^2 / r$

$Fn = mg – mv^2/r = m (g – v^2/r) = (0.5)(9.8 – 2.05^2 / 0.95) = 2.7$ N

(c) To stop the cart at point A, all of the kinetic energy that would have existed here needs to be removed by the work of friction which does negative work to remove the energy.

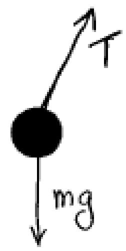
$W_{fk} = – K_A$

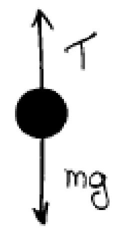
$W_{fk} = – ½ m v_A{}^2 = – ½ (0.5)(2.05^2) = – 1.1$ J

(d) The car is rolling over a hill at point A and when F_n becomes zero the car just barely loses contact with the track. Based on the equation from part (c) the larger the quantity (mv^2 / r) the more likely the car is to lose contact with the track (since more centripetal force would be required to keep it there) … to increase this quantity either the velocity could be increased or the radius could be decreased. To increase the velocity of the car, make the initial hill higher to increase the initial energy. To decrease the radius, simply shorten the hill length.

B2005B2.

FBD i) ii)





(b) Apply energy conservation?

$U_{top} = K_{bottom}$

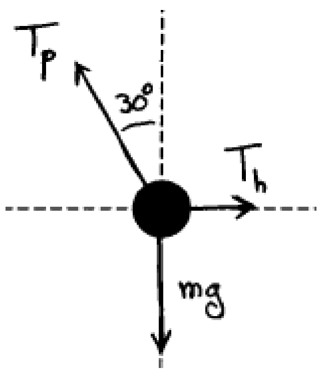
$mgh = ½ m v^2$ $(9.8)(.08) – ½ v^2$ v = 1.3 m/s

(c) $F_{net(c)} = mv^2/r$

$F_t – mg = mv^2/r$ $F_t = mv^2/r + mg$ $(0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $F_t = 0.93$ N

2005B2.

(a) FBD



(b) Apply

$F_{net(X)} = 0$ $F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $T_P \sin 30 = T_H$

$T_P = 20.37$ N $T_H = 10.18$ N

(c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$

$g(L – L \cos θ) = ½ v^2$

$(10) (2.3 – 2.3 \cos 30) = ½ v^2$ $v_{bottom} = 2.5$ m/s

B2006B2.

(a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$ $Mgh = ½ (M) (3.5v_o)^2$ $h = 6.125 v_o^2 / g$

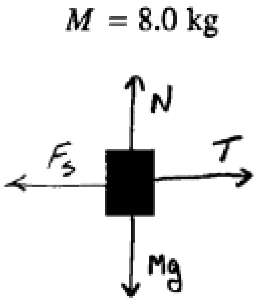
(b) $W_{NC} = ΔK \; (K_f-K_i) \quad K_f = 0$

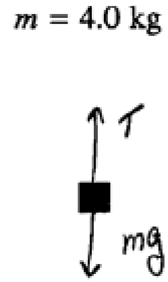
$– f_k d = 0 – ½ (1.5M)(2v_o)^2$

$μ_k (1.5 M) g (d) = 3Mv_o^2$ $μ_k = 2v_o^2 / gD$

2006B1.

(a) FBD





(b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

(c) Tension in string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k (0.05)$ $k = 780$ N/m

(d) 4 kg mass is in free fall. $D = v_i t + ½ g t^2$ $-0.7 = 0 + ½ (-9.8)t^2$ $t = 0.38$ sec

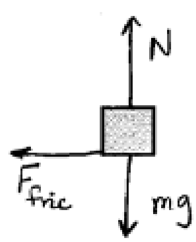
(e) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $½ k \Delta x^2 = ½ mv^2$ $½ (780) (0.05) = ½ (8) v^2$ $v = 0.49$ m/s

---

B2008B2.

(a) $d = v_i t + ½ a t^2$ $(55) = (25)(3) + ½ a (3)^2$ $a = -4.4$ m/s^2

(b) FBD



(c) using the diagram above and understanding that the static friction is actually responsible for decelerating the box to match the deceleration of the truck, we apply F_{net}
$F_{net} = ma$
$-f_s = -\mu_s F_n = ma$ $-\mu_s mg = ma$ $-\mu_s = a/g$ $-\mu_s = -4.4 / 9.8$ $\mu_s = 0.45$

Static friction applied to keep the box at rest relative to the truck bed.

(d) Use the given info to find the acceleration of the truck $a = \Delta v / t = 25/10 = 2.5$ m/s^2
To keep up with the trucks acceleration, the crate must be accelerated by the spring force, apply F_{net}
$F_{net} = ma$ $F_{sp} = ma$ $k\Delta x = ma$ $(9200)(\Delta x) = (900)(2.5)$ $\Delta x = 0.24$ m

(e) If the truck is moving at a constant speed the net force is zero. Since the only force acting directly on the crate is the spring force, the spring force must also become zero therefore the Δx would be zero and is **LESS** than before. Keep in mind the crate will stay on the frictionless truck bed because its inertia will keep it moving forward with the truck (remember you don't necessarily need forces to keep things moving)

2008B2.

(a) In a connected system, we must first find the acceleration of the system as a whole. The spring is internal when looking at the whole system and can be ignored.

$F_{net} = ma$ $(4) = (10)\,a$ $a = 0.4\ m/s^2$ → the acceleration of the whole system and also of each individual block when looked at separate

Now we look at just the 2 kg block, which has only the spring force acting on its FBD horizontal direction.

$F_{net} = ma$ $F_{sp} = (2)(.4)$ $F_{sp} = 0.8\ N$

(b) Use $F_{sp} = k\Delta x$ $0.8 = (80)\ \Delta x$ $\Delta x = 0.01\ m$

(c) Since the same force is acting on the same total mass and $F_{net} = ma$, the acceleration is the same

(d) The spring stretch will be MORE. This can be shown mathematically by looking at either block. Since the 8 kg block has only the spring force on its FBD we will look at that one.

$F_{sp} = ma$ $k\Delta x = ma$ $(80)(\Delta x) = (8)(0.4)$ $\Delta x = 0.04\ m$

(e) When the block A hits the wall it instantly stops, then block B will begin to compress the spring and transfer its kinetic energy into spring potential energy. Looking at block B energy conservation:

$K_b = U_{sp}$ $\frac{1}{2}\, m\, v_b^2 = \frac{1}{2}\, k\ \Delta x^2$ $(8)(0.5)^2 = (80)\Delta x^2$ $\Delta x = 0.16\ m$

2009B1.

(a) Apply energy conservation. All of the spring potential becomes gravitational potential

$U_{sp} = U$

$\frac{1}{2} k \Delta x^2 = mgh$ $\frac{1}{2} k x^2 = mgh$ $h = kx^2 / 2mg$

(b) You need to make a graph that is of the form y = m x, with the slope having "k" as part of it and the y and x values changing with each other. Other constants can also be included in the slope as well to make the y and x variables simpler. h is dependent on the different masses used so we will make h our y value and use m as part of our x value. Rearrange the given equation so that is it of the form y = mx with h being y and mass related to x.

We get y = m x

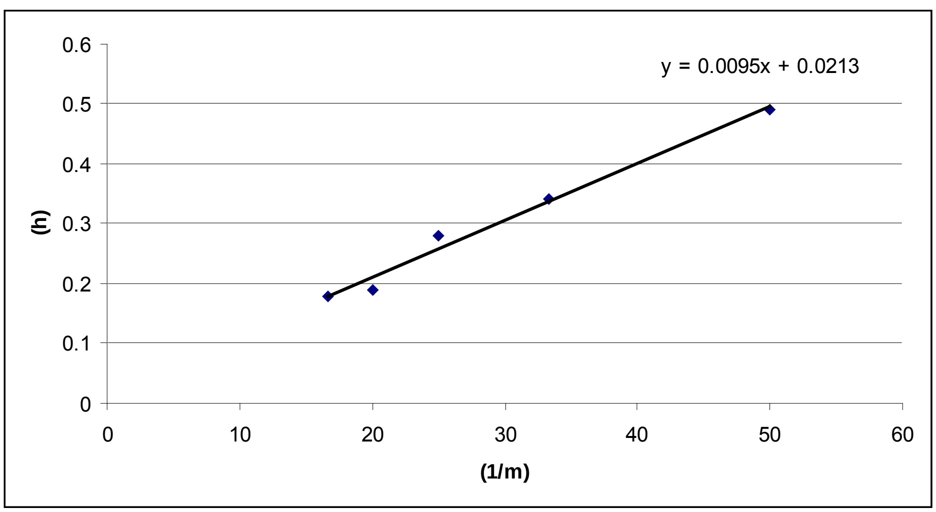
$$h = \left(\frac{kx^2}{2g}\right)\frac{1}{m}$$

so we use h as y and the value 1/m as x and graph it.

(note: we lumped all the things that do not change together as the constant slope term. Once we get a value for the slope, we can set it equal to this term and solve for k)

1/m	m (kg)	h (m)
50	0.020	0.49
33.33	0.030	0.34
25	0.040	0.28
20	0.050	0.19
16.67	0.060	0.18
X values		Y values

(c) Graph



(d) The slope of the best fit line is 0.01

We set this slope equal to the slope term in our equation, plug in the other known values and then solve it for k

$$0.01 = \left(\frac{kx^2}{2g}\right)$$

$$0.01 = \left(\frac{k(0.02)^2}{2(9.8)}\right)$$

Solving gives us k = 490 N/m

(e) - Use a stopwatch, or better, a precise laser time measurement system (such as a photogate), to determine the time it takes the toy to leave the ground and raise to the max height (same as time it takes to fall back down as well). Since its in free fall, use the down trip with v_i=0 and apply $d = \frac{1}{2} g t^2$ to find the height.
- Or, videotape it up against a metric scale using a high speed camera and slow motion to find the max h.

C1973M2

(a) Apply work-energy theorem

$W_{nc} = \Delta ME$

$W_{fk} = \Delta K$ $(K_f - K_i)$ $K_f = 0$

$-f_k d = -½ mv_i^2$ $-f_k (0.12) = -½ (0.030)(500)^2$ $f_k = 31250$ N

(b) Find find acceleration $-f_k = ma$ $-(31250) = (0.03)a$ $a = -1.04\text{x}10^6$ m/s^2

Then use kinematics $v_f = v_i + at$ $0 = 500 + (-1.04\text{x}10^6)t$ $t = 4.8\text{x}10^{-4}$ sec

---

C1982M1

(a) Apply energy conservation, set the top of the spring as h=0, therefore H at start = Lsin θ = 6 sin30 = 3 m

$U_{top} = K_{bot}$ $mgh = ½ mv^2$ $(9.8)(3) = ½ (v^2)$ $v = 7.67$ m/s

(b) Set a new position for h=0 at the bottom of the spring. Apply energy conservation comparing the h=0 position and the initial height location. Note: The initial height of the box will include both the y component of the initial distance along the inclined plane plus the y component of the compression distance Δx.

$h = L \sin\theta + \Delta x \sin\theta$

$U_{top} = U_{sp}(bot)$

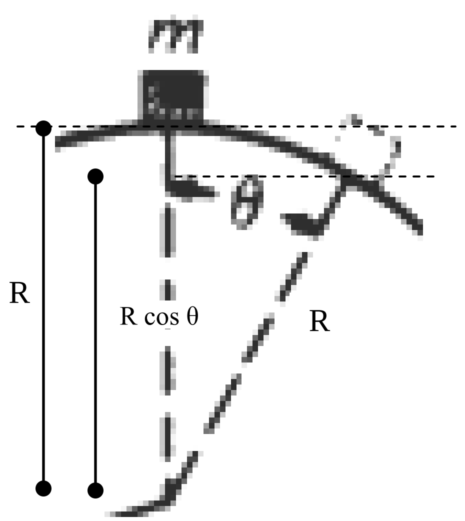
$mgh = ½ k \Delta x^2$

$mg(L \sin\theta + \Delta x \sin\theta) = ½ k \Delta x^2$

$(20)(9.8)(6 \sin 30 + 3 \sin 30) = ½ k (3)^2$ $k = 196$ N/m

(c) The speed is NOT a maximum when the block first hits the spring. Although the spring starts to push upwards against the motion of the block, the upwards spring force is initially less than the x component of the weight pushing down the incline (F_{gx}) so there is still a net force down the incline which makes the box accelerate and gain speed. This net force will decrease as the box moves down and the spring force increases. The maximum speed of the block will occur when the upwards spring force is equal in magnitude to the force down the incline such that Fnet is zero and the box stops accelerating down the incline. Past this point, the spring force becomes greater and there is a net force acting up the incline which slows the box until it eventually and momentarily comes to rest in the specified location.

C1983M3.



$h = R - R\cos\theta = R(1 - \cos\theta)$

i) $K_2 = U_{top}$

$K_2 = mg(R(1 - \cos\theta))$

ii) From, $K = ½ m v^2 = mgR(1 - \cos\theta)$ … $v^2 = 2gR(1 - \cos\theta)$

Then $a_c = v^2 / R =$ $2g(1 - \cos\theta)$

C1985M1

(a) We use $F_{net} = 0$ for the initial brink of slipping point. $F_{gx} - f_k = 0$ $mg \sin\theta = \mu_s(F_n)$

$mg \sin\theta = \mu_s\, mg \cos\theta$ $\mu_s = \tan\theta$

(b) Note: we cannot use the friction force from part a since this is the static friction force, we would need kinetic friction. So instead we must apply W_{nc} = energy loss = $\Delta K + \Delta U + \Delta U_{sp}$. ΔK is zero since the box starts and ends at rest, but there is a loss of gravitational U and a gain of spring U so those two terms will determine the loss of energy, setting final position as h=0. Note that the initial height would be the y component of the total distance traveled (d+x) so h= (d+x)sin θ

$U_f - U_i + U_{sp(F)} - U_{sp(i)}$

$0 - mgh + \frac{1}{2} k\Delta x^2 - 0$ $\frac{1}{2} kx^2 - mg(d+x)\sin\theta$

(c) To determine the coefficient of kinetic friction, plug the term above back into the work-energy relationship, sub in –Work of friction as the work term and then solve for μ_k

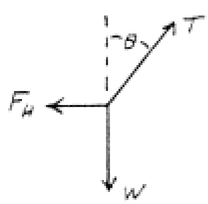
$W_{NC} = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$ $-f_k(d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$-\mu_k\, mg \cos\theta\, (d+x) = \frac{1}{2} kx^2 - mg(d+x)\sin\theta$

$\mu_k = [mg(d+x)\sin\theta - \frac{1}{2} kx^2] / [mg\,(d+x)\cos\theta]$

C1987M1

(a)



$F_{net(y)} = 0$

$T \cos\theta - W = 0$ $T = W / \cos\theta$

(b) Apply SIMULTANEOUS EQUATIONS

$F_{net(y)} = 0$ $F_{net(x)} = 0$

$T \cos\theta - W = 0$ $T \sin\theta - F_h = 0$

Sub T into X equation to get F_h $F_h = W \tan\theta$

(c) Using the same geometry diagram as solution 1975B7 solve for the velocity at the bottom using energy conservation

$U_{top} = K_{bot}$

$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos\theta)}$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(T - W) = m(2gL(1-\cos\theta)) / L$

$T = W + 2mg - 2mg\cos\theta$

$T = W + 2W - 2W\cos\theta = W(3 - 2\cos\theta)$

C1988M2

(a) The graph is one of force vs Δx so the slope of this graph is the spring constant. Slope = 200 N/m

(b) Since there is no friction, energy is conserved and the decrease in kinetic energy will be equal to the gain in spring potential $|\Delta K| = U_{sp(f)} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2}(200)(0.1)^2 = 1$J.

Note: This is the same as the area under the line since the area would be the work done by the conservative spring force and the work done by a conservative force is equal to the amount of energy transferred.

(c) Using energy conservation. $K_i = U_{sp(f)}$ $\frac{1}{2} mv_o^2 = 1$ J $\frac{1}{2}(5)\, v_o^2 = 1$ $v_o = 0.63$ m/s

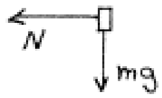
---

C1989M1

(a) Apply energy conservation from point A to point C setting point C as h=0 location
(note: to find h as shown in the diagram, we will have to add in the initial 0.5m below h=0 location)

$U_A = K_C$ $mgh_a = ½ m v_c^2$ $(0.1)(9.8)(h_a) = ½ (0.1)(4)^2$ $h_a = 0.816m$
$h = h_a + 0.5 m = 1.32 m$

(b)



(c) Since the height at B and the height at C are the same, they would have to have the same velocities $v_b = 4 m/s$

(d) $F_{net(c)} = mv^2 / r$ $F_n = (0.1)(4)^2/(0.5) = 3.2 N$

(e) Using projectile methods … $V_{iy} = 4\sin30 = 2 m/s$ Then $v_{fy}^2 = v_{iy}^2 + 2 a d_y$
$(0) = (2)^2 + 2(–9.8)(d_y)$ $d_y = 0.2$
$h_{max} = d_y$ + initial height $= 0.7 m$

Alternatively you can do energy conservation setting h=0 at point C. Then $K_c = U_{top} + K_{top}$ keeping in mind that at the top the block has a kinetic energy related to its velocity there which is the same as v_x at point C.

(f) Since the block will have the same total energy at point C as before but it will lose energy on the track the new initial height h is larger than before. To find the loss of energy on the track, you can simply subtract the initial energies in each case. $U_{new} – U_{old} = mgh_{net} - mgh_{old}$ $(0.1)(9.8)(2-1.32) = 0.67$ J lost.

C1989M3

(a) Apply energy conservation from start to top of spring using h=0 as top of spring.
$U = K$ $mgh = ½ m v^2$ $(9.8)(0.45) = ½ v^2$ $v = 3 m/s$

(b) At equilibrium the forces are balances $F_{net} = 0$ $F_{sp} = mg =(2)(9.8) = 19.6 N$

(c) Using the force from part b, $F_{sp} = k \Delta x$ $19.6 = 200 \Delta x$ $\Delta x = 0.098 m$

(d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the start position is now increased by the amount of Δx found in part c $h_{new} = h+\Delta x = 0.45 + 0.098 = 0.548 m$)
$U_{top} = U_{sp} + K$
$mgh = ½ k \Delta x^2 + ½ mv^2$ $(2)(9.8)(0.548) = ½ (200)(0.098)^2 + ½ (2)(v^2)$ $v = 3.13 m/s$

(e) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

C1990M2

(a) Energy conservation, $K_{bot} = U_{top}$ $½ m v^2 = mgh$ $½ (v_o^2) = gh$ $h = v_o^2 / 2g$

(b) Work-Energy theorem. $W_{nc} = \Delta K + \Delta U$ $(U_i = 0, K_f = 0)$
$– f_k d = (mgh – 0) + (0 – ½ m v_o^2)$ $– (\mu_k mg\cos \theta) h_2 / \sin \theta = mgh_2 – ½ mv_o^2$

$\mu mg \cos \theta h_2 / \sin \theta + mgh_2 = ½ mv_o^2$ $h_2 (\mu g \cos \theta / \sin \theta + g) = ½ v_o^2$
$h_2 = v_o^2 / (2g(\mu \cot \theta+1))$

C1991M1

(a) Apply energy conservation.

$K_{bottom} = U_p + K_p$ $\quad$ $\frac{1}{2} mv_{bot}^2 = mgh_p + K_p$

$\frac{1}{2}\, 3m (v_o/3)^2 = 3mg(r) + K_p$ $\quad$ $K_p = mv_o^2/6 - 3mgr$

(b) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set $F_n=0$ at the top of the loop so that only mg is acting down on the block. Then apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $\quad$ $3mg = 3m\, v^2 /r$ $\quad$ $v = \sqrt{rg}$

(c) Energy conservation, top of loop to bottom of loop

$U_{top} + K_{top} = K_{bot}$

$mgh + \frac{1}{2} m\, v_{top}^2 = \frac{1}{2} m\, v_{bot}^2$ $\quad$ $g(2r) + \frac{1}{2} (\sqrt{rg})^2 = \frac{1}{2} (vo')^2$ $\quad$ $v_o' = \sqrt{5gr}$

C1993M1

- since there is friction on the surface the whole time, this is not an energy conservation problem, use work-energy.

(a) $U_{sp} = \frac{1}{2} k\, \Delta x^2 = \frac{1}{2} (400)(0.5)^2 = 50$ J

(b) Using work-energy

$W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)} - U_{sp(i)}) + (K_f - K_i)$

$- f_k d = (0 - 50J) + (\frac{1}{2} m\, v_f^2 - 0)$

$- \mu\, mg\, d = \frac{1}{2} mv_f^2 - 50$

$- (0.4)(4)(9.8)(0.5) = \frac{1}{2} (4)(v_c^2) - 50$ $\quad$ $v_c = 4.59$ m/s

(c) $W_{nc} = (K_f - K_i)$

$-f_k d = (0 - \frac{1}{2} m\, v_i^2)$ $\quad$ $- \mu\, mg\, d = - \frac{1}{2} m\, v_i^2$ $\quad$ $(0.4)(6)(9.8)\, d = \frac{1}{2} (6)(3)^2$ $\quad$ $d = 1.15$ m

---

C2002M2

(a) Energy conservation, potential top = kinetic bottom $\quad$ $v = \sqrt{2gh}$

(b) Energy conservation, potential top = spring potential $\quad$ $U=U_{sp}$ $\quad$ $(2m)gh = \frac{1}{2} k\, x_m^2$

$$x_m = 2\sqrt{\frac{mgh}{k}}$$

C2004M1

(a) Energy conservation with position B set as h=0. $U_a = K_b$ $\quad$ $v_b = \sqrt{2gL}$

(b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = mv_b^2 / r$ $\quad$ $F_t - mg = m(2gL) / L$ $\quad$ $F_t = 3mg$

(c) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + \frac{1}{2} g\, t^2$ $\quad$ $L = 0 + gt^2 / 2$

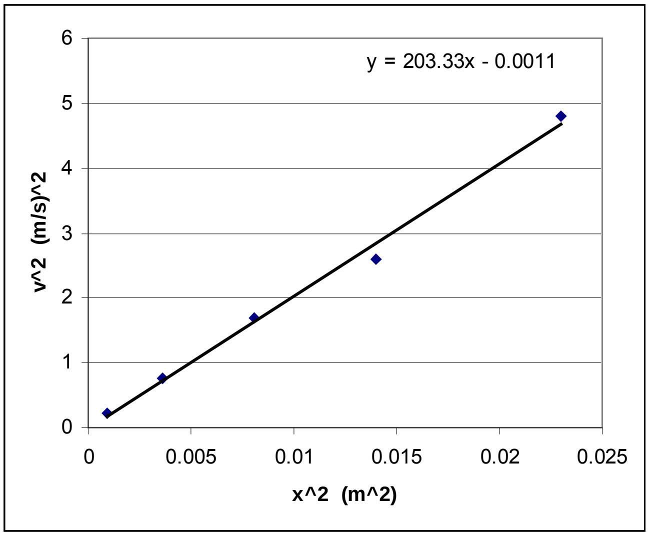
$t = \sqrt{\frac{2L}{g}}$ $\quad$ Then use $v_x = d_x / t$ $\quad$ $d_x = v'\sqrt{\frac{2L}{g}}$ $\quad$ total distance $\quad$ $x = v'\sqrt{\frac{2L}{g}} + L$

total distance includes the initial horizontal displacement L so it is added to the range

C2007M3

(a) Spring potential energy is converted into kinetic energy $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

(b) (c) i)



ii) using the equation above and rearrange to the form $y = m x$ with v^2 as y and x^2 as x

$y = m \quad x$
$v^2 = (k/m)\, x^2$

Slope = 200 = k /m

200 = (40) / m

m = 0.2 kg

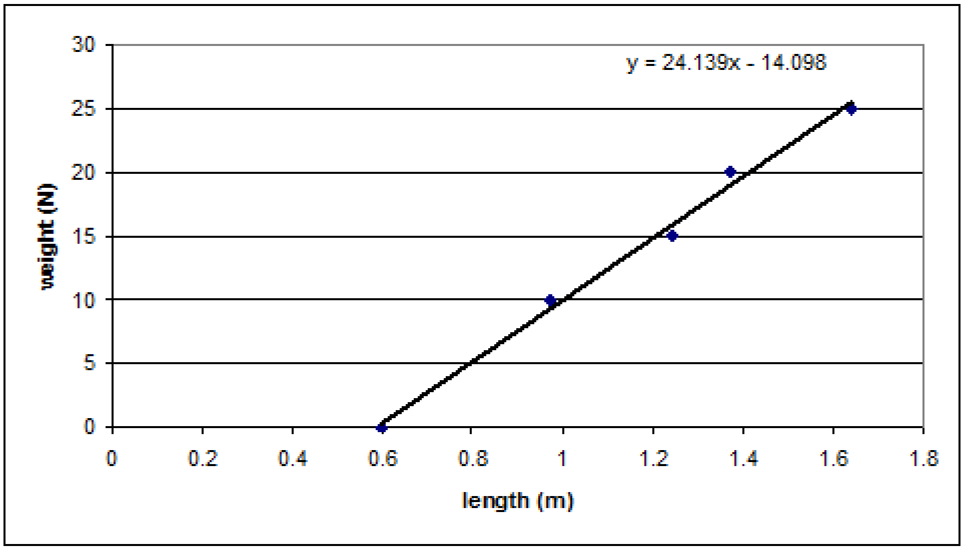
(d) Now you start with spring potential and gravitational potential and convert to kinetic. Note that at position A the height of the glider is given by h + the y component of the stretch distance x. $h_{initial} = h + x \sin\theta$

$U + U_{sp} = K$

$mgh + \frac{1}{2} k x^2 = \frac{1}{2} m v^2 \qquad mg(h + x \sin\theta) + \frac{1}{2} kx^2 = \frac{1}{2} mv^2$

C2008M3

(a)



(b) The slope of the line is $F / \Delta x$ which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

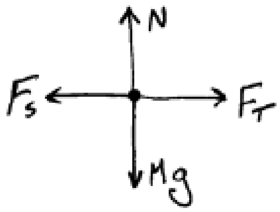
$mgh = \frac{1}{2} k \Delta x^2$ $\quad m(9.8)(1.5) = \frac{1}{2}(24)(0.9)^2$ $\quad m = 0.66$ kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $\quad F_{sp} = (0.66)(9.8)$ $\quad F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $\quad 6.5 = (24) \Delta x$ $\quad \Delta x = 0.27$ m

Supplemental

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

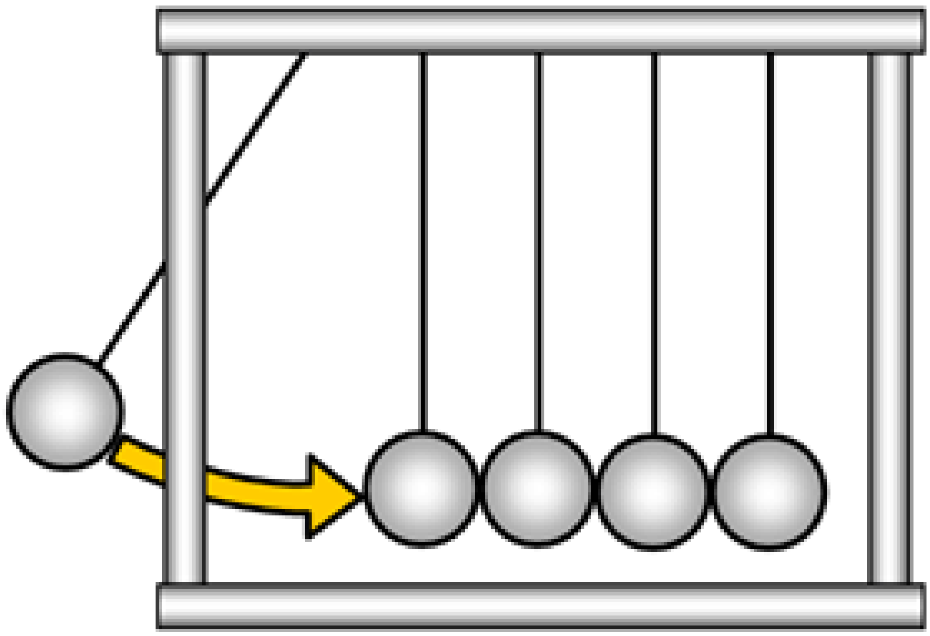
$\frac{1}{2} k \Delta x^2 = \frac{1}{2} k\Delta x_2{}^2 + \frac{1}{2} mv^2$
$k (\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$ $v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$

(d) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get
$F_{net} = ma$ $F_{sp} - f_k = ma$
From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$

Chapter 5

Momentum and Impulse



AP Physics Multiple Choice Practice – Momentum and Impulse

1. A car of mass m, traveling at speed v, stops in time t when maximum braking force is applied. Assuming the braking force is independent of mass, what time would be required to stop a car of mass 2m traveling at speed v?
(A) ½ t (B) t (C) √2 t (D) 2t (E) 4t

2. A block of mass M is initially at rest on a frictionless floor. The block, attached to a massless spring with spring constant k, is initially at its equilibrium position. An arrow with mass m and velocity v is shot into the block. The arrow sticks in the block. What is the maximum compression of the spring?

$(A)\ v\sqrt{\frac{m}{k}}$ $(B)\ v\sqrt{\frac{k}{m}}$ $(C)\ v\sqrt{\frac{m+M}{k}}$ $(D)\ \frac{(m+M)v}{\sqrt{mk}}$ $(E)\ \frac{mv}{\sqrt{(m+M)k}}$

3. How long must a 100 N net force act to produce a change in momentum of 200 kg·m/s?
(A) 0.25 s (B) 0.50 s (C) 1.0 s (D) 2.0 s (E) 4.0 s

4. Which is a vector quantity
(A) energy (B) mass (C) impulse (D) power (E) work

5. When the velocity of a moving object is doubled, its _____ is also doubled.
(A) acceleration (B) kinetic energy (C) mass (D) momentum (E) potential energy.

6. Two objects, P and Q, have the same momentum. Q can have more kinetic energy than P if it has:
(A) More mass than P (B) The same mass as P (C) More speed than P
(D) The same speed at P (E) Q can not have more kinetic energy than P

7. A spring is compressed between two objects with unequal masses, m and M, and held together. The objects are initially at rest on a horizontal frictionless surface. When released, which of the following is true?
(A) Kinetic energy is the same as before begin released.
(B) The total final kinetic energy is zero.
(C) The two objects have equal kinetic energy.
(D) The speed of one object is equal to the speed of the other.
(E) The total final momentum of the two objects is zero.

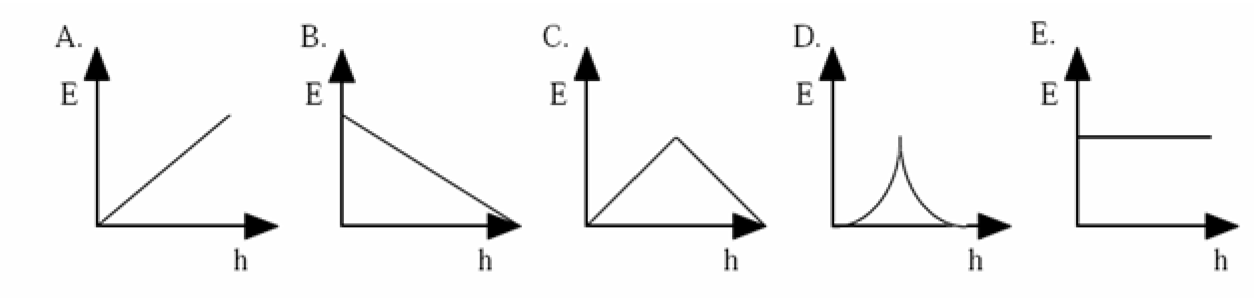
8. Net Impulse is best related to
(A) momentum (B) change in momentum (C) kinetic energy
(D) change in kinetic energy (E) none of the above

9. Two football players with mass 75 kg and 100 kg run directly toward each other with speeds of 6 m/s and 8 m/s respectively. If they grab each other as they collide, the combined speed of the two players just after the collision would be:
(A) 2 m/s (B) 3.4 m/s (C) 4.6 m/s (D) 7.1 m/s (E) 8 m/s

10. A 30 kg child who is running at 4 m/s jumps onto a stationary 10 kg skateboard. The speed of the child and the skateboard is approximately:
(A) 3 m/s (B) 4 m/s (C) 5 m/s (D) 6 m/s (E) 7 m/s

11. A 5000 kg freight car moving at 4 km/hr collides and couples with an 8000 kg freight car which is initially at rest. The approximate common final speed of these two cars is
(A) 1 km/h (B) 1.3 km/h (C) 1.5 km/h (D) 2.5 km/h (E) 4 km/h

12. A rubber ball is held motionless a height h_o above a hard floor and released. Assuming that the collision with the floor is elastic, which one of the following graphs best shows the relationship between the total energy E of the ball and its height h above the surface.



13. Two carts are held together. Cart 1 is more massive than Cart 2. As they are forced apart by a compressed spring between them, which of the following will have the same magnitude for both carts.
(A) acceleration (B) change of velocity (C) force (D) speed (E) velocity

14. If the unit for force is F, the unit for velocity is v and the unit for time t, then the unit for momentum is
(A) Ft (B) Ftv (C) Ft^2v (D) Ft / v (E) Fv / t

15. A ball with a mass of 0.50 kg and a speed of 6 m/s collides perpendicularly with a wall and bounces off with a speed of 4 m/s in the opposite direction. What is the magnitude of the impulse acting on the ball?
(A) 13 J (B) 1 Ns (C) 5 Ns (D) 2 m/s (E) 10 m/s

16. A cart with mass 2m has a velocity v before it strikes another cart of mass 3m at rest. The two carts couple and move off together with a velocity of
(A) v/5 (B) 2v/5 (C) 3v/5 (D) 2v/3 (E) $(2/5)^{1/2}$ v

17. Consider two laboratory carts of different masses but identical kinetic energies and the three following statements.
I. The one with the greatest mass has the greatest momentum
II. The same impulse was required to accelerate each cart from rest
III. Both can do the same amount of work as they come to a stop
Which of the above statements would be correct?
(A) I only (B) II only (C) III only (D) I and II only E) I and III only

18. A mass m has speed v. It then collides with a stationary object of mass 2m. If both objects stick together in a perfectly inelastic collision, what is the final speed of the newly formed object?
(A) v / 3 (B) v / 2 (C) 2v / 3 (D) v (E) 3v / 2

19. A Freight car is moving freely along a railroad track at 7 m/s and collides with a tanker car that is at rest. After the collision, the two cars stick together and continue to move down the track. What is the magnitude of the final velocity of the cars if the freight car has a mass of 1200 kg and the tanker car has a mass of 1600 kg?
(A) 0 m/s (B) 1 m/s (C) 3 m/s (D) 4 m/s (E) 6 m/s

20. A 50 kg skater at rest on a frictionless rink throws a 2 kg ball, giving the ball a velocity of 10 m/s. Which statement describes the skater's subsequent motion?
(A) 0.4 m/s in the same direction as the ball.
(B) 0.4 m/s in the opposite direction of the ball
(C) 2 m/s in the same direction as the ball
(D) 4 m/s in the same direction as the ball
(E) 4 m/s in the opposite direction of the ball

21. A certain particle undergoes erratic motion. At every point in its motion, the direction of the particles momentum is ALWAYS
(A) the same as the direction of its velocity
(B) the same as the direction of its acceleration
(C) the same as the direction of the net force
(D) the same as the direction of the kinetic energy vector
(E) none of the above

22. A student initially at rest on a frictionless frozen pond throws a 1 kg hammer in one direction. After the throw, the hammer moves off in one direction while the student moves off in the other direction. Which of the following correctly describes the above situation?
(A) The hammer will have the momentum with the greater magnitude
(B) The student will have the momentum with the greater magnitude
(C) The hammer will have the greater kinetic energy
(D) The student will have the greater kinetic energy
(E) The student and the hammer will have equal but opposite amounts of kinetic energy

23. The net force on a rocket with a weight of 1.5×10^4 N is 2.4×10^4 N. How much time is needed to increase the rockets speed from 12 m/s to 36 m/s near the surface of the Earth at takeoff?
(A) 0.62 s (B) 0.78 s (C) 1.5 s (D) 3.8 s (E) 15 s

24. A 50 kg gymnast falls freely from a height of 4 meters on to a trampoline. The trampoline then bounces her back upward with a speed equal to the speed at which she first struck the trampoline. What is the average force the trampoline applies to the gymnast.
(A) 50 N (B) 200 N (C) 500 N (D) 2000 N (E) More information is required

25. Two toy cars with different masses originally at rest are pushed apart by a spring between them. Which of the following statements would NOT be true?
(A) both toy cars will acquire equal but opposite momenta
(B) both toy cars will acquire equal kinetic energies
(C) the more massive toy car will acquire the least speed
(D) the smaller toy car will experience an acceleration of the greatest magnitude

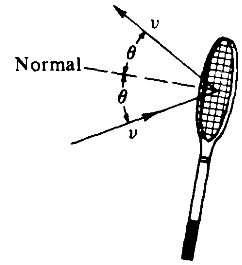
26. A bat striking a 0.125 kg baseball is in contact with the ball for a time of 0.03 seconds. The ball travels in a straight line as it approaches and then leaves the bat. If the ball arrives at the bat with a speed of 4.5 m/s and leaves with a speed of 6.5 m/s in the opposite direction, what is the magnitude of the average force acting on the ball?
(A) 8.33 N (B) 18.75 N (C) 27.08 N (D) 45.83 N (E) 458 N

27. An arrow is shot through an apple. If the 0.1 kg arrow changes speed by 10 m/s during the collision (from 30 m/s to 20 m/s) and the apple goes from rest to a speed of 2 m/s during the collisions, then the mass of the apple must be
(A) 0.2 kg (B) 0.5 kg (C) 0.8 kg (D) 1 kg (E) 2 kg

28. A railroad flatcar of mass 2,000 kilograms rolls to the right at 10 meters per second and collides with a flatcar of mass 3,000 kilograms that is rolling to the left at 5 meters per second. The flatcars couple together. Their speed after the collision is
(A) 1 m/s (B) 2.5 m/ s (C) 5 m/ s (D) 7 m/ s (E) 7.5 m/ s

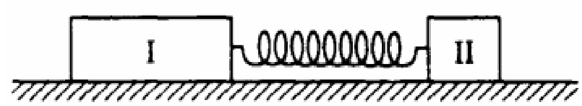
29. Which of the following quantities is a scalar that is always positive or zero?
(A) Power (B) Work (C) Kinetic energy (D) Linear momentum (E) Angular momentum

30. A tennis ball of mass m rebounds from a racquet with the same speed v as it had initially as shown. The magnitude of the momentum change of the ball is
(A) 0 (B) mv (C) 2mv (D) 2mv sinθ (E) 2mv cosθ



31. Two bodies of masses 5 and 7 kilograms are initially at rest on a horizontal frictionless surface. A light spring is compressed between the bodies, which are held together by a thin thread. After the spring is released by burning through the thread, the 5 kilogram body has a speed of 0.2 m/s. The speed of the 7 kilogram body is (in m/s) (A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{1}{\sqrt{35}}$ (D) $\frac{1}{5}$ (E) $\frac{7}{25}$

32. A satellite of mass M moves in a circular orbit of radius R at a constant speed v. Which of the following must be true?
I. The net force on the satellite is equal to MR and is directed toward the center of the orbit.
II. The net work done on the satellite by gravity in one revolution is zero.
III. The angular momentum of the satellite is a constant.
(A) I only (B) III only (C) I and II only (D) II and III only (E) I, II, and III



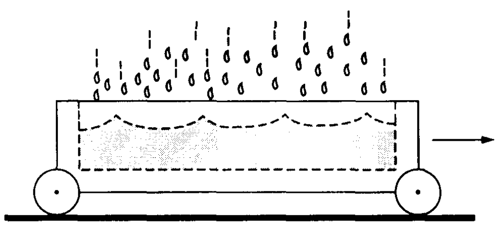
33. Two pucks are firmly attached by a stretched spring and are initially held at rest on a frictionless surface, as shown above. The pucks are then released simultaneously. If puck I has three times the mass of puck II, which of the following quantities is the same for both pucks as the spring pulls the two pucks toward each other?
(A) Speed (B) Velocity (C) Acceleration (D) Kinetic energy (E) Magnitude of momentum

34. Which of the following is true when an object of mass m moving on a horizontal frictionless surface hits and sticks to an object of mass M > m, which is initially at rest on the surface?
(A) The collision is elastic.
(B) All of the initial kinetic energy of the less massive object is lost.
(C) The momentum of the objects that are stuck together has a smaller magnitude than the initial momentum of the less-massive object.
(D) The speed of the objects that are stuck together will be less than the initial speed of the less massive object.
(E) The direction of motion of the objects that are stuck together depends on whether the hit is a head-on collision.

35. Two objects having the same mass travel toward each other on a flat surface each with a speed of 1.0 meter per second relative to the surface. The objects collide head-on and are reported to rebound after the collision, each with a speed of 2.0 meters per second relative to the surface. Which of the following assessments of this report is most accurate?
(A) Momentum was not conserved therefore the report is false.
(B) If potential energy was released to the objects during the collision the report could be true.
(C) If the objects had different masses the report could be true.
(D) If the surface was inclined the report could be true.
(E) If there was no friction between the objects and the surface the report could be true.

36. A solid metal ball and a hollow plastic ball of the same external radius are released from rest in a large vacuum chamber. When each has fallen 1m, they both have the same
(A) inertia (B) speed (C) momentum (D) kinetic energy (E) change in potential energy

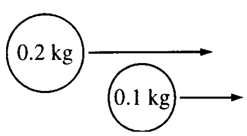
37. A railroad car of mass *m* is moving at speed *v* when it collides with a second railroad car of mass *M* which is at rest. The two cars lock together instantaneously and move along the track. What is the speed of the cars immediately after the collision?
(A) $v/2$ (B) mv/M (C) Mv/m (D) $(m + M)v/m$ (E) $mv/(m+M)$



38. An open cart on a level surface is rolling without frictional loss through a vertical downpour of rain, as shown above. As the cart rolls, an appreciable amount of rainwater accumulates in the cart. The speed of the cart will

(A) increase because of conservation of momentum
(B) increase because of conservation of mechanical energy
(C) decrease because of conservation of momentum
(D) decrease because of conservation of mechanical energy
(E) remain the same because the raindrops are falling perpendicular to the direction of the cart's motion

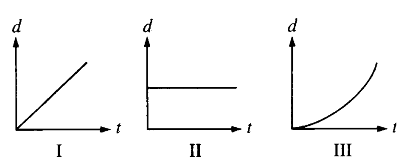
39. A 2 kg object moves in a circle of radius 4 m at a constant speed of 3 m/s. A net force of 4.5 N acts on the object. What is the angular momentum of the object with respect to an axis perpendicular to the circle and through its center?
(A) 9 N m/kg (B) 12 m^2/s (C) 13.5 kg m^2/s^2 (D) 18 N m/kg (E) 24 kg m^2/s.



40. Two objects of mass 0.2 kg and 0.1 kg, respectively, move parallel to the x-axis, as shown above. The 0.2 kg object overtakes and collides with the 0.1 kg object. Immediately after the collision, the y-component of the velocity of the 0.2 kg object is 1 m/s upward. What is the y-component of the velocity of the 0.1 kg object immediately after the collision'?

(A) 2 m/s downward (B) 0.5 m/s downward (C) 0 m/s
(D) 0.5 m/s upward (E) 2 m/s upward

<u>Questions 41-42</u>
Three objects can only move along a straight, level path. The graphs below show the position *d* of each of the objects plotted as a function of time *t*.



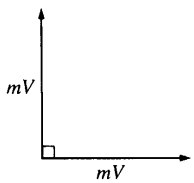
41. The magnitude of the momentum of the object is increasing in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

42. The sum of the forces on the object is zero in which of the cases?
(A) II only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

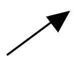
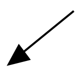
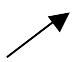
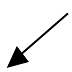
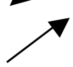
43. A ball of mass 0.4 kg is initially at rest on the ground. It is kicked and leaves the kicker's foot with a speed of 5.0 m/s in a direction 60° above the horizontal. The magnitude of the impulse imparted by the ball to the foot is most nearly

(A) 1 N s (B) $\sqrt{3}$ N s (C) 2 N s (D) $\frac{2}{\sqrt{3}}$ N s (E) 4 N s

44. Two people of unequal mass are initially standing still on ice with negligible friction. They then simultaneously push each other horizontally. Afterward, which of the following is true?
(A) The kinetic energies of the two people are equal.
(B) The speeds of the two people are equal.
(C) The momenta of the two people are of equal magnitude.
(D) The center of mass of the two-person system moves in the direction of the less massive person.
(E) The less massive person has a smaller initial acceleration than the more massive person.



45. A stationary object explodes, breaking into three pieces of masses *m*, *m*, and *3m*. The two pieces of mass *m* move off at right angles to each other with the same magnitude of momentum mV, as shown in the diagram above. What are the magnitude and direction of the velocity of the piece having mass *3m* ?

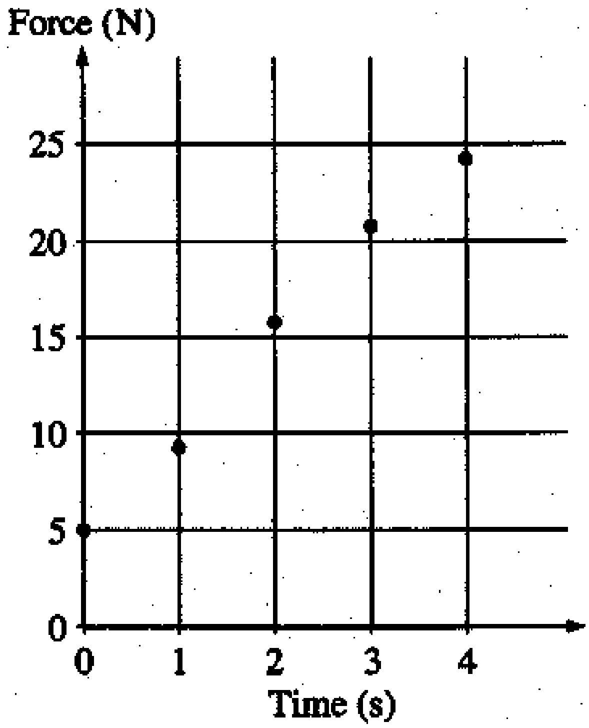
	Magnitude	Direction
(A)	$\frac{V}{\sqrt{3}}$	
(B)	$\frac{V}{\sqrt{3}}$	
(C)	$\frac{\sqrt{2}V}{3}$	
(D)	$\frac{\sqrt{2}V}{3}$	
(E)	$\sqrt{2}V$	

46. A ball is thrown straight up in the air. When the ball reaches its highest point, which of the following is true?

(A) It is in equilibrium
(B) It has zero acceleration.
(C) It has maximum momentum.
(D) It has maximum kinetic energy.
(E) None of the above

47. An empty sled of mass M moves without friction across a frozen pond at speed v_o. Two objects are dropped vertically into the sled one at a time: first an object of mass m and then an object of mass 2m. Afterward the sled moves with speed v_f. What would be the final speed of the sled if the objects were dropped into it in reverse order?
(A) $v_f / 3$
(B) $v_f / 2$
(C) v_f
(D) $2v_f$
(E) $3v_f$

Questions 48-49



A student obtains data on the magnitude of force applied to an object as a function of time and displays the data on the graph above.

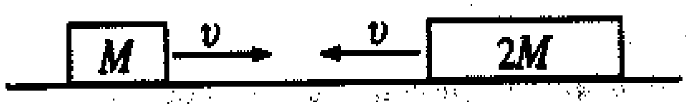
48. The slope of the "best fit" straight line is most nearly
   (A) 5 N/s
   (B) 6 N/s
   (C) 7 N/s
   (D) 8 N/s
   (E) 10 N/s

49. The increase in the momentum of the object between t=0 s and t=4 s is most nearly
   (A) 40 N·s
   (B) 50 N·s
   (C) 60 N·s
   (0) 80 N·s
   (E) 100 N·s

50. How does an air mattress protect a stunt person landing on the ground after a stunt?
   (A) It reduces the kinetic energy loss of the stunt person.
   (B) It reduces the momentum change of the stunt person.
   (C) It increases the momentum change of the stunt person.
   (D) It shortens the stopping time of the stunt person and increases the force applied during the landing.
   (E) It lengthens the stopping time of the stunt person and reduces the force applied during the landing.

51. Two objects, A and B, initially at rest, are "exploded" apart by the release of a coiled spring that was compressed between them. As they move apart, the velocity of object A is 5 m/s and the velocity of object B is – 2 m/s. The ratio of the mass of object A to the mass object B, m_a/m_b is

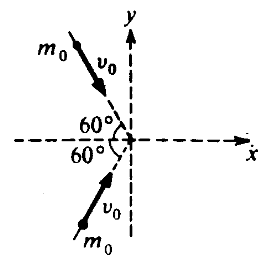
(A) 4/25 (B) 2/5 (C) 1/1 (D) 5/2 (E) 25/4



52. The two blocks of masses M and 2M shown above initially travel at the same speed v but in opposite directions. They collide and stick together. How much mechanical energy is lost to other forms of energy during the collision?

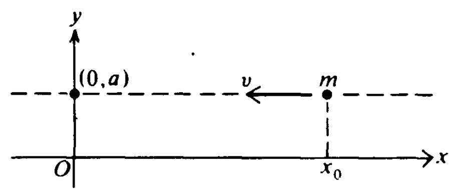
(A) Zero
(B) 1/2 M v^2
(C) 3/4 M v^2
(D) 4/3 M v^2
(E) 3/2 M v^2

53. Two particles of equal mass m_o, moving with equal speeds v_o along paths inclined at 60° to the x-axis as shown, collide and stick together. Their velocity after the collision has magnitude



(A) $\frac{v_0}{4}$ (B) $\frac{v_0}{2}$ (C) $\frac{\sqrt{2}v_0}{2}$ (D) $\frac{\sqrt{3}v_0}{2}$ (E) v_o

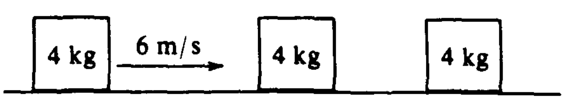
54. A particle of mass m moves with a constant speed v along the dashed line y = a. When the x-coordinate of the particle is x_o, the magnitude of the angular momentum of the particle with respect to the origin of the system is



(A) zero (B) mva (C) mvx_o

(D) $mv\sqrt{x^2 + a^2}$ (E) $\frac{mva}{\sqrt{x^2 + a^2}}$

Questions 55 and 56



A 4-kilogram mass has a speed of 6 meters per second on a horizontal frictionless surface, as shown above. The mass collides head-on with an identical 4-kilogram mass initially at rest and sticks. The combined masses then collide head-on and stick to a third 4-kilogram mass initially at rest.

55. The final speed of the first 4-kilogram mass is
(A) 0 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s (E) 6 m/s

56. The final speed of the two 4-kilogram masses that stick together is
(A) 0 m/s (B) 2 m/s (C) 3 m/s (D) 4 m/s (E) 6 m/s

57. A projectile of mass M_1 is fired horizontally from a spring gun that is initially at rest on a frictionless surface. The combined mass of the gun and projectile is M_2. If the kinetic energy of the projectile after firing is K, the gun will recoil with a kinetic energy equal to

(A) K (B) $\frac{M_2}{M_1}K$ (C) $\frac{M_1^2}{M_2^2}K$ D) $\frac{M_1}{M_2 - M_1}K$ (E) $\sqrt{\frac{M_1}{M_2 - M_1}}K$

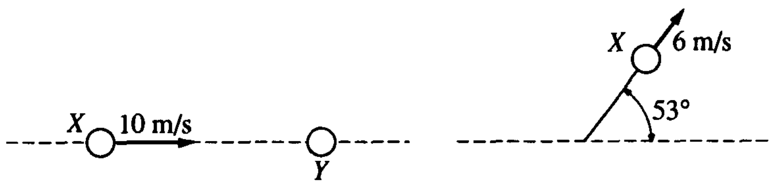
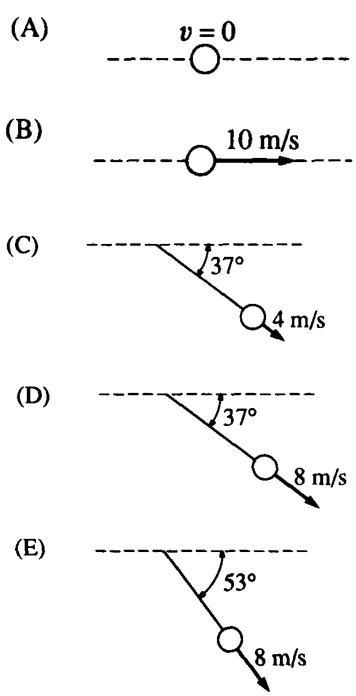
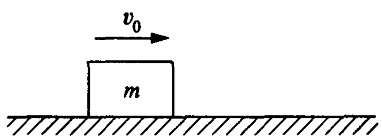


Figure I Figure II

58. Two balls are on a frictionless horizontal tabletop. Ball X initially moves at 10 meters per second, as shown in Figure I above. It then collides elastically with identical ball Y which is initially at rest. After the collision, ball X moves at 6 meters per second along a path at 53° to its original direction, as shown in Figure II above. Which of the following diagrams best represents the motion of ball Y after the collision?

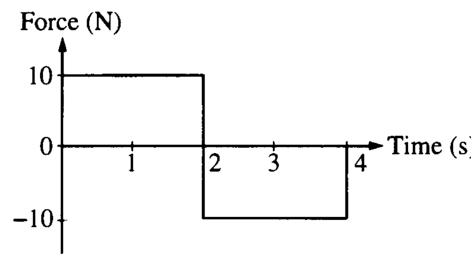


59. If one knows only the constant resultant force acting on an object and the time during which this force acts, one can determine the
(A) change in momentum of the object (B) change in velocity of the object
(C) change in kinetic energy of the object (D) mass of the object (E) acceleration of the object

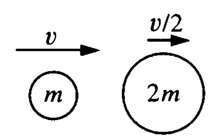


60. An object of mass m is moving with speed v_o to the right on a horizontal frictionless surface, as shown above, when it explodes into two pieces. Subsequently, one piece of mass 2/5 m moves with a speed $v_o / 2$ to the left. The speed of the other piece of the object is
(A) $v_o / 2$ (B) $v_o / 3$ (C) $7v_o / 5$ (D) $3v_o / 2$ (E) $2v_o$

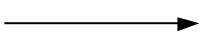
61. The graph shows the force on an object of mass M as a function of time. For the time interval 0 to 4 s, the total change in the momentum of the object is
(A) 40 kg m/s (B) 20 kg m/s
(C) 0 kg m/s (D) – 20 kg m/s
(E) indeterminable unless the mass M of the object is known



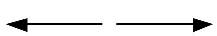
62. As shown in the top view, a disc of mass m is moving horizontally to the right with speed v on a table with negligible friction when it collides with a second disc of mass 2m The second disc is moving horizontally to the right with speed v/2 at the moment of impact The two discs stick together upon impact. The speed of the composite body immediately after the collision is
(A) v/3 (B) v/2 (C) 2v/3 (D) 3v/2 (E) 2v



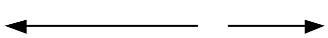
Top View

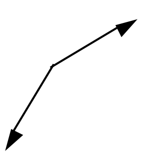
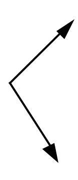


63. An object having an initial momentum that may be represented by the vector above strikes an object that is initially at rest. Which of the following sets of vectors may represent the momenta of the two objects after the collision?

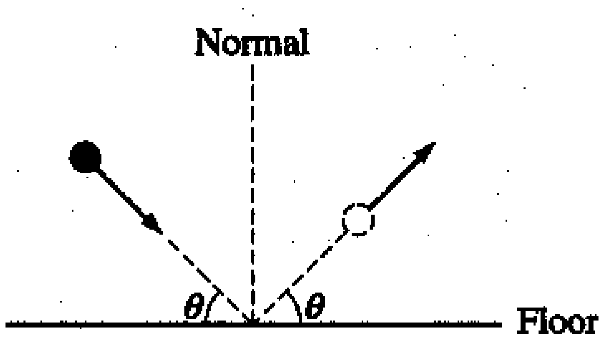
(A) 

(B) 

(C) 

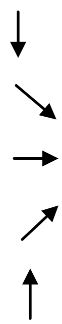
 

(D) (E)



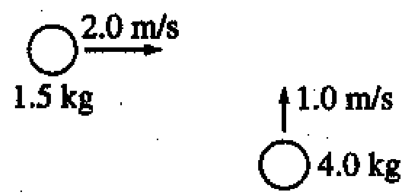
64. A 2 kg ball collides with the floor at an angle θ and rebounds at the same angle and speed as shown above. Which of the following vectors represents the impulse exerted on the ball by the floor?

(A)
(B)
(C)
(D)
(E)



<u>Questions 65-66</u>

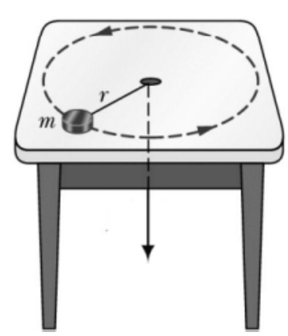
Two pucks moving on a frictionless air table are about to collide, as shown. The 1.5 kg puck is moving directly east at 2.0 m/s. The 4.0 kg puck is moving directly north at 1.0 m/s.



65. What is the total kinetic energy of the two-puck system before the collision?
(A) √13 J (B) 5.0 J (C) 7.0 J (D) 10 J (E) 11 J

66. What is the magnitude of the total momentum of the two-puck system after the collision?
(A) 1.0 kg·m/s (B) 3.5 kg·m/s (C) 5.0 kg·m/s (D) 7.0 kg·m/s (E) 5.5√5 kg·m/s

67. An object m, on the end of a string, moves in a circle on a horizontal frictionless table as shown. As the string is pulled very slowly through a small hole in the table, which of the following is correct for an observer measuring from the hole in the table?
(A) The angular momentum of m remains constant.
(B) The angular momentum of m decreases.
(C) The kinetic energy of m remains constant
(D) The kinetic energy of m decreases
(E) None of the above occurs.

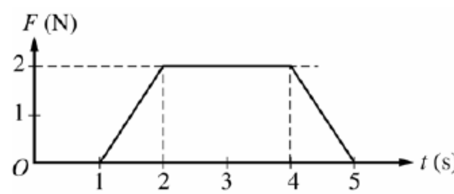


68. A car of mass 900 kg is traveling at 20 m/s when the brakes are applied. The car then comes to a complete stop in 5 s. What is the average power that the brakes produce in stopping the car?
(A) 1800 W (B) 3600 W (C) 7200 W (D) 36,000 W (E) 72,000 W

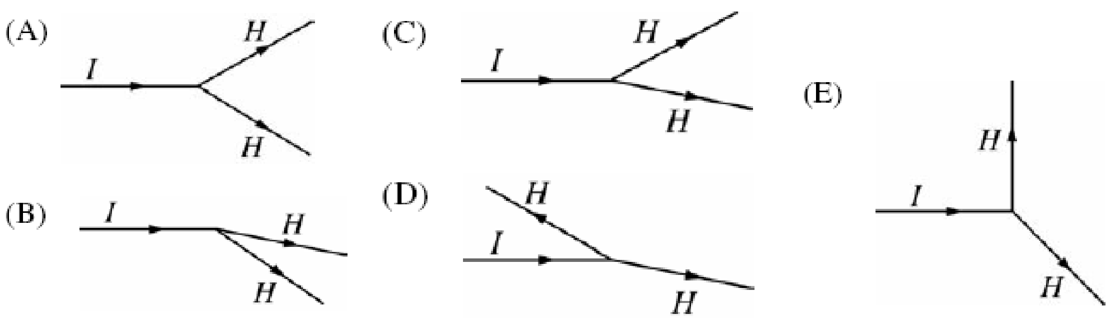
69. A boy of mass m and a girl of mass 2m are initially at rest at the center of a frozen pond. They push each other so that she slides to the left at speed v across the frictionless ice surface and he slides to the right. What is the total work done by the children?
(A) Zero (B) mv (C) mv^2 (D) $2mv^2$ (E) $3mv^2$

70. An object of mass M travels along a horizontal air track at a constant speed v and collides elastically with an object of identical mass that is initially at rest on the track. Which of the following statements is true for the two objects after the impact?
(A) The total momentum is Mv and the total kinetic energy is ½ Mv^2
(B) The total momentum is Mv and the total kinetic energy is less than ½ Mv^2
(C) The total momentum is less than Mv and the total kinetic energy is ½ Mv^2
(D) The momentum of each object is ½ Mv
(E) The kinetic energy of each object is ¼ Mv^2

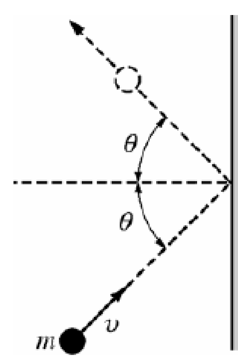
71. A 2 kg object initially moving with a constant velocity is subjected to a force of magnitude F in the direction of motion. A graph of F as a function of time t is shown. What is the increase, if any, in the velocity of the object during the time the force is applied?
(A) 0 m/s (B) 2.0 m/s (C) 3.0 m/s (D) 4.0 m/s (E) 6.0 m/s



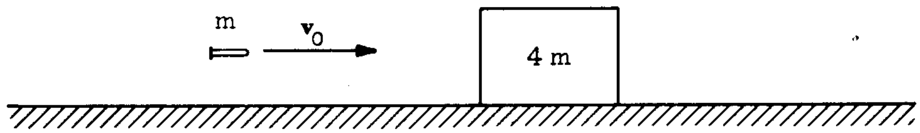
72. A disk slides to the right on a horizontal, frictionless air table and collides with another disk that was initially stationary. The figures below show a top view of the initial path I of the sliding disk and a hypothetical path H for each disk after the collision. Which figure shows an impossible situation?



73. A ball of mass m with speed v strikes a wall at an angle θ with the normal, as shown. It then rebounds with the same speed and at the same angle. The impulse delivered by the ball to the wall is
(A) zero (B) mv sin θ (C) mv cos θ (D) 2mv sin θ (E) 2mv cos θ



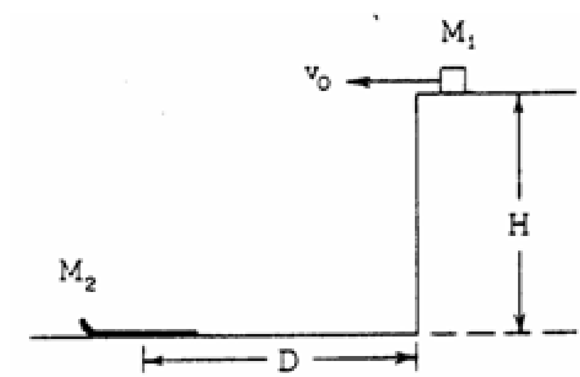
AP Physics Free Response Practice – Momentum and Impulse



1976B2.

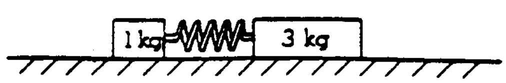
A bullet of mass m and velocity v_o is fired toward a block of mass 4m. The block is initially at rest on a frictionless horizontal surface. The bullet penetrates the block and emerges with a velocity of $\frac{v_o}{3}$

(a) Determine the final speed of the block.
(b) Determine the loss in kinetic energy of the bullet.
(c) Determine the gain in the kinetic energy of the block.



1978B2. A block of mass M_1 travels horizontally with a constant speed v_o on a plateau of height H until it comes to a cliff. A toboggan of mass M_2 is positioned on level ground below the cliff as shown above. The center of the toboggan is a distance D from the base of the cliff.

(a) Determine D in terms of v_o, H, and g so that the block lands in the center of the toboggan.
(b) The block sticks to the toboggan which is free to slide without friction. Determine the resulting velocity of the block and toboggan.



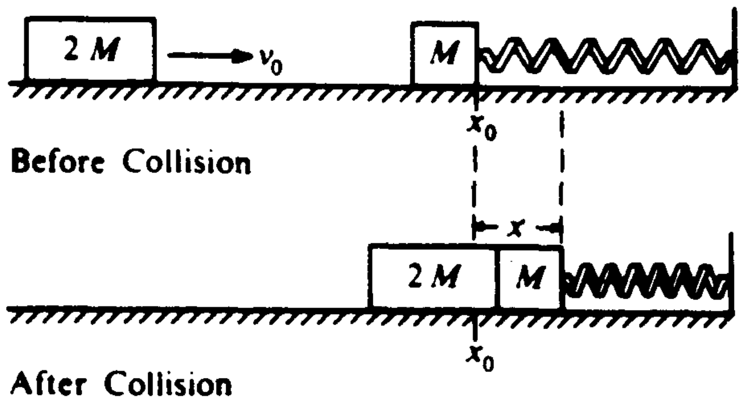
1981B2. A massless spring is between a 1-kilogram mass and a 3-kilogram mass as shown above, but is not attached to either mass. Both masses are on a horizontal frictionless table.

In an experiment, the 1-kilogram mass is held in place and the spring is compressed by pushing on the 3-kilogram mass. The 3-kilogram mass is then released and moves off with a speed of 10 meters per second.

a. Determine the minimum work needed to compress the spring in this experiment.

In a different experiment, the spring is compressed again exactly as above, but this time both masses are released simultaneously and each mass moves off separately at unknown speeds.

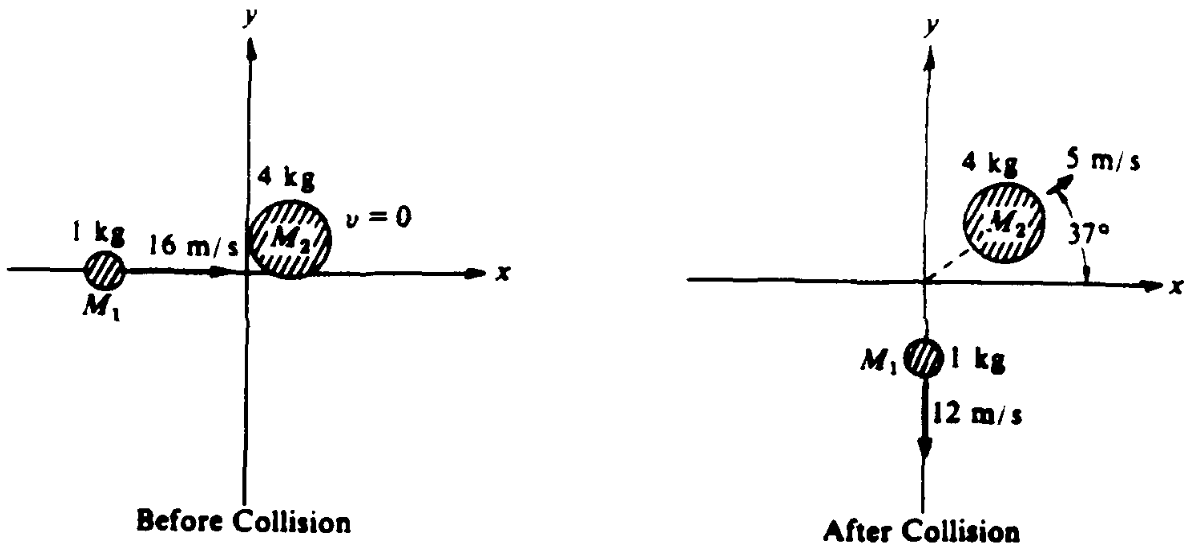
b. Determine the final velocity of each mass relative to the cable after the masses are released.



1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact

b. x, the maximum distance the spring is compressed



View From Above

1984B2. Two objects of masses M_1 = 1 kilogram and M_2 = 4 kilograms are free to slide on a horizontal frictionless surface. The objects collide and the magnitudes and directions of the velocities of the two objects before and after the collision are shown on the diagram above. (sin 37° = 0.6, cos 37° = 0.8, tan 37° = 0.75)

a. Calculate the x and y components (p_x and p_y, respectively) of the momenta of the two objects before and after the collision, and write your results in the proper places in the following table.

	M_1 = 1 kg		M_2 = 4 kg	
	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$	$p_x\left(\frac{kg\cdot m}{s}\right)$	$p_y\left(\frac{kg\cdot m}{s}\right)$
Before Collision				
After Collision				

b. Show, using the data that you listed in the table, that linear momentum is conserved in this collision.

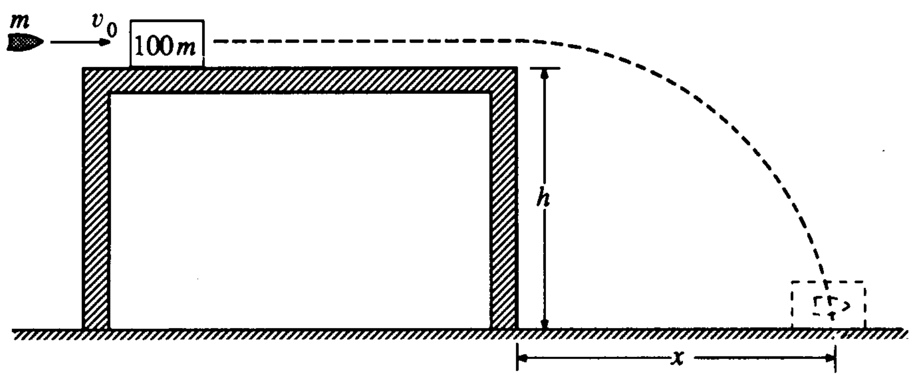
c. Calculate the kinetic energy of the two-object system before and after the collision.

d. Is kinetic energy conserved in the collision?

Before Collision   Immediately After Collision

1985B1. A 2-kilogram block initially hangs at rest at the end of two 1-meter strings of negligible mass as shown on the left diagram above. A 0.003-kilogram bullet, moving horizontally with a speed of 1000 meters per second, strikes the block and becomes embedded in it. After the collision, the bullet/ block combination swings upward, but does not rotate.

a. Calculate the speed v of the bullet/ block combination just after the collision.
b. Calculate the ratio of the initial kinetic energy of the bullet to the kinetic energy of the bullet/ block combination immediately after the collision.
c. Calculate the maximum vertical height above the initial rest position reached by the bullet/block combination.



1990B1. A bullet of mass m is moving horizontally with speed v_o when it hits a block of mass 100m that is at rest on a horizontal frictionless table, as shown above. The surface of the table is a height h above the floor. After the impact, the bullet and the block slide off the table and hit the floor a distance x from the edge of the table. Derive expressions for the following quantities in terms of m, h, v_o, and appropriate constants:

a. the speed of the block as it leaves the table
b. the change in kinetic energy of the bullet-block system during impact
c. the distance x

Suppose that the bullet passes through the block instead of remaining in it.

d. State whether the time required for the block to reach the floor from the edge of the table would now be greater, less, or the same. Justify your answer.
e. State whether the distance x for the block would now be greater, less, or the same. Justify your answer.

1992B2. A 30-kilogram child moving at 4.0 meters per second jumps onto a 50-kilogram sled that is initially at rest on a long, frictionless, horizontal sheet of ice.

a. Determine the speed of the child-sled system after the child jumps onto the sled.

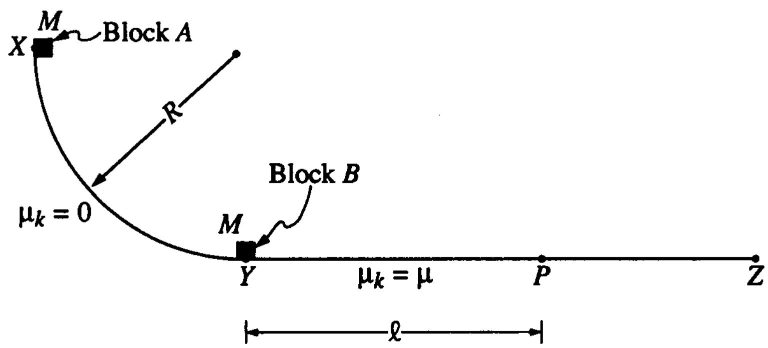
b. Determine the kinetic energy of the child-sled system after the child jumps onto the sled.

After coasting at constant speed for a short time, the child jumps off the sled in such a way that she is at rest with respect to the ice.

c. Determine the speed of the sled after the child jumps off it.

d. Determine the kinetic energy of the child-sled system when the child is at rest on the ice.

e. Compare the kinetic energies that were determined in parts (b) and (d). If the energy is greater in (d) than it is in (b), where did the increase come from? If the energy is less in (d) than it is in (b), where did the energy go?



Side View

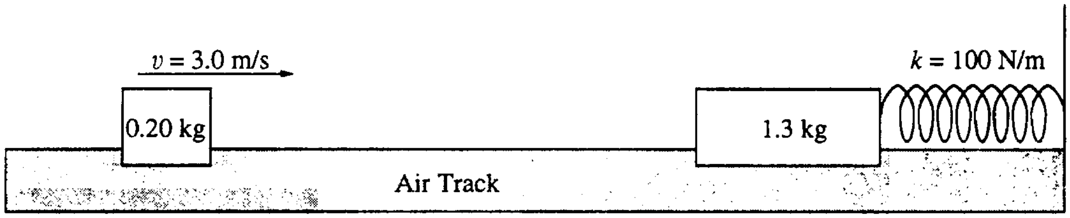
1994B2. A track consists of a frictionless arc XY, which is a quarter-circle of radius R, and a rough horizontal section YZ. Block A of mass M is released from rest at point X, slides down the curved section of the track, and collides instantaneously and inelastically with identical block B at point Y. The two blocks move together to the right, sliding past point P, which is a distance *L* from point Y. The coefficient of kinetic friction between the blocks and the horizontal part of the track is μ Express your answers in terms of M, *L*, μ, R, and g.

a. Determine the speed of block A just before it hits block B.

b. Determine the speed of the combined blocks immediately after the collision.

c. Assuming that no energy is transferred to the track or to the air surrounding the blocks. Determine the amount of internal energy transferred in the collision

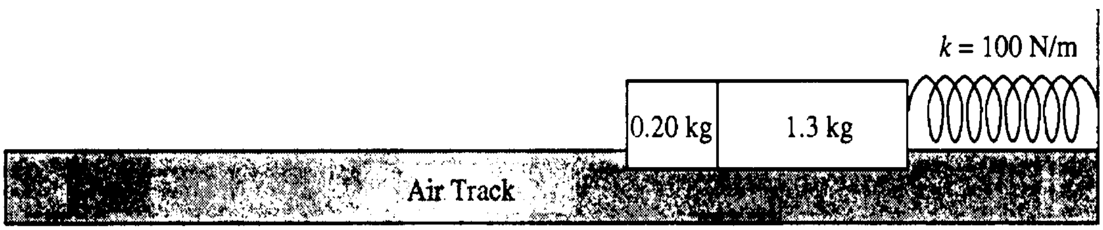
d. Determine the additional thermal energy that is generated as the blocks move from Y to P



1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

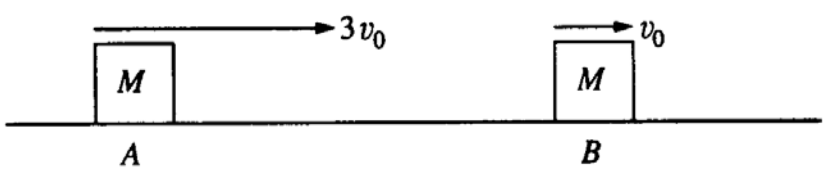
a. Determine the following for the 0.20-kilogram mass immediately before the impact.
   i. Its linear momentum ii. Its kinetic energy

b. Determine the following for the combined masses immediately after the impact.
   i. The linear momentum ii. The kinetic energy



After the collision, the two masses compress the spring as shown.

c. Determine the maximum compression distance of the spring.



1996B1. Two identical objects A and B of mass M move on a one-dimensional, horizontal air track. Object B initially moves to the right with speed v_o. Object A initially moves to the right with speed $3v_o$, so that it collides with object B. Friction is negligible. Express your answers to the following in terms of M and v_o.

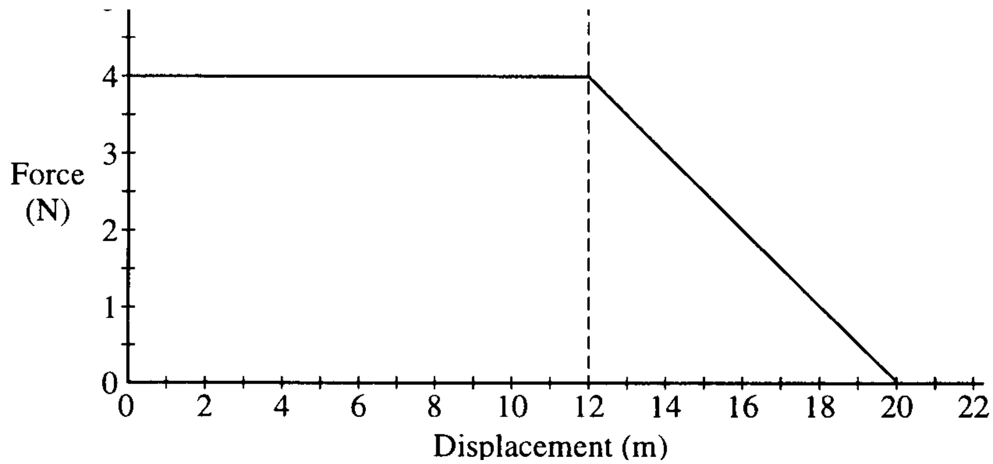
a. Determine the total momentum of the system of the two objects.

b. A student predicts that the collision will be totally inelastic (the objects stick together on collision). Assuming this is true, determine the following for the two objects immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)

When the experiment is performed, the student is surprised to observe that the objects separate after the collision and that object B subsequently moves to the right with a speed $2.5v_o$.

c. Determine the following for object A immediately after the collision.
   i. The speed
   ii. The direction of motion (left or right)

d. Determine the kinetic energy dissipated in the actual experiment.



1997B1. A 0.20 kg object moves along a straight line. The net force acting on the object varies with the object's displacement as shown in the graph above. The object starts from rest at displacement x = 0 and time t = 0 and is displaced a distance of 20 m. Determine each of the following.

a. The acceleration of the particle when its displacement x is 6 m.

b. The time taken for the object to be displaced the first 12 m.

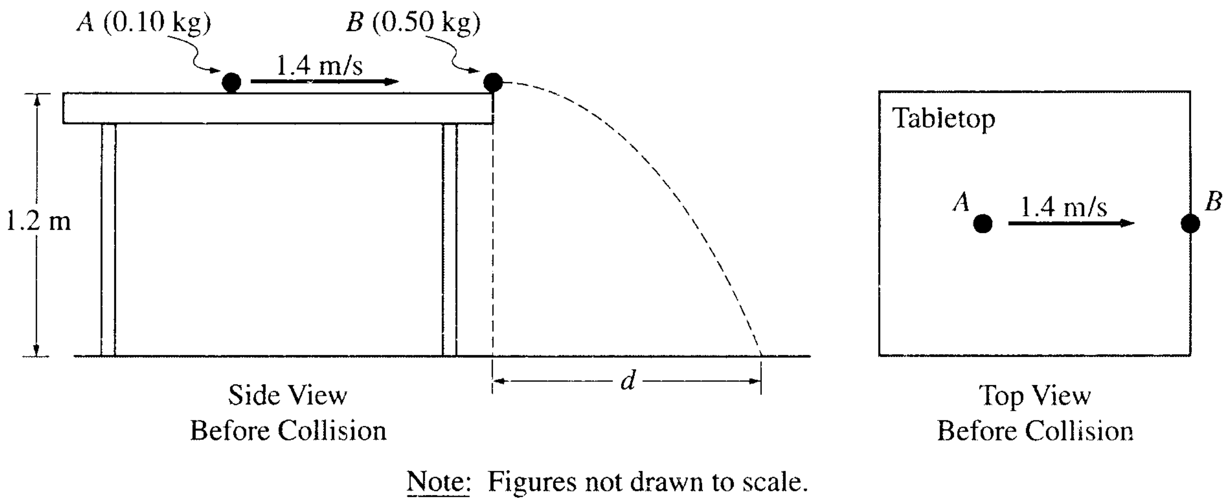
c. The amount of work done by the net force in displacing the object the first 12 m.

d. The speed of the object at displacement x = 12 m.

e. The final speed of the object at displacement x = 20 m.

f. The change in the momentum of the object as it is displaced from x = 12 m to x = 20 m

---



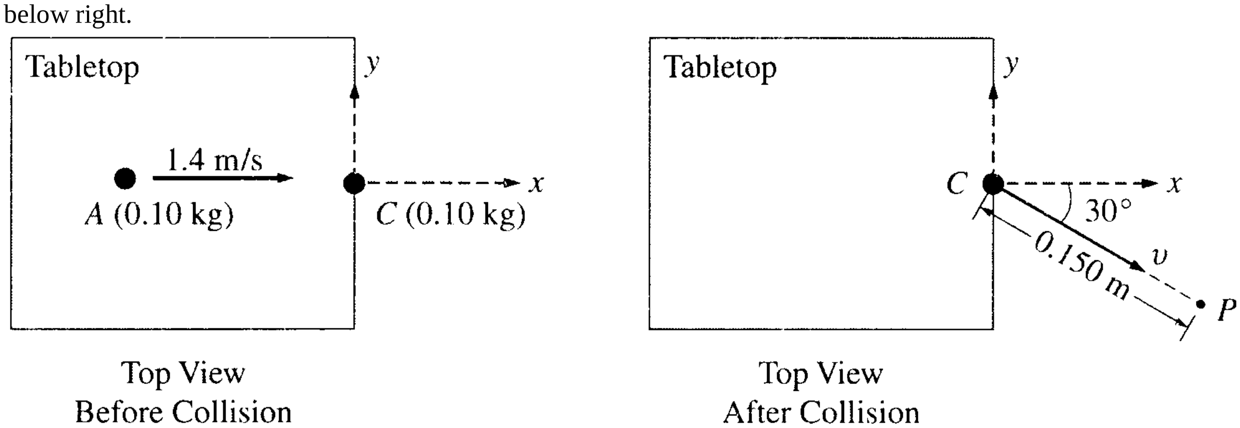
2001B2. An incident ball A of mass 0.10 kg is sliding at 1.4 m/s on the horizontal tabletop of negligible friction as shown above. It makes a head-on collision with a target ball B of mass 0.50 kg at rest at the edge of the table. As a result of the collision, the incident ball rebounds, sliding backwards at 0.70 m/s immediately after the collision.

a. Calculate the speed of the 0.50 kg target ball immediately after the collision.

The tabletop is 1.20 m above a level, horizontal floor. The target ball is projected horizontally and initially strikes the floor at a horizontal displacement *d* from the point of collision.

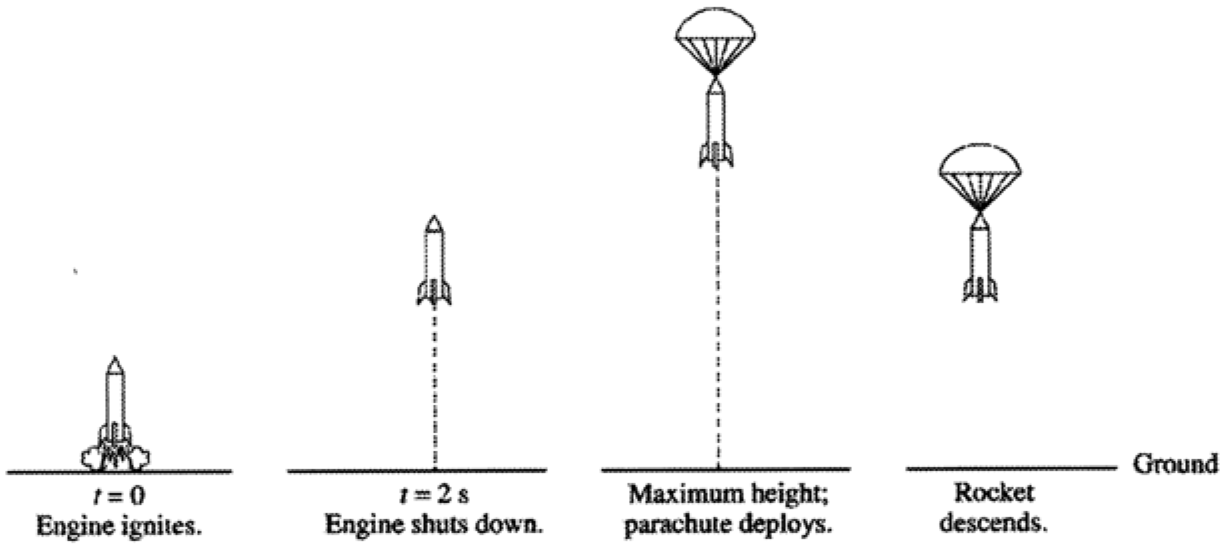
b. Calculate the horizontal displacement

In another experiment on the same table, the target ball B is replaced by target ball C of mass 0.10 kg. The incident ball A again slides at 1.4 m/s, as shown below left, but this time makes a glancing collision with the target ball C that is at rest at the edge of the table. The target ball C strikes the floor at point P, which is at a horizontal displacement of 0.15 m from the point of the collision, and at a horizontal angle of 30° from the +x-axis, as shown below right.



c. Calculate the speed v of the target ball C immediately after the collision.

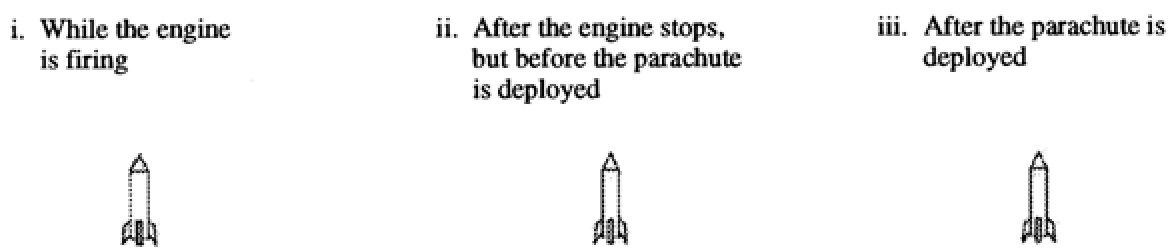
d. Calculate the y-component of incident ball A's momentum immediately after the collision.



Note: Figures not drawn to scale.

2002B1. A model rocket of mass 0.250 kg is launched vertically with an engine that is ignited at time t = 0, as shown above. The engine provides an impulse of 20.0 N•s by firing for 2.0 s. Upon reaching its maximum height, the rocket deploys a parachute, and then descends vertically to the ground.

(a) On the figures below, draw and label a free-body diagram for the rocket during each of the following intervals.

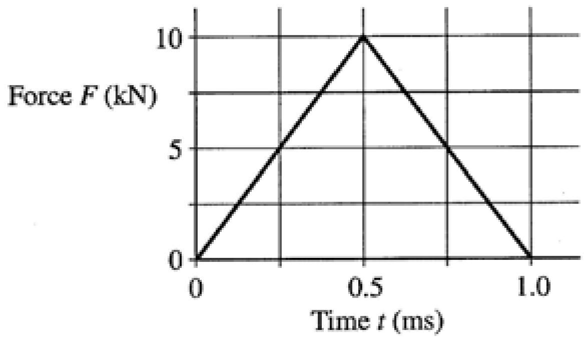


(b) Determine the magnitude of the average acceleration of the rocket during the 2 s firing of the engine.

(c) What maximum height will the rocket reach?

(d) At what time after t = 0 will the maximum height be reached?

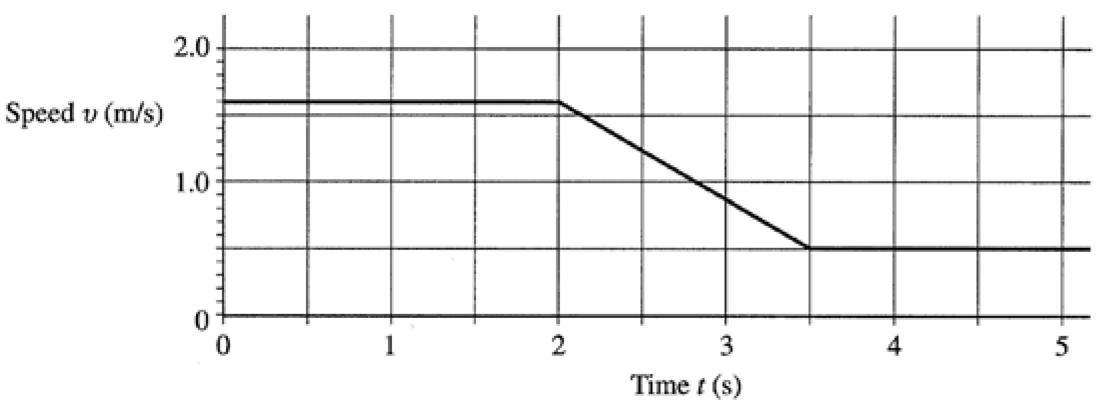
2002B1B. A 2.0 kg frictionless cart is moving at a constant speed of 3.0 m/s to the right on a horizontal surface, as shown above, when it collides with a second cart of undetermined mass m that is initially at rest. The force F of the collision as a function of time t is shown in the graph below, where $t = 0$ is the instant of initial contact. As a result of the collision, the second cart acquires a speed of 1.6 m/s to the right. Assume that friction is negligible before, during, and after the collision.



(a) Calculate the magnitude and direction of the velocity of the 2.0 kg cart after the collision.

(b) Calculate the mass m of the second cart.

After the collision, the second cart eventually experiences a ramp, which it traverses with no frictional losses. The graph below shows the speed v of the second cart as a function of time t for the next 5.0 s, where $t = 0$ is now the instant at which the carts separate.

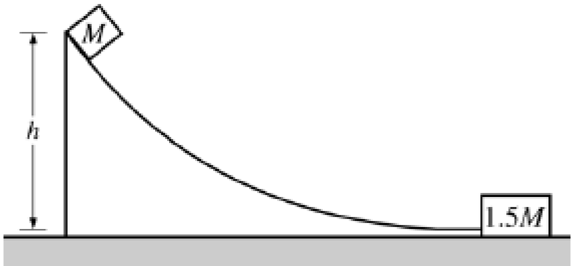


(c) Calculate the acceleration of the cart at $t = 3.0$ s.

(d) Calculate the distance traveled by the second cart during the 5.0 s interval after the collision (0 s < t < 5.0 s).

(e) State whether the ramp goes up or down **and** calculate the maximum elevation (above or below the initial height) reached by the second cart on the ramp during the 5.0 s interval after the collision (0 s < t < 5.0 s).

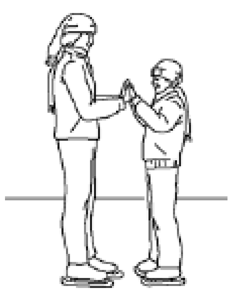
2006B2B



A small block of mass M is released from rest at the top of the curved frictionless ramp shown above. The block slides down the ramp and is moving with a speed $3.5v_o$ when it collides with a larger block of mass $1.5M$ at rest at the bottom of the incline. The larger block moves to the right at a speed $2\ v_o$ immediately after the collision. Express your answers to the following questions in terms of the given quantities and fundamental constants.

(a) Determine the height h of the ramp from which the small block was released.
(b) Determine the speed of the small block after the collision.
(c) The larger block slides a distance D before coming to rest. Determine the value of the coefficient of kinetic friction μ between the larger block and the surface on which it slides.
(d) Indicate whether the collision between the two blocks is elastic or inelastic. Justify your answer.

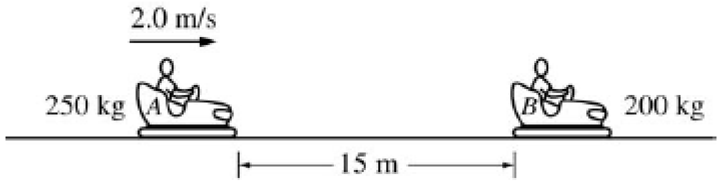
2008B1B



A 70 kg woman and her 35 kg son are standing at rest on an ice rink, as shown above. They push against each other for a time of 0.60 s, causing them to glide apart. The speed of the woman immediately after they separate is 0.55 m/s. Assume that during the push, friction is negligible compared with the forces the people exert on each other.

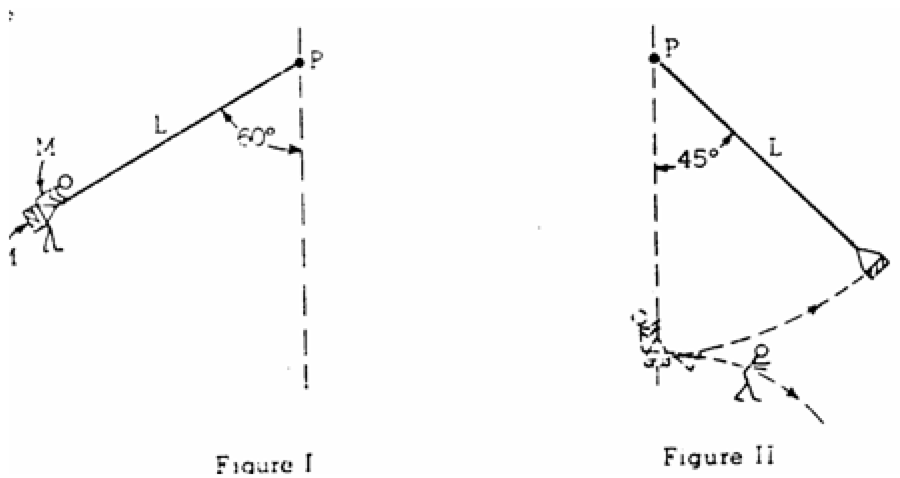
(a) Calculate the initial speed of the son after the push.
(b) Calculate the magnitude of the average force exerted on the son by the mother during the push.
(c) How do the magnitude and direction of the average force exerted on the mother by the son during the push compare with those of the average force exerted on the son by the mother? Justify your answer.
(d) After the initial push, the friction that the ice exerts cannot be considered negligible, and the mother comes to rest after moving a distance of 7.0 m across the ice. If their coefficients of friction are the same, how far does the son move after the push?

2008B1



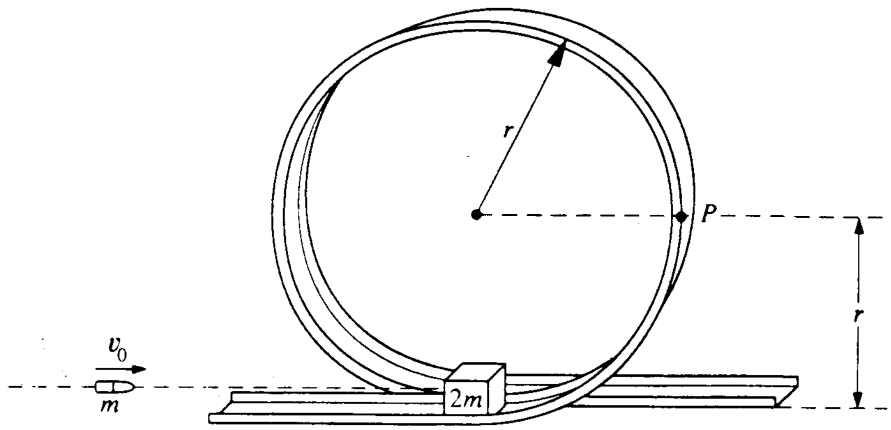
Several students are riding in bumper cars at an amusement park. The combined mass of car *A* and its occupants is 250 kg. The combined mass of car *B* and its occupants is 200 kg. Car *A* is 15 m away from car *B* and moving to the right at 2.0 m/s, as shown, when the driver decides to bump into car *B*, which is at rest.

(a) Car *A* accelerates at 1.5 m/s^2 to a speed of 5.0 m/s and then continues at constant velocity until it strikes car *B*. Calculate the total time for car *A* to travel the 15 m.
(b) After the collision, car *B* moves to the right at a speed of 4.8 m/s .
i. Calculate the speed of car *A* after the collision.
ii. Indicate the direction of motion of car *A* after the collision.
____ To the left ____ To the right ____ None; car *A* is at rest.
(c) Is this an elastic collision?
____ Yes ____ No
Justify your answer.



C1981M2. A swing seat of mass M is connected to a fixed point P by a massless cord of length L. A child also of mass M sits on the seat and begins to swing with zero velocity at a position at which the cord makes a 60° angle with the vertical is shown in Figure I. The swing continues down until the cord is exactly vertical at which time the child jumps off in a horizontal direction. The swing continues in the same direction until its cord makes a 45° angle with the vertical as shown in Figure II: at that point it begins to swing in the reverse direction.

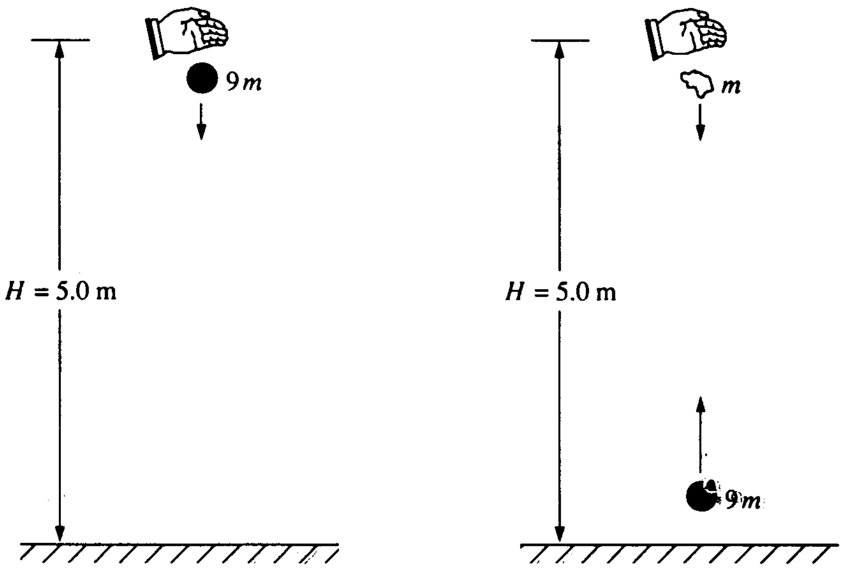
a) Determine the speed of the child and seat just at the lowest position prior to the child's dismount from the seat
b) Determine the speed if the seat immediately after the child dismounts
c) Determine the speed of the child immediately after he dismounts from the swing?



C1991M1. A small block of mass 2m initially rests on a track at the bottom of the circular, vertical loop-the-loop shown above, which has a radius r. The surface contact between the block and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_o and remains embedded in the block as the block and bullet circle the loop.

Determine each of the following in terms of m, v_o, r, and g.

a. The speed of the block and bullet immediately after impact
b. The kinetic energy of the block and bullet when they reach point P on the loop
c. The speed v_{min} of the block at the top of the loop to remain in contact with track at all times
d The new required entry speed v_o' of the block and bullet at the bottom of the loop such that the conditions in part c apply.
e. The new initial speed of the bullet to produce the speed from part d above.



C1992M1. A ball of mass 9m is dropped from rest from a height H = 5.0 meters above the ground, as shown above on the left. It undergoes a perfectly elastic collision with the ground and rebounds. At the instant that the ball rebounds, a small blob of clay of mass m is released from rest from the original height H, directly above the ball, as shown above on the right. The clay blob, which is descending, collides with the ball 0.5 seconds later, which is ascending. Assume that $g = 10\ m/s^2$, that air resistance is negligible, and that the collision process takes negligible time.

a. Determine the speed of the ball immediately before it hits the ground.

b. Determine the rebound speed of the ball immediately after it collides with the ground, justify your answer.

c. Determine the height above the ground at which the clay-ball collision takes place.

d. Determine the speeds of the ball and the clay blob immediately before the collision.

e. If the ball and the clay blob stick together on impact, what is the magnitude and direction of their velocity immediately after the collision?

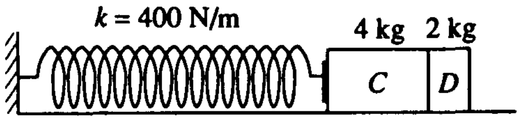


Figure I

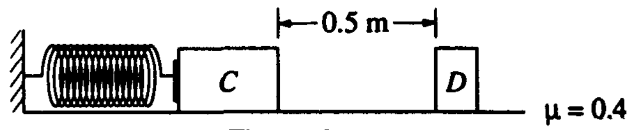


Figure II

C1993M1. A massless spring with force constant k = 400 newtons per meter is fastened at its left end to a vertical wall, as shown in Figure 1. Initially, block C (mass m_c = 4.0 kilograms) and block D (mass m_D = 2.0 kilograms) rest on a horizontal surface with block C in contact with the spring (but not compressing it) and with block *D* in contact with block C. Block C is then moved to the left, compressing the spring a distance of 0.50 meter, and held in place while block *D* remains at rest as shown in Figure 11. (Use g = 10 m/s^2.)

a. Determine the elastic energy stored in the compressed spring.

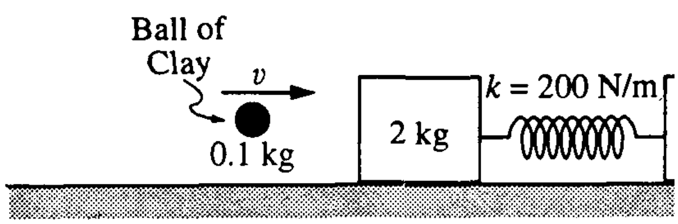
Block C is then released and accelerates to the right, toward block *D.* The surface is rough and the coefficient of friction between each block and the surface is μ = 0.4. The two blocks collide instantaneously, stick together, and move to the right. Remember that the spring is not attached to block C. Determine each of the following.

b. The speed v_c of block C just before it collides with block *D*

c. The speed v_f of blocks C and *D* just after they collide

d. The horizontal distance the blocks move before coming to rest

---



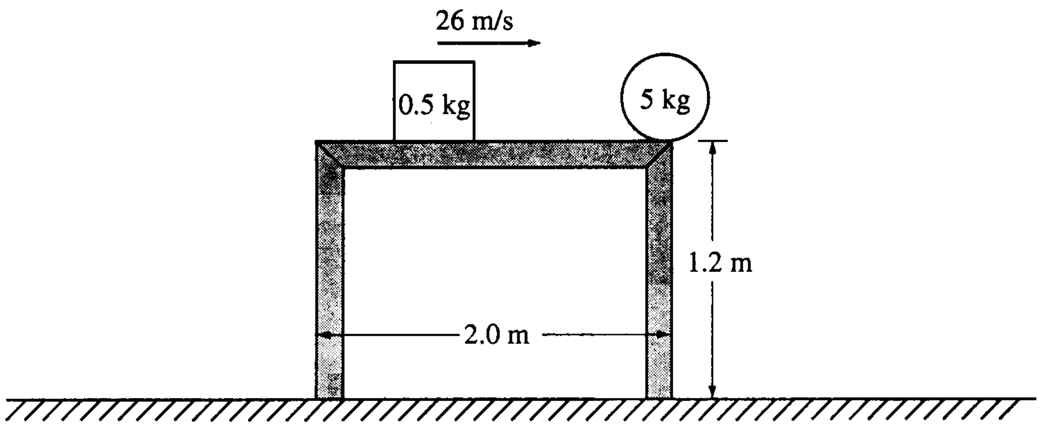
C1994M1. A 2-kilogram block is attached to an ideal spring (for which k = 200 N/m) and initially at rest on a horizontal frictionless surface, as shown in the diagram above.

In an initial experiment, a 100-gram (0.1 kg) ball of clay is thrown at the 2-kilogram block. The clay is moving horizontally with speed v when it hits and sticks to the block. The spring is attached to a wall as shown. As a result, the spring compresses a maximum distance of 0.4 meters.

a. Calculate the energy stored in the spring at maximum compression.

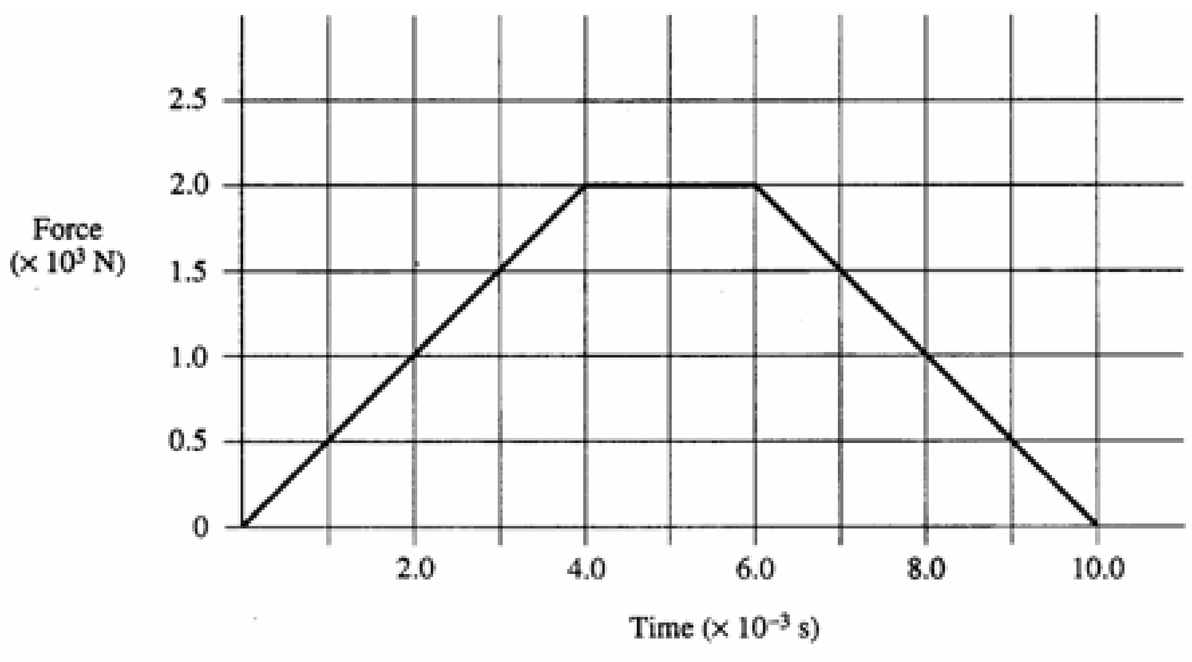
b. Calculate the speed of the clay ball and 2-kilogram block immediately after the clay sticks to the block but before the spring compresses significantly.

c. Calculate the initial speed v of the clay.

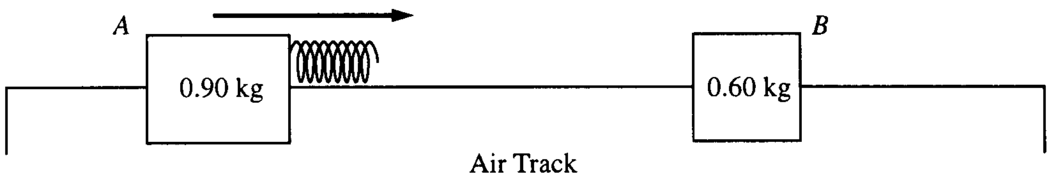


Note: Figure not drawn to scale.

C1995M1. A 5-kilogram ball initially rests at the edge of a 2-meter-long, 1.2-meter-high frictionless table, as shown above. A hard plastic cube of mass 0.5 kilogram slides across the table at a speed of 26 meters per second and strikes the ball, causing the ball to leave the table in the direction in which the cube was moving. The figure below shows a graph of the force exerted **on the ball** by the cube as a function of time.

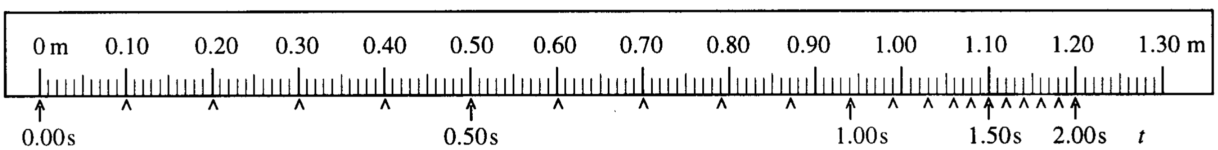


a. Determine the total impulse given to the ball.
b. Determine the horizontal velocity of the ball immediately after the collision.
c. Determine the following for the cube immediately after the collision.
   i. Its speed
   ii. Its direction of travel (right or left), if moving
d. Determine the kinetic energy dissipated in the collision.
e. Determine the distance between the two points of impact of the objects with the floor.



C1998M1. Two gliders move freely on an air track with negligible friction, as shown above. Glider A has a mass of 0.90 kg and glider B has a mass of 0.60 kg. Initially, glider A moves toward glider B, which is at rest. A spring of negligible mass is attached to the right side of glider A. Strobe photography is used to record successive positions of glider A at 0.10 s intervals over a total time of 2.00 s, during which time it collides with glider B.

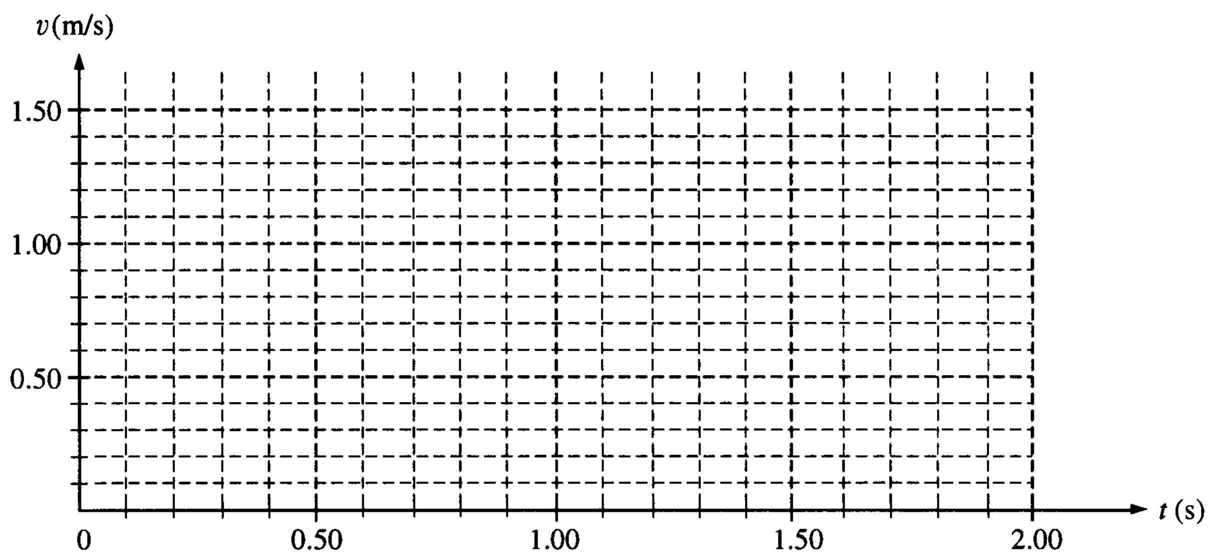
The following diagram represents the data for the motion of glider A. Positions of glider A at the end of each 0.10s interval are indicated by the symbol **ʌ** against a metric ruler. The total elapsed time t after each 0.50 s is also indicated.



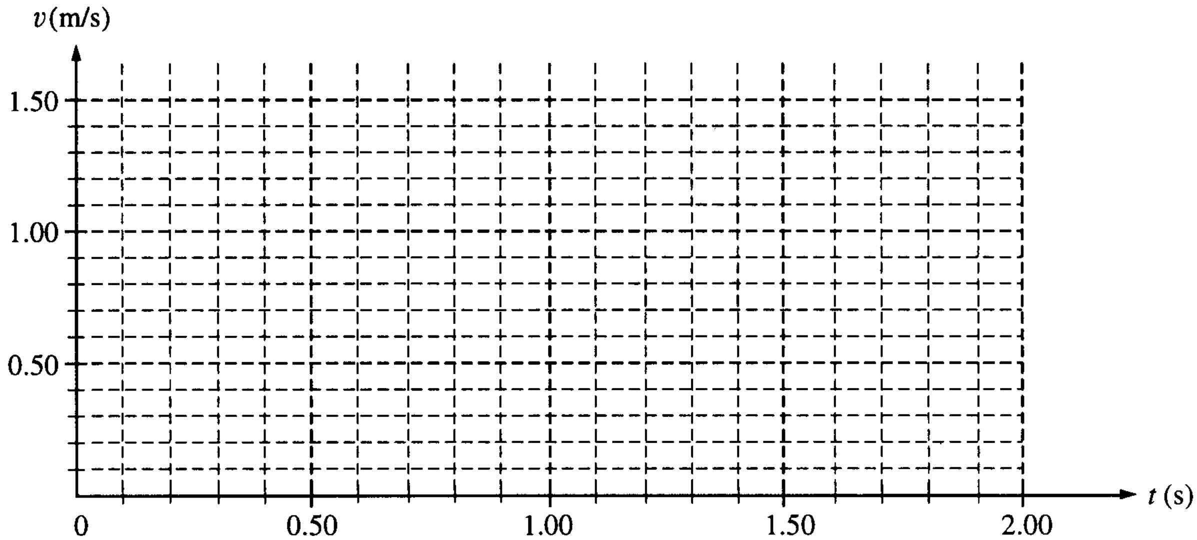
a. Determine the average speed of glider A for the following time intervals.

i. 0.0 s to 0.30 s ii. 0.90 s to 1.10 s iii. 1.70 s to 1.90 s

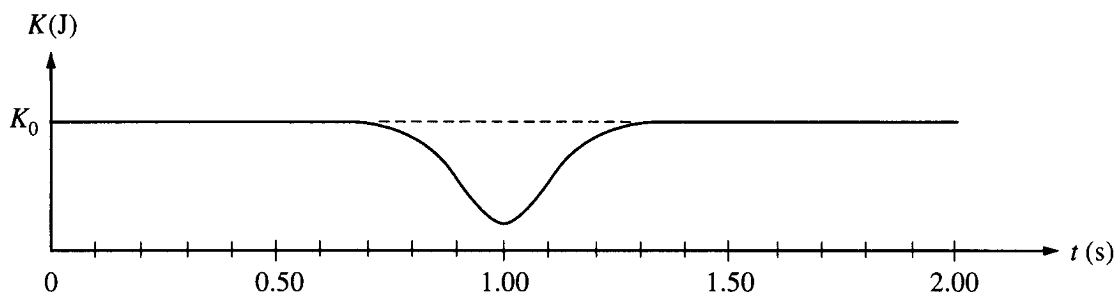
b. On the axes below, sketch a graph, consistent with the data above, of the speed of glider A as a function of time t for the 2.00 s interval.



c. i. Use the data to calculate the speed of glider B immediately after it separates from the spring.
ii. On the axes below, sketch a graph of the speed of glider B as a function of time t.



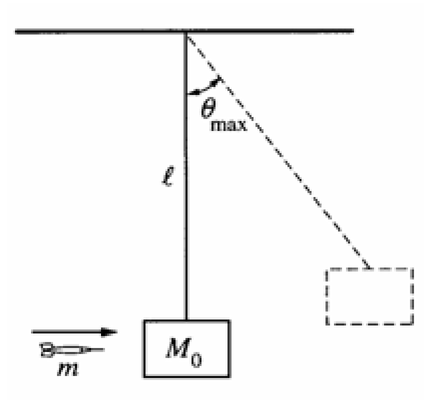
A graph of the total kinetic energy K for the two-glider system over the 2.00 s interval has the following shape. K_o is the total kinetic energy of the system at time t = 0.



d. i. Is the collision elastic? Justify your answer.

ii. Briefly explain why there is a minimum in the kinetic energy curve at t = 1.00 s.

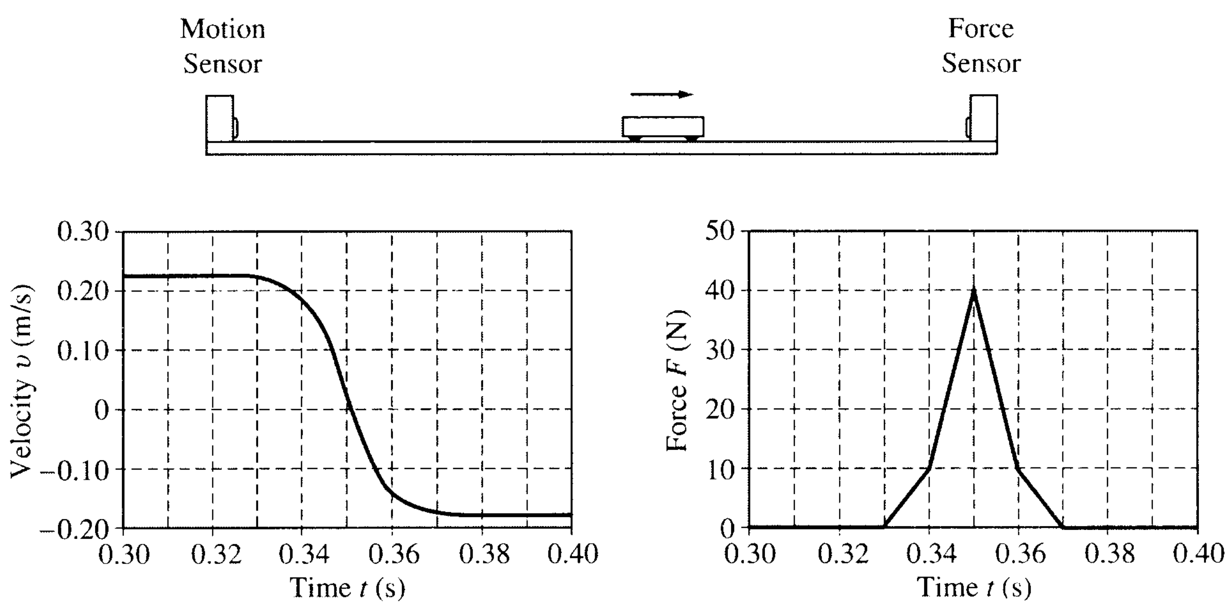
C1999M1. In a laboratory experiment, you wish to determine the initial speed of a dart just after it leaves a dart gun. The dart, of mass m, is fired with the gun very close to a wooden block of mass M_o which hangs from a cord of length *l* and negligible mass, as shown. Assume the size of the block is negligible compared to *l*, and the dart is moving horizontally when it hits the left side of the block at its center and becomes embedded in it. The block swings up to a maximum angle from the vertical. Express your answers to the following in terms of m, M_0, *l*, θ_{max}, and g.



a. Determine the speed v_o of the dart immediately before it strikes the block.
b. The dart and block subsequently swing as a pendulum. Determine the tension in the cord when it returns to the lowest point of the swing.
c. At your lab table you have only the following additional equipment.

Meter stick	Stopwatch	Set of known masses	Protractor
5 m of string	Five more blocks of mass M_0	Spring	

Without destroying or disassembling any of this equipment, design another practical method for determining the speed of the dart just after it leaves the gun. Indicate the measurements you would take, and how the speed could be determined from these measurements.



2001M1. A motion sensor and a force sensor record the motion of a cart along a track, as shown above. The cart is given a push so that it moves toward the force sensor and then collides with it. The two sensors record the values shown in the following graphs.

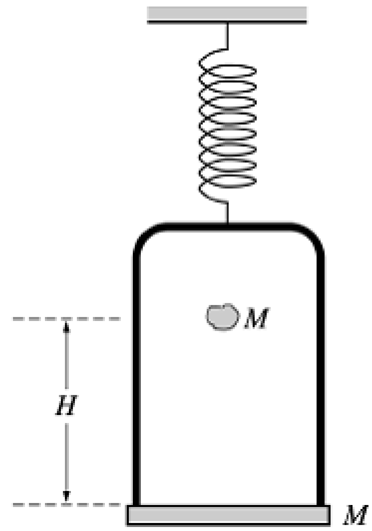
a. Determine the cart's average acceleration between t = 0.33 s and t = 0.37 s.
b. Determine the magnitude of the change in the cart's momentum during the collision.
c. Determine the mass of the cart.
d. Determine the energy lost in the collision between the force sensor and the cart

---

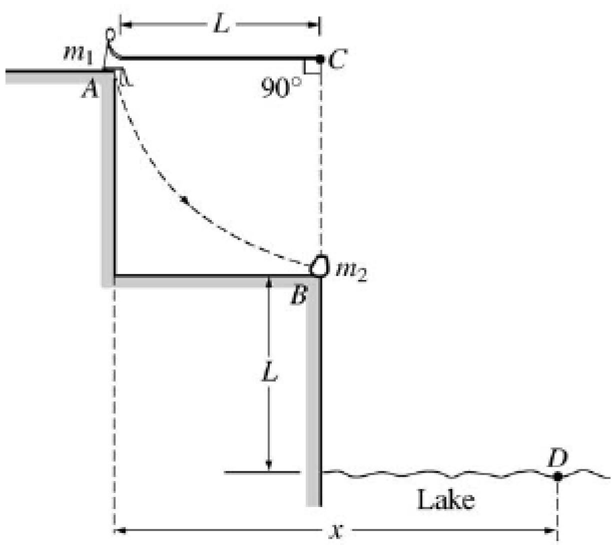
C2003M2.

An ideal massless spring is hung from the ceiling and a pan suspension of total mass M is suspended from the end of the spring. A piece of clay, also of mass M, is then dropped from a height H onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) Determine the speed of the clay at the instant it hits the pan.
(b) Determine the speed of the pan just after the clay strikes it.
(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.



C2004M1.



A rope of length L is attached to a support at point C. A person of mass m_1 sits on a ledge at position A holding the other end of the rope so that it is horizontal and taut, as shown. The person then drops off the ledge and swings down on the rope toward position B on a lower ledge where an object of mass m_2 is at rest. At position B the person grabs hold of the object and simultaneously lets go of the rope. The person and object then land together in the lake at point D, which is a vertical distance L below position B. Air resistance and the mass of the rope are negligible. Derive expressions for each of the following in terms of m_1, m_2, L, and g.

(a) The speed of the person just before the collision with the object
(b) The tension in the rope just before the collision with the object
(c) The speed of the person and object just after the collision
(d) The total horizontal displacement x of the person from position A until the person and object land in the water at point D.

ANSWERS - AP Physics Multiple Choice Practice – Momentum and Impulse

	Solution	Answer
1.	Based on $Ft = m\Delta v$, doubling the mass would require twice the time for same momentum change	D
2.	Two step problem. I) find velocity after collision with arrow. II) now use energy conservation. $K_i = U_{sp(f)}$ $m_a v_{ai} = (m_a+m_b) v_f$ $v_f = mv / (m+M)$ $\frac{1}{2}(m+M)v_f^2 = \frac{1}{2} k \Delta x^2$, sub in v_f from I	E
3.	Use $J=\Delta p$ $Ft=\Delta p$ $(100)t=200$ $t=2$	D
4.	Definition. Impulse, just like momentum, needs a direction and is a vector	C
5.	Since $p=mv$, by doubling v you also double p	D
6.	Since the momentum is the same, that means the quantity $m_1v_1 = m_2v_2$. This means that the mass and velocity change proportionally to each other so if you double m_1 you would have to double m_2 or v_2 on the other side as well to maintain the same momentum. Now we consider the energy formula $KE= \frac{1}{2} mv^2$ since the v is squared, it is the more important term to increase in order to make more energy. So if you double the mass of 1, then double the velocity of 2, you have the same momentum but the velocity of 2 when squared will make a greater energy, hence we want more velocity in object 2 to have more energy.	C
7.	Due to momentum conservation, the total before is zero therefore the total after must also be zero	E
8.	Definition. $J_{net} = \Delta p$	B
9.	Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(75)(6) + (100)(-8) = (175) v_f$	A
10.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(30)(4) = (40)v_f$	A
11.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(5000)(4) = (13000)v_f$	C
12.	Energy is conserved during fall and since the collision is elastic, energy is also conserved during the collision and always has the same total value throughout.	E
13.	To conserve momentum, the change in momentum of each mass must be the same so each must receive the same impulse. Since the spring is in contact with each mass for the same expansion time, the applied force must be the same to produce the same impulse.	C
14.	Momentum is equivalent to impulse which is Ft	A
15.	Use $J=\Delta p$ $J = mv_f - mv_i$ $J = (0.5)(-4) - (0.5)(6)$	C
16.	Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(2m)(v) = (5m) v_f$	B
17.	First of all, if the kinetic energies are the same, then when brought to rest, the non conservative work done on each would have to be the same based on work-energy principle. Also, since both have the same kinetic energies we have $\frac{1}{2} m_1v_1^2 = \frac{1}{2} m_2v_2^2$ … since the velocity is squared an increase in mass would need a proportionally smaller decrease in velocity to keep the terms the same and thus make the quantity mv be higher for the larger mass. This can be seen through example: If mass m_1 was double mass m_2 its velocity would be $v / \sqrt{2}$ times in comparison to mass m_2's velocity. So you get double the mass but less than half of the velocity which makes a larger mv term.	E

18. Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (3m)\ v_f$ A

19. Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(1200)(7) = (2800)v_f$ C

20. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (50)(v_{1f}) + (2)(10)$ B

21. Since p=mv and both p and v are vectors, they must share the same direction A

22. Explosion, momentum before is zero and after must also be zero. To have equal momentum the heavier student must have a much smaller velocity and since that smaller velocity is squared it has the effect of making the heavier object have less energy than the smaller one C

23. Use $J=\Delta p$ $Ft = mv_f - mv_i$ $Ft = m(v_f - v_i)$ … *note: since m is not given we will plug in F_g / g with g as 10 to be used in the impulse equation.* C
$(24000)(t) = (15000 / 10 m/s^2)(36-12)$

24. This is a rather involved question. First find speed of impact using free fall or energy. Define up as positive and Let v_1 = trampoline impact velocity and v_2 be trampoline rebound velocity. With that $v_1 = \sqrt{80}$ and $v_2 = -\sqrt{80}$. Now analyze the impact with the pad using $J_{net} = \Delta p$ $F_{net}\ t = mv_2 - mv_1$ At this point we realize we need the time in order to find the F_{net} and therefore cannot continue. If the time was given, you could find the F_{net} and then use $F_{net} = F_{pad} - mg$ to find F_{pad}. E

25. Based on momentum conservation both carts have the same magnitude of momentum "mv" but based on $K = ½\ m\ v^2$ the one with the larger mass would have a directly proportional smaller velocity that then gets squared. So by squaring the smaller velocity term it has the effect of making the bigger mass have less energy. This can be shown with an example of one object of mass m and speed v compared to a second object of mass 2m and speed v/2. The larger mass ends up with less energy even through the momenta are the same. B

26. Use $J=\Delta p$ $Ft = mv_f - mv_i$ $Ft = m(v_f - v_i)$ $F\ (0.03) = (0.125)(-6.5 - 4.5)$ D

27. Momentum conservation. $p_{before} = p_{after}$ $m_1v_{1i} = m_1v_{1f} + m_2v_{2f}$ $(0.1)(30) = (0.1)(20)+(m_a)(2)$ B

28. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $(2000)(10) + (3000)(-5) = (5000)\ v_f$ A

29. Kinetic energy has no direction and based on $K = ½\ m\ v^2$ must always be + C

30. A 2d collision must be looked at in both x-y directions always. Since the angle is the same and the v is the same, v_y is the same both before and after therefore there is no momentum change in the y direction. All of the momentum change comes from the x direction. E
$v_{ix} = v \cos\theta$ and $v_{fx} = -v\cos\theta$. $\Delta p = mv_{fx} - mv_{ix}$ … $-mv\cos\theta - mv\cos\theta$

31. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = (7)(v_{1f}) + (5)(0.2)$ B

32. In a circle at constant speed, work is zero since the force is parallel to the incremental distance moved during revolution. Angular momentum is given by mvr and since none of those quantities are changing it is constant. However the net force is NOT = MR, its Mv^2/R D

33. Since the momentum before is zero, the momentum after must also be zero. Each mass must have equal and opposite momentum to maintain zero total momentum. E

34. In a perfect inelastic collision with one of the objects at rest, the speed after will always be less no matter what the masses. The 'increase' of mass in 'mv' is offset by a decrease in velocity D

35. Since the total momentum before and after is zero, momentum conservation is not violated, however the objects gain energy in the collision which is not possible unless there was some energy input which could come in the form of inputting stored potential energy in some way. B

36. The plastic ball is clearly lighter so anything involving mass is out, this leaves speed which makes sense based on free-fall B

37. Perfect inelastic collision. $m_1v_{1i} = m_{tot}(v_f)$ … $(m)(v) = (m+M)\, v_f$ E

38. As the cart moves forward it gains mass due to the rain but in the x direction the rain does not provide any impulse to speed up the car so its speed must decrease to conserve momentum C

39. Angular momentum is given by the formula $L = mvr = (2)(3)(4)$ E

40. 2D collision. Analyze the y direction. Before $p_y = 0$ so after p_y must equal 0. A
$0 = m_1v_{1fy} + m_2v_{2fy}$ $\quad 0 = (0.2)(1) + (0.1)(V_{2fy})$

41. Momentum increases if velocity increases. In a d-t graph, III shows increasing slope (velocity) B

42. The net force is zero if velocity (slope) does not change, this is graphs I and II C

43. Since the impulse force is applied in the same direction (60°) as the velocity, we do not need to use components but use the 60° inclined axis for the impulse momentum problem. In that direction. $J = \Delta p$ $\quad J = mv_f - mv_i = m(v_f - v_i) = (0.4)(0 - 5)$ C

44. Initially, before the push, the two people are at rest and the total momentum is zero. After, the total momentum must also be zero so each man must have equal and opposite momenta. C

45. Since the initial object was stationary and the total momentum was zero it must also have zero total momentum after. To cancel the momentum shown of the other two pieces, the 3m piece would need an x component of momentum $p_x = mV$ and a y component of momentum $p_y = mV$ giving it a total momentum of $\sqrt{2}\, mV$ using Pythagorean theorem. Then set this total momentum equal to the mass * velocity of the 3rd particle. D
$\sqrt{2}\, mV = (3m)\, V_{m3}$ and solve for V_{m3}

46. None of the statements are true. I) it is accelerating so is not in equilibrium, II) Its acceleration is –9.8 at all times, III) Its momentum is zero because its velocity is momentarily zero, IV) Its kinetic energy is also zero since its velocity is momentarily zero. E

47. Its does not matter what order to masses are dropped in. Adding mass reduces momentum proportionally. All that matters is the total mass that was added. This can be provided by finding the velocity after the first drop, then continuing to find the velocity after the second drop. Then repeating the problem in reverse to find the final velocity which will come out the same C

48. Stupid easy. Find slope of line A

49. Increase in momentum is momentum change which C

50. Basic principle of impulse. Increased time lessens the force of impact. E

51. Explosion. $p_{before} = 0 = p_{after}$ … $0 = m_1v_{1f} + m_2v_{2f}$ … $0 = m_1(5) + m_2(-2)$ B

52. Perfect inelastic collision. $m_1v_{1i} + m_2v_{2i} = m_{tot}(v_f)$ … $Mv + (-2Mv) = (3M)\, v_f$ gives $v_f = v/3$. Then to find the energy loss subtract the total energy before – the total energy after D
$[\, \tfrac{1}{2} Mv^2 + \tfrac{1}{2} (2M)v^2 \,] - \tfrac{1}{2} (3M)\, (v/3)^2 \;=\; 3/6\, Mv^2 + 6/6\, Mv^2 - 1/6\, Mv^2$

53. 2D collision. The y momentums are equal and opposite and will cancel out leaving only the x momentums which are also equal and will add together to give a total momentum equal to twice the x component momentum before hand. $p_{before} = p_{after}$ $2m_o v_o \cos 60 = (2m_o)\, v_f$ — B

54. Angular momentum is given by L = mvr = mva — B

55. Perfect inelastic collision. $m_1 v_{1i} = m_{tot}(v_f)$ … $(4)(6) = (8)v_f$ — C

56. Perfect inelastic collision. $m_1 v_{1i} = m_{tot}(v_f)$ … $(8)(3) = (12)v_f$ — B

57. First use the given kinetic energy of mass M1 to determine the projectile speed after. — D
$K= ½M_1 v_{1f}^2$ … $v_{1f} = \sqrt{(2K/M_1)}$. Now solve the explosion problem with $p_{before}=0 = p_{after}$.
Note that the mass of the gun is $M_2–M_1$ since M_2 was given as the total mass.
$0 = M_1 v_{1f} + (M_2–M_1)v_{2f}$ … now sub in from above for v_{1f} .
$M_1(\sqrt{(2K/M_1)}) = –(M_2–M_1)\, v_{2f}$ and find v_{2f} … $v_{2f} = –M_1(\sqrt{(2K/M_1)}) / (M_2–M_1)$.
Now sub this into $K_2 = ½ (M_2–M_1)\, v_{2f}^2$ and simplify

58. Since there is no y momentum before, there cannot be any net y momentum after. The balls have equal masses so you need the y velocities of each ball to be equal after to cancel out the momenta. By inspection, looking at the given velocities and angles and without doing any math, the only one that could possibly make equal y velocities is choice D — D

59. Definition. $J_{net} = \Delta p$ $F_{net}\, t = \Delta p$ — A

60. Explosion with initial momentum. $p_{before} = p_{after}$ $mv_o = m_a v_{af} + m_b v_{bf}$ — E
$mv_o = (2/5\ m)(–v_o / 2) + (3/5\ m)(v_{bf})$ … solve for v_{bf}

61. The area of the Ft graph is the impulse which determines the momentum change. Since the net impulse is zero, there will be zero total momentum change. — C

62. Perfect inelastic collision. $m_1 v_{1i} + m_2 v_{2i} = m_{tot}(v_f)$ … $(m)(v) + (2m)(v / 2) = (3m)v_f$ — C

63. The total momentum vector before must match the total momentum vector after. Only choice E has a possibility of a resultant that matches the initial vector. — E

64. Since the angle and speed are the same, the x component velocity has been unchanged which means there could not have been any x direction momentum change. The y direction velocity was reversed so there must have been an upwards y impulse to change and reverse the velocity. — E

65. Simply add the energies $½ (1.5)(2)^2 + ½ (4)(1)^2$ — B

66. Total momentum before must equal total momentum after. Before, there is an x momentum of (2)(1.5)=3 and a y momentum of (4)(1)=4 giving a total resultant momentum before using the Pythagorean theorem of 5. The total after must also be 5. — C

67. Just as linear momentum must be conserved, angular momentum must similarly be conserved. Angular momentum is given by L = mvr, so to conserve angular momentum, these terms must all change proportionally. In this example, as the radius decreases the velocity increases to conserve momentum. — A

68. To find the breaking force, use impulse-momentum. $J = \Delta p$ $Ft = mv_f - mv_i$ D

$F(5) = 0 - (900)(20)$ $F = -3600$ N

The average velocity of the car while stopping is found with $\bar{v} = \frac{v_i + v_f}{2}$ = 10 m/s

Then find the power of that force $P = F\bar{v}$ = (3600)(10) = 36000 W

69. Each child does work by pushing to produce the resulting energy. This kinetic energy is input through the stored energy in their muscles. To transfer this energy to each child, work is done. The amount of work done to transfer the energy must be equal to the amount of kinetic energy gained. Before hand, there was zero energy so if we find the total kinetic energy of the two students, that will give us the total work done. First, we need to find the speed of the boy using momentum conservation, explosion: E

$p_{before} = 0 = p_{after}$ $0 = m_b v_b + m_g v_g$ $0 = (m)(v_b) = (2m)(v_g)$ so $v_b = 2v$

Now we find the total energy $K_{tot} = K_b + K_g = ½\, m(2v)^2 + ½\, 2m(v)^2 = 2mv^2 + mv^2 = 3mv^2$

70. Since it is an elastic collision, the energy after must equal the energy before, and in all collisions momentum before equals momentum after. So if we simply find both the energy before and the momentum before, these have the same values after as well. $p = Mv$, $K = ½\, Mv^2$ A

71. The area under the F-t graph will give the impulse which is equal to the momentum change. With the momentum change we can find the velocity change. C

J = area = 6 Then $J = \Delta p = m\Delta v$ $6 = (2)\Delta v$ $\Delta v = 3$ m/s

72. This is a 2D collision. Before the collision, there is no y momentum, so in the after condition the y momenta of each disk must cancel out. In choice B, both particles would have Y momentum downwards making a net Y momentum after which is impossible. B

73. This is the same as question 30 except oriented vertically instead of horizontally. E

1976B2.
a) Apply momentum conservation. $p_{before} = p_{after}$ $mv_o = (m)(v_o/3) + (4m)(v_{f2})$ $v_{f2} = v_o / 6$

b) $KE_f – KE_i = ½ mv_o^2 – ½ m (v_o / 3)^2 = 4/9\ mv_o^2$

c) $KE = ½ (4m)(v_o / 6)^2 = 1/18\ mv_o^2$

1978B1.

a) Projectile methods. Find t in y direction. $d_y = v_{iy}t + ½ a t^2$ $t = \sqrt{\frac{2H}{g}}$

D is found with $v_x = d_x / t$ $D = v_ot$ $v_o\sqrt{\frac{2H}{g}}$

b) Apply momentum conservation in the x direction. $p_{before(x)} = p_{after(x)}$ $M_1v_o = (M_1+M_2)v_f$ $v_f = M_1v_0 / (M_1+M_2)$

1981B2.

a) The work to compress the spring would be equal to the amount of spring energy it possessed after compression. After releasing the mass, energy is conserved and the spring energy totally becomes kinetic energy so the kinetic energy of the mass when leaving the spring equals the amount of work done to compress the spring $W = ½ m v^2 = ½ (3) (10)^2 = 150$ J

b) Apply momentum conservation to the explosion
$p_{before} = 0 = p_{after}$ $0 = m_1v_{1f} + m_2v_{2f}$ $0 = (1)v_{1f} + (3)v_{2f}$ $v_{1f} = 3 v_{2f}$

Apply energy conservation … all of the spring energy is converted into the kinetic energy of the masses
$150\text{ J} = K_1 + K_2$ $150 = ½ m v_{1f}^2 + ½ m v_{2f}^2$ sub in above for v_{2f}
$150 = ½ (1)(3v_{2f})^2 + ½ (3)(v_{2f})^2$
$v_{2f} = 5$ m/s $v_{1f} = 15$ m/s

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $½ (3M)(2/3\ v_o)^2 = ½ k \Delta x^2$ $\sqrt{\frac{4Mv_o^2}{3k}}$

1984B2.

a) Before the collision there is only an x direction momentum of mass M_1 … $p_x = m_1v_{1x} = 16$, all the rest are 0
After the collision, M_1 has y direction momentum = $m_1v_{1fy} = 12$ and M_2 has x and y direction momentums.
Using trig to find the x and y velocities of mass M_2 … $v_x = 5 \cos 37 = 3$, and $v_y = 5 \sin 37 = 3.75$.
Then plug into mv to get each x and y momentum after.

	M_1 = 1 kg		M_2 = 4 kg	
	p_x (kg m /s)	p_y (kg m /s)	p_x (kg m /s)	p_y (kg m /s)
Before	16	0	0	0
After	0	-12	16	12

b) SUM = 16 -12 16 12
When adding x's before they = x's after 16=16, when adding y's before they equal y's after |-12|=12

c) Kinetic Energy Before
$K = ½ m_1v_{1ix}^2$
$K = ½ (1)(16)^2 = 128$ J

Kinetic Energy After
$K = ½ m_1v_{1fy}^2 + ½ m_2v_2^2$
$K = ½ (1)(12)^2 + ½ (4)(5)^2 = 122$ J

d) From above, K is not conserved.

1985B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_{1i} = (m+M)v_f$ $v_f = 1.5$ m/s

b) KE_i / KE_f $½ m v_{1i}^2 / ½ (m+M)vf^2 = 667$

c) Apply conservation of energy of combined masses K = U $½ (m+M)v^2 = (m+M)gh$ h = 0.11 m

---

1990B1.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $m_1v_o = (101m)v_f$ $v_f = v_o / 101$

b) $\Delta K = K_f – K_i = ½ (101m)v_f^2 – ½ mv_o^2 = ½ (101m)(v_o/101)^2 – ½ mv_o^2 = – (50/101) mv_o^2$

c) Using projectile methods. Find t in y direction. $d_y = v_{iy}t + ½ a t^2$ $t = \sqrt{\frac{2h}{g}}$

D is found with $v_x = d_x / t$ $D = v_xt$ $\frac{v_0}{101}\sqrt{\frac{2h}{g}}$

d) The velocity of the block would be different but the change in the x velocity has no impact on the time in the y direction due to independence of motion. v_{iy} is still zero so t is unchanged.

e) In the initial problem, all of the bullets momentum was transferred to the block. In the new scenario, there is less momentum transferred to the block so the block will be going slower. Based on $D = v_xt$ with the same time as before but smaller velocity the distance x will be smaller.

1992 B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$
$m_1 v_{1i} = (m_1+m_2)v_f$ $(30)(4) = (80)v_f$ $v_f = 1.5$ m/s

b) $K = ½ (m_1+m_2) v_f^2 = ½ (80)(1.5)^2 = 90$ J

c) Apply momentum conservation explosion.
$p_{before} = p_{after}$ $(m_1+m_2)v = m_1 v_{1f} + m_2 v_{2f}$ $(80)(1.5) = 0 + (50)v_{2f}$ $v_{2f} = 2.4$ m/s

d) $K = ½ m_2 v_{2f}^2 = ½ (50)(2.4)^2 = 144$ J

e) By inspection the energy in d is greater. The energy increased due to an energy input from the work of the child's muscles in pushing on the sled.

1994 B2.

a) Apply energy conservation top to bottom. $U = K$ $mgh = ½ mv^2$ $(gR) = ½ v^2$ $v = \sqrt{2gR}$

b) Apply momentum conservation

$p_{before} = p_{after}$ $m_a v_{ai} = (m_a+m_b)v_f$ $M(\sqrt{2gR}) = 2Mv_f$ $v_f = \dfrac{\sqrt{2gR}}{2}$

c) The loss of the kinetic energy is equal to the amount of internal energy transferred

$\Delta K = K_f - K_i = ½\, 2M \left(\dfrac{\sqrt{2gR}}{2}\right)^2 - ½\, M\left(\sqrt{2gR}\right)^2$ $= -MgR / 2$ lost ➔ $MgR / 2$ internal energy gain.

d) Find the remaining kinetic energy loss using work-energy theorem which will be equal the internal energy gain.
$W_{nc} = \Delta K$ $-f_k d = -\mu F_n d = -\mu(2m)gL = -2\mu MgL$, kinetic loss = internal E gain ➔ $2\mu MgL$

1995 B1.

a) i) $p = mv$ $= (0.2)(3) = 0.6$ kg m/s
ii) $K = ½ mv^2$ $= ½ (0.2)(3)^2 = 0.9$ J

b) i.) Apply momentum conservation $p_{before} = p_{after} = 0.6$ kg m/s
ii) First find the velocity after using the momentum above
$0.6 = (1.3+0.2)\, v_f$ $v_f = 0.4$ m/s $K = ½ (m_1+m_2) v_f^2 = ½ (1.3+0.2)(0.4)^2 = 0.12$ J

c) Apply energy conservation $K = U_{sp}$ $0.12 \text{ J} = ½ k\Delta x^2 = ½ (100)\, \Delta x^2$ $\Delta x = 0.05$ m

---

1996 B1.

a) p_{tot} = M(3v_o) + (M)(v_o) = 4mv_o

b) i) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$
4Mv_o = (m_1+m_2)v_f 4Mv_o = (2M) v_f v_f = 2v_o
ii) Since they are both moving right they would have to be moving right after

c) i) Apply momentum conservation $p_{before} = p_{after}$
4Mv_o = m_1v_{1f} + m_2v_{2i} 4Mv_o = Mv_{af} + M(2.5v_o) v_{af} = 1.5 v_o
ii) As before, the would have to be moving right.

d) $\Delta K = K_f - K_i = (½\, m_a v_{af}^2 + ½\, m_b v_{bf}^2) - (½\, m_a v_{ai}^2 + ½\, m_b v_{bi}^2) = 4.25\, Mv_o^2 - 5\, Mv_o^2 = -0.75\, Mv_o^2$

1997 B1.

a) The force is constant, so simple F_{net} = ma is sufficient. (4) = (0.2) a a = 20 m/s^2

b) Use $d = v_i t + ½\, a t^2$ 12 = (0) + ½ (20) t^2 t = 1.1 sec

c) W = Fd W = (4 N) (12 m) = 48 J

d) Using work energy theorem W = ΔK (K_i = 0) W = Kf – Ki
W = ½ m v_f^2
Alternatively, use $v_f^2 = v_i^2 + 2ad$ 48J = ½ (0.2) (v_f^2) v_f = 21.9 m/s

e) The area under the triangle will give the extra work for the last 8 m
½ (8)(4) = 16J + work for first 12 m (48J) = total work done over 20 m = 64 J

Again using work energy theorem W = ½ m v_f^2 64 J = ½ (0.2) v_f^2 v_f = 25.3 m/s

Note: if using F = ma and kinematics equations, the acceleration in the last 8 m would need to be found using the average force over that interval.

f) The momentum change can simply be found with $\Delta p = m\Delta v = m(v_f - v_i)$ = 0.2 (25.3 – 21.9) = 0.68 kg m/s

---

2001B2.

a) Apply momentum conservation $p_{before} = p_{after}$
$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ (0.1)(1.4) = (0.1)(–0.7) + (0.5)v_{bf} v_{bf} = 0.42 m/s

b) Using projectile methods. Find t in y direction. $d_y = v_{iy}t + ½\, a t^2$ – 1.2m = 0 + ½ (–9.8) t^2 t = 0.49
D is found with $v_x = d_x / t$ D = v_xt (0.42)(0.49) D = 0.2 m

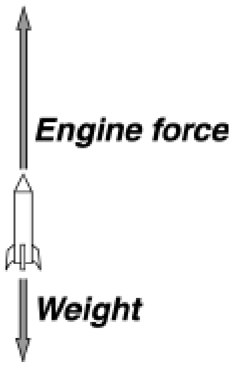
c) The time of fall is the same as before since it's the same vertical distance. t = 0.49 s
The velocity of ball C leaving the table can be found using projectile methods. v_x = d / t = 0.15/0.49 = 0.31 m/s

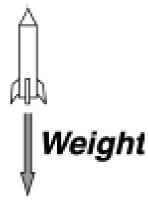
d) Looking that the y direction. $p_{y(before)} = p_{y(after)}$
0 = $p_{ay} - p_{cy}$ 0 = $p_{ay} - m_c v_{cy}$ 0 = p_{ay} – (0.1)(0.31)sin30 p_{ay} = 0.015 kg m/s

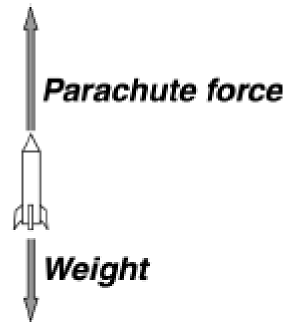
---

2002B1.

a)

i. 

ii. 

iii. 

b) $J_{engine} = F_{eng}\, t$ $(20) = F_{eng}\,(2)$ $F_{eng} = 10$ N

$F_{net} = ma$ $(F_{eng} – mg) = ma$ $(10 – 0.25(9.8)) = (0.25)a$ $a = 30$ m/s^2

c) Find distance traveled in part (i) $d_1 = v_i t + ½\, a\, t^2 = 0 + ½\,(30)(2)^2 = 60$ m
Find distance in part (ii) free fall.
first find velocity at end of part (i) = v_i for part ii then find distance traveled in part ii
$v_{1f} = v_{1i} + a_1 t_1 = (0) + (30)(2) = 60$ m/s $v_{2f}^2 = v_{2i}^2 + 2gd_2 = (60)^2 + 2(–9.8)(d_2)$
$d_2 = 184$ m

$d_{total} = 244$ m

d) Find time in part ii. $v_{2f} = v_{2i} + gt$ $0 = 60 + –9.8\, t$ $t = 6.1$ s
then add it to the part I time (2 s) total time ➔ 8.1 sec

2002B1B

a) The graph of force vs time uses area to represent the Impulse and the impulse equals change in momentum.
Area = 2 x ½ bh = (0.5 ms)(10kN). Milli and kilo cancel each other out. Area = 5 Ns = J

VERY IMPORTANT – Based on the problem, the force given and therefore impulse is actually negative because the graph is for the 2 kg cart and clearly the force would act opposite the motion of the cart.

$J = \Delta p = mv_f – mv_i$ $(–5) = (2)(v_f) – 2(3)$ $v_f = 0.5$ m/s (for the 2 kg cart)

b) Apply momentum conservation $p_{before} = p_{after}$
$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(2)(3) = (2)(0.5) + (m_b)(1.6)$ $m_b = 3.125$ kg

c) slope = acceleration = $\Delta y / \Delta x = (0.5 – 1.6) / (3.5 – 3) = – 0.73$ m/s^2

d) distance = area under line, using four shapes.
0-2 rectangle, 2-3.5 triangle top + rectangle bottom, 3.5-5, rectangle ➔ 5.5 m

e) Since the acceleration is negative the cart is slowing so it must be going up the ramp. Use energy conservation to find the max height. $K_{bot} = U_{top}$ $½\, mv^2 = mgh$ $½\,(1.6)^2 = (9.8)\, h$ $h = 0.13$ m

2006B2B.

a) Apply energy conservation

$U_{top} = K_{bottom}$

$mgh = ½ m v^2$ $Mgh = ½ (M) (3.5v_o)^2$ $h = 6.125 v_o^2 / g$

b) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(M)(3.5v_o) = (M)v_{af} + (1.5M)(2v_o)$ $v_{af} = ½ v_o$

c) $W_{NC} = \Delta K$ $(K_f - K_i)$ $K_f = 0$

$- f_k d = 0 - ½ (1.5M)(2v_o)^2$

$\mu_k (1.5 M) g (d) = 3Mv_o^2$ $\mu_k = 2v_o^2 / gD$

d) Compare the kinetic energies before and after

Before	After	
$K = ½ M (3.5v_o)^2$	$½ M(½ v_o)^2 + ½ (1.5M)(2v_o)^2$	there are not equal so its inelastic

---

2008B1B.

a) Apply momentum conservation to the explosion $p_{before} = 0 = p_{after}$

$0 = m_a v_{af} + m_b v_{bf}$ $0 = (70)(- 0.55) + (35)(v_{bf})$ $v_{bf} = 1.1$ m/s

b) $J_{son} = \Delta p_{son}$ $F_{on\text{-}son} t = m(v_f - v_i)$ $F (0.6) = (35)(0 - 1.1) =$ $F = - 64$ N

c) Based on newtons third law action/reaction, the force on the son must be the same but in the opposite direction as the force on the mother.

d) On the son $W_{fk} = \Delta K$ $- f_k d = ½ mv_f^2 - ½ mv_i^2$ $- \mu mg\, d = ½ m (0 - v_i^2)$ $d = v_i^2 / (2\mu)$

This would be the same formula for the mother's motion with a different initial velocity. Since the mass cancels out we see the distance traveled is proportional to the velocity squared. The boy moves at twice the speed of the mother, so based on this relationship should travel 4 x the distance. The mother traveled 7 m so the son would have a sliding distance of 28 m.
(Alternatively, you could plug in the numbers for the mother to solve for μ and then plug in again using the same value of μ and the sons velocity to find the distance. μ is the same for both people.)

2008B1.

a) First determine the time to travel while the car accelerates. $v_{1f} = v_{1i} + a_1 t_1$ $(5) = (2) + (1.5)\, t_1$ $t_1 = 2$ sec
Also determine the distance traveled while accelerating $d_1 = v_{1i} t_1 + ½ a_1 t_1^2$ $d_1 = (2)(2) + ½ (1.5)(2)^2 = 7$ m

This leaves 8 m left for the constant speed portion of the trip.
The velocity at the end of the 7m is the average constant velocity for the second part of the trip

$v_2 = d_2 / t_2$ $5 = 8 / t_2$ $t_2 = 1.6$ sec ➔ total time $= t_1 + t_2 =$ 3.6 seconds

b) i) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(250)(5) = (250)v_{af} + (200)(4.8)$ $v_{af} = 1.2$ m/s

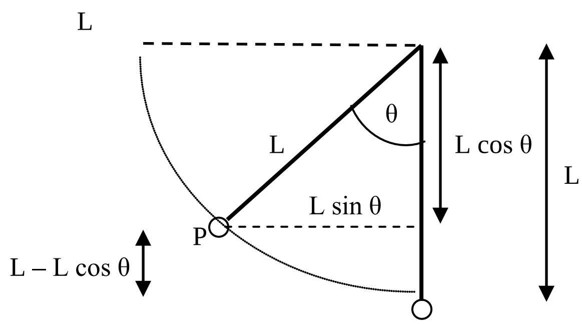
ii) Since the velocity is + the car is moving right

c) Check kinetic energy before vs after

$K_i = ½ (250)(5)^2 = 3125$ J $K_f = ½ (250) (1.2)^2 + ½ (200)(4.8)^2 = 2484$ J

Since the energies are not the same, it is inelastic

C1981M2.



a) $U_{top} = K_{bot}$
$mgh = \frac{1}{2} mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$
$$v = \sqrt{2g(L - \frac{L}{2})}$$
$$v = \sqrt{gL}$$

b) Use the max rise height on the opposite site to find the seats speed
$K_{bot} = U_{top}$ $\frac{1}{2} mv^2 = mgh$

$$v = \sqrt{2g(L - L\cos 45)}$$
$$v = \sqrt{2g(L - \frac{\sqrt{2}L}{2})}$$
$$v = \sqrt{2gL\,(1 - \frac{\sqrt{2}}{2})} \qquad v = \sqrt{gL\,(2 - \sqrt{2})}$$

c) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $$(2m)\sqrt{gL} = m\,v_{af} + m(\sqrt{gL\,(2-\sqrt{2})}) \qquad = \sqrt{gL}\left(2 - \sqrt{(2-\sqrt{2})}\right)$$

C1991M1.

(a) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$mv_o = (m+2m)\, v_f$ $v_f = v_o / 3$

(b) Apply energy conservation.
$K_{bottom} = U_p + K_p$ $\frac{1}{2} mv_{bot}^2 = mgh_p + K_p$
$\frac{1}{2} 3m\,(v_o/3)^2 = 3mg(r) + K_p$ $K_p = mv_o^2/6 - 3mgr$

(c) The minimum speed to stay in contact is the limit point at the top where F_n just becomes zero. So set F_n=0 at the top of the loop so that only mg is acting down on the block. The apply $F_{net(C)}$

$F_{net(C)} = mv^2 / r$ $3mg = 3m\,v^2 / r$ $v = \sqrt{rg}$

(d) Energy conservation top of loop to bottom of loop
$U_{top} + K_{top} = K_{bot}$
$mgh + \frac{1}{2} m\,v_{top}^2 = \frac{1}{2} m\,v_{bot}^2$ $g(2r) + \frac{1}{2}(\sqrt{rg})^2 = \frac{1}{2}(vo')^2$ $$v_o' = \sqrt{5gr}$$

(e) Apply momentum conservation, perfect inelastic with v_f as the speed found above and v_i unknown

$p_{before} = p_{after}$ $mv_b' = (m+2m)\, v_f$ $v_b' = 3v_f =$ $$v_o' = 3\sqrt{5gr}$$

C1992M1.

a) $U_{top} = K_{bot}$ $mgh = ½ mv^2$ $(10)(5) = ½ v^2$ $v = 10$ m/s

b) Since the ball hits the ground elastically, it would rebound with a speed equal to that it hit with 10 m/s

c) Free fall of clay $d = v_i t + ½ gt^2 = 0 + ½ (–10)(0.5)^2$
$d = –1.25$ m displaced down, so height from ground would be 3.75 m

d) Clay free fall (down) $v_f = v_i + gt = 0 +(–10)(0.5) = –5$ m/s speed = 5 m/s
Ball free fall (up) $v_f = v_i + gt = 10 + (–10)(0.5) = 5$ m/s speed = 5 m/s

e) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai} + m_b v_{bi} = (m_a+m_b) v_f$ $(9m)(5) + (m)(–5) = (10m) v_f$ $v_f = 4$ m/s, up (since +)

C1993M1. - since there is friction on the surface the whole time, energy conservation cannot be used

a) $U_{sp} = ½ k \Delta x^2 = ½ (400)(0.5)^2 = 50$ J

b) Using work-energy $W_{nc} = \Delta U_{sp} + \Delta K = (U_{sp(f)}–U_{sp(i)}) + (K_f – K_i)$
$– f_k d = (0 – 50J) + (½ m v_f^2 – 0)$
$– \mu mg d = ½ mv_f^2 – 50$
$– (0.4)(4)(9.8)(0.5) = ½ (4)(v_c^2) – 50$ $v_c = 4.59$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ci}= (m_c+m_d) v_f$ $(4)(4.59) = (4+2) v_f$ $v_f = 3.06$ m/s

d) $W_{nc} = (K_f – K_i)$
$– f_k d = (0 – ½ m v_i^2)$ $– \mu mg d = – ½ m v_i^2$ $(0.4)(6)(9.8) d = ½ (6)(3.06)^2$ $d = 1.19$ m

---

C1994M1.

a) $U_{sp} = ½ k \Delta x^2 = ½ (200)(0.4)^2 = 16$ J

b) Apply energy conservation $K_{before\ compression} = U_{sp-after\ compression}$
$½ (m_a+m_b) v^2 = U_{sp}$ $½ (0.1+2) v^2 = 16$ $v = 3.9$ m/s

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$
$m_a v_{ai}= (m_a+m_b) v_f$ $(0.1) v_{ai} = (0.1+2) (3.9)$ $v_{ai} = 81.9$ m/s

---

C1995M1.

a) In the F vs t curve the impulse is the area under the curve. Area of triangle + rectangle + triangle = 12 Ns

b) $J_{on\text{-}ball} = \Delta p_{ball}$ $J = m(v_{bf} - v_{bi})$ $12 = 5(v_{bf} - 0)$ $v_{bf} = 2.4$ m/s

c) i) Due to action reaction, the force on the cube is the same as that on the ball but in the opposite direction so the impulse applied to it is –12 Ns. $J_{on\text{-}cube} = \Delta p_{cube}$ $J = m(v_{cf} - v_{ci})$ $-12 = 0.5(v_{cf} - 26)$ $v_{cf} = 2$ m/s

ii) since +, moving right

d) $\frac{1}{2} mv_{cf}^2 + \frac{1}{2} mv_{bf}^2 - \frac{1}{2} mv_{ci}^2 = 154$ J

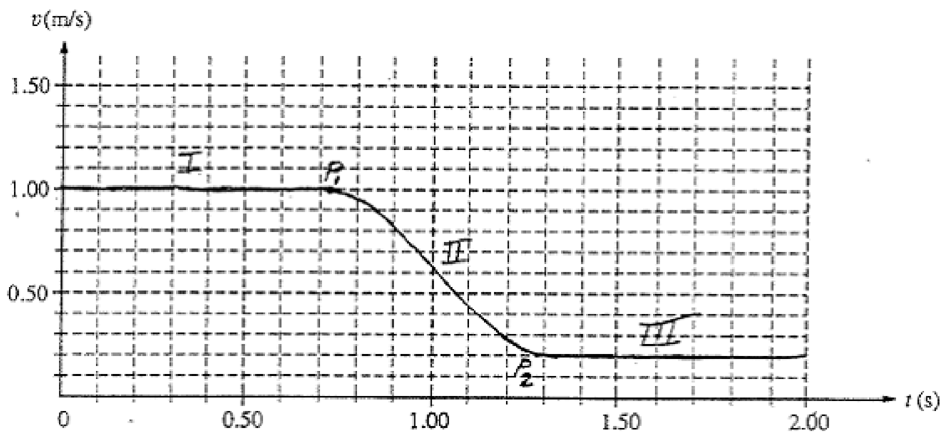
e) Using projectiles … both take same time to fall since $v_{iy} = 0$ for both and distance of fall same for both
$d_y = v_{iy}t + \frac{1}{2} g\, t_2$ $-1.2 = 0 + \frac{1}{2}(-9.8)\, t^2$ $t = 0.5$ sec

Each d_x is found using $d_x = v_x t$ for each respetive speed of cube and ball.
Gives d_x(cube) = 1m d_x(ball) = 1.2 m so they are spaced by 0.2 m when they hit.

C1998M1.

a) use v = d / t for each interval   i) 1 m/s ii) 0.6 m/s iii) 0.2 m/s

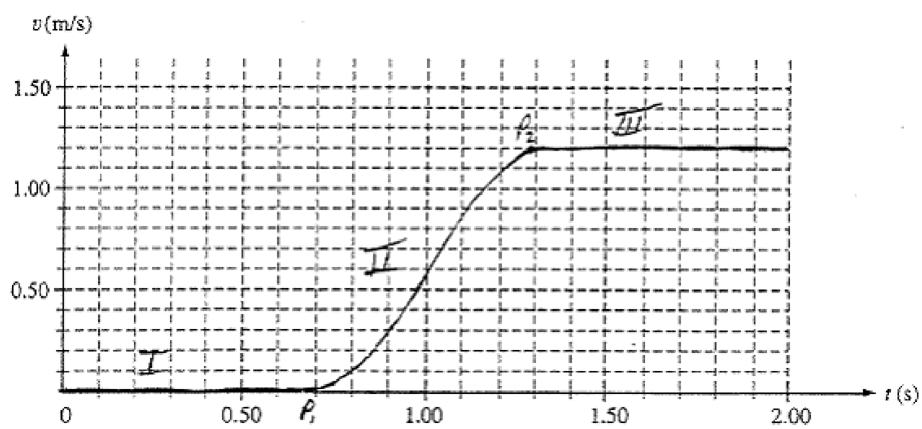
b) Based on the pattern of the **Λ** shapes of the ruler we can see the glider moves at a constant speed up until 0.70 s where the spacings start to change and it decelerates up until around the 1.3 second time where the speed becomes constant again. So the first constant speed is the initial velocity of the glider (1 m/s) and the second constant speed is the final velocity of the glider after the collision (0.2 m/s)



c)) i) Apply momentum conservation $p_{before} = p_{after}$

$m_a v_{ai} = m_a v_{af} + m_b v_{bf}$ $(0.9)(1) = (0.9)(0.2) + (0.6)(v_{bf})$ $v_{bf} = 1.2$ m/s

ii) Glider B is at rest up until 0.7 seconds where the collision accelerates to a final constant speed of 1.2 m/s



d) i) The collision is elastic because the kinetic energy before and after is the same

ii) The kinetic energy becomes a minimum because the energy is momentarily transferred to the spring

C1991M1. - The geometry of this problem is similar to C1981M2 in this document.

a) First determine the speed of the combined dart and block using energy conservation.

$K_{bot} = U_{top}$

$\frac{1}{2} mv^2 = mgh$ Then apply momentum conservation bullet to block collision

$$v = \sqrt{2g(L - L\cos\theta)}$$

$$v = \sqrt{2gL(1-\cos\theta)}$$

perfect inelastic …$p_{before} = p_{after}$

$mv_o = (m+M_o)\ v$ $$v_0 = \frac{(m+M_o)}{m}\sqrt{2gL(1-\cos\theta)}$$

b) Apply $F_{net(c)} = mv^2 / r$, at the lowest point (tension acts upwards weight acts down)

$F_t - mg = mv^2/r$ $F_t = m(g+v^2/r)$ substitute v from above

$F_t = (m+M_o)(g + 2gL(1 - \cos\theta)/L) = (m+M_o)(g + 2g - 2g\cos\theta) = (m+M_o)\, g\, (3 - 2\cos\theta)$

c) One way would be to hang the spring vertically, attach the five known masses, measure the spring stretch, and use these results to find the spring constant based on F=kΔx. Then attach the block to the spring and measure the spring stretch again. Fire the dart vertically at the block and measure the maximum distance traveled. Similar to the problem above, use energy conservation to find the initial speed of the block+dart then use momentum conservation in the collision to find the darts initial speed.

C2001M1.

a) Pick velocity from the graph and use $a = (v_f - v_i)/t$ $a = -10\ m/s^2$

b) The area of the force time graph gives the impulse which equals the momentum change. You can break the graph into three triangles and 1 rectangle and find the area = 0.6 Ns = 0.6 kg m/s of momentum change

c) Using the value above. $\Delta p = m(v_f - v_i)$ $-0.6 = m(-0.22 - 0.18)$ m = 1.5 kg.
The force sensor applies a – momentum since it would push in the negative direction as the cart collides with it.

d) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = \frac{1}{2}(1.5)(0.18^2 - 0.22^2) = -0.012$ J

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ $mgh = \frac{1}{2} mv^2$ $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\, v_f$ $M(\sqrt{2gH}) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Even though the position shown has an unknown initial stretch and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

Apply energy conservation $K + U = U_{sp\ (gained)}$ $\frac{1}{2} mv^2 + mgh = \frac{1}{2} k\Delta x^2$
Plug in mass (2m), h = H/2 and Δx = H/2 ➔ $\frac{1}{2}(2m)v^2 + (2m)g(H/2) = \frac{1}{2} k(H/2)^2$
plug in v_f from part b $m(2gH/4) + mgH = kH^2/8$ ….

Both sides * (1/H) ➔ $mg/2 + mg = kH/8$ ➔ $3/2\ mg = kH/8$ $k = 12mg/H$

C2004M1.

a) Energy conservation with position B set as h=0. $U_a = K_b$ $v_b = \sqrt{2gL}$

b) Forces at B, F_t pointing up and mg pointing down. Apply $F_{net(c)}$

$F_{net(C)} = m_1 v_b^2 / r$ $F_t - m_1 g = m_1(2gL) / L$ $F_t = 3m_1 g$

c) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$m_1 v_{1i} = (m_1+m_2)\, v_f$ $v_f = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}$

d) Projectile. First find time to travel from B to D using the y direction equations

$d_y = v_{iy}t + ½\, g\, t^2$ $L = 0 + gt^2 / 2$

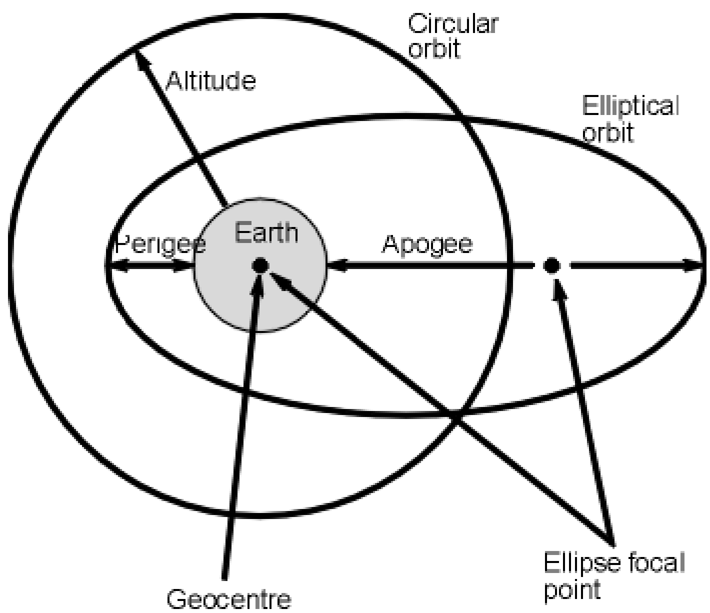
$t = \sqrt{\dfrac{2L}{g}}$ Then use $v_x = d_x / t$ $d_x = \dfrac{m_1}{(m_1+m_2)}\sqrt{2gL}\sqrt{\dfrac{2L}{g}} = \dfrac{m_1}{(m_1+m_2)}2L$

The dx found is measured from the edge of the second lower cliff so the total horizontal distance would have to include the initial x displacement (L) starting from the first cliff.

➔ $\dfrac{m_1}{(m_1+m_2)}2L + L = L\left[\dfrac{2m_1}{(m_1+m_2)} + 1\right]$

Chapter 6

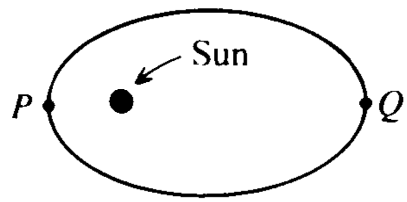
Gravitation



AP Physics Multiple Choice Practice – Gravitation

1. Each of five satellites makes a circular orbit about an object that is much more massive than any of the satellites. The mass and orbital radius of each satellite are given below. Which satellite has the greatest speed?

	Mass	Radius
(A)	½m	R
(B)	m	½R
(C)	m	R
(D)	m	2R
(E)	2m	R



*2. An asteroid moves in an elliptic orbit with the Sun at one focus as shown above. Which of the following quantities increases as the asteroid moves from point P in its orbit to point Q?
(A) Speed (B) Angular momentum (C) Total energy (D) Kinetic energy (E) Potential energy

3. Two planets have the same size, but different masses, and no atmospheres. Which of the following would be the same for objects with equal mass on the surfaces of the two planets?
I. The rate at which each would fall freely
II. The amount of mass each would balance on an equal-arm balance
III. The amount of momentum each would acquire when given a certain impulse
(A) I only (B) III only (C) I and II only (D) II and III only (E) I, II, and III

4. A person weighing 800 newtons on Earth travels to another planet with twice the mass and twice the radius of Earth. The person's weight on this other planet is most nearly
(A) 400 N (B) $\frac{800}{\sqrt{2}}$ N (C) 800 N (D) $800\sqrt{2}$ (E) 1,600 N

5. Mars has a mass 1/10 that of Earth and a diameter 1/2 that of Earth. The acceleration of a falling body near the surface of Mars is most nearly
(A) 0.25 m/s^2 (B) 0.5 m/s^2 (C) 2 m/s^2 (D) 4 m/s^2 (E) 25 m/s^2

6. A satellite of mass M moves in a circular orbit of radius R at a constant speed v. Which of the following must be true?
I. The net force on the satellite is equal to Mv^2/R and is directed toward the center of the orbit.
II. The net work done on the satellite by gravity in one revolution is zero.
III. The angular momentum of the satellite is a constant.
(A) I only (B) III only (C) I and II only (D) II and III only (E) I, II, and III

7. If Spacecraft X has twice the mass of Spacecraft Y, then true statements about X and Y include which of the following?
I. On Earth. X experiences twice the gravitational force that Y experiences.
II. On the Moon, X has twice the weight of Y.
III. When both are in the same circular orbit, X has twice the centripetal acceleration of Y.
(A) I only (B) III only (C) I and II only (D) II and III only (E) I, II, and III

8. The two spheres pictured above have equal densities and are subject only to their mutual gravitational attraction. Which of the following quantities must have the same magnitude for both spheres?
(A) Acceleration (B) Velocity (C) Kinetic energy
(D) Displacement from the center of mass (E) Gravitational force

9. The planet Mars has mass 6.4×10^{23} kilograms and radius 3.4×10^{6} meters. The acceleration of an object in free-fall near the surface of Mars is most nearly
(A) zero (B) 1.0 m/s^2 (C) 1.9 m/s^2 (D) 3.7 m/s^2 (E) 9.8 m/s^2

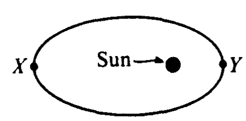
10. An object has a weight *W* when it is on the surface of a planet of radius *R*. What will be the gravitational force on the object after it has been moved to a distance of *4R* from the center of the planet?
(A) 16W (B) 4W (C) W (D) 4 (E) 1/16 W

11. A new planet is discovered that has twice the Earth's mass and twice the Earth's radius. On the surface of this new planet, a person who weighs 500 N on Earth would experience a gravitational force of
(A) 125 N (B) 250 N (C) 500 N (D) 1000 N (E) 2000 N

*12. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are shorter. (B) Both are the same. (C) Both are longer.
(D) The period of the mass on the spring is shorter, that of the pendulum is the same.
(E) The period of the pendulum is shorter; that of the mass on the spring is the same.

13. A satellite of mass m and speed v moves in a stable, circular orbit around a planet of mass *M*. What is the radius of the satellite's orbit?
(A) $\frac{GM}{mv}$ (B) $\frac{Gv}{mM}$ (C) $\frac{GM}{v^2}$ (D) $\frac{GmM}{v}$ (E) $\frac{GmM}{v^2}$

14. The mass of Planet X is one-tenth that of the Earth, and its diameter is one-half that of the Earth. The acceleration due to gravity at the surface of Planet X is most nearly
(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 7 m/s^2 (E) 10 m/s^2

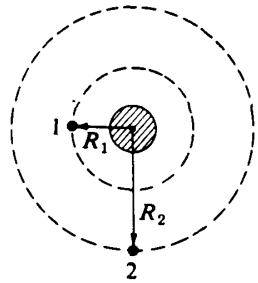


*15. A satellite travels around the Sun in an elliptical orbit as shown above. As the satellite travels from point X to point Y. which of the following is true about its speed and angular momentum?

	Speed	Angular Momentum
(A)	Remains constant	Remains constant
(B)	Increases	Increases
(C)	Decreases	Decreases
(D)	Increases	Remains constant
(E)	Decreases	Remains constant

16. A newly discovered planet, "Cosmo," has a mass that is 4 times the mass of the Earth. The radius of the Earth is R_e. The gravitational field strength at the surface of Cosmo is equal to that at the surface of the Earth if the radius of Cosmo is equal to
(A) $\frac{1}{2}R_e$ (B) R_e (C) $2R_e$ (D) $\sqrt{R_e}$ (E) R_e^2

17. The radius of the Earth is approximately 6,000 kilometers. The acceleration of an astronaut in a perfectly circular orbit 300 kilometers above the Earth would be most nearly
(A) 0 m/s^2 (B) 0.05 m/s^2 (C) 5 m/s^2 (D) 9 m/s^2 (E) 11 m/s^2



18. Two artificial satellites, 1 and 2, orbit the Earth in circular orbits having radii R_1 and R_2, respectively, as shown above. If $R_2 = 2R_1$, the accelerations a_2 and a_1 of the two satellites are related by which of the following?
(A) $a_2 = 4a_1$ (B) $a_2 = 2a_1$ (C) $a_2 = a_1$ (D) $a_2 = a_1/2$ (E) $a_2 = a_1/4$

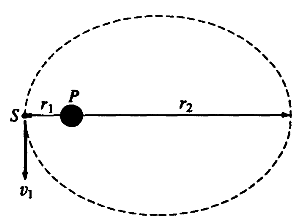
19. A satellite moves in a stable circular orbit with speed v_o at a distance R from the center of a planet. For this satellite to move in a stable circular orbit a distance $2R$ from the center of the planet, the speed of the satellite must be

(A) $\frac{v_0}{2}$ (B) $\frac{v_0}{\sqrt{2}}$ (C) v_o (D) $\sqrt{2v_0}$ (E) $2v_o$

20. If F_1 is the magnitude of the force exerted by the Earth on a satellite in orbit about the Earth and F_2 is the magnitude of the force exerted by the satellite on the Earth, then which of the following is true?
(A) F_1 is much greater than F_2. (B) F_1 is slightly greater than F_2.
(C) F_1 is equal to F_2. (D) F_2 is slightly greater than F_1 (E) F_2 is much greater than F_1

21. A newly discovered planet has twice the mass of the Earth, but the acceleration due to gravity on the new planet's surface is exactly the same as the acceleration due to gravity on the Earth's surface. The radius of the new planet in terms of the radius R of Earth is

(A) ½R (B) $\frac{\sqrt{2}}{2}$ R (C) $\sqrt{2}$ R (D) 2R (E) 4R



*22. A satellite S is in an elliptical orbit around a planet P, as shown above, with r_1 and r_2 being its closest and farthest distances, respectively, from the center of the planet. If the satellite has a speed v_1 at its closest distance, what is its speed at its farthest distance?

(A) $\frac{r_1}{r_2}v_1$ (B) $\frac{r_2}{r_1}v_1$ (C) $(r_2 - r_2)v_1$ (D) $\frac{r_1+r_2}{2}v_1$ (E) $\frac{r_2-r_1}{r_1+r_2}v_1$

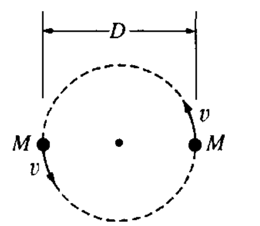
Questions 23 – 24

A ball is tossed straight up from the surface of a small, spherical asteroid with no atmosphere. The ball rises to a height equal to the asteroid's radius and then falls straight down toward the surface of the asteroid.

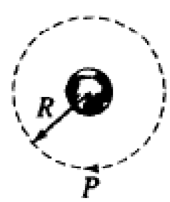
23. What forces, if any, act on the ball while it is on the way up?
(A) Only a decreasing gravitational force that acts downward
(B) Only an increasing gravitational force that acts downward
(C) Only a constant gravitational force that acts downward
(D) Both a constant gravitational force that acts downward and a decreasing force that acts upward
(E) No forces act on the ball.

24. The acceleration of the ball at the top of its path is
(A) at its maximum value for the ball's flight
(B) equal to the acceleration at the surface of the asteroid
(C) equal to one-half the acceleration at the surface of the asteroid
(D) equal to one-fourth the acceleration at the surface of the asteroid
(E) zero

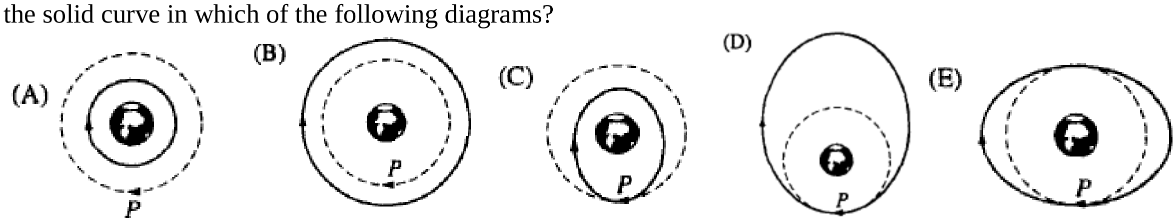
25. A satellite of mass M moves in a circular orbit of radius R with constant speed v. True statements about this satellite include which of the following?
I. Its angular speed is v/R.
II. Its tangential acceleration is zero.
III. The magnitude of its centripetal acceleration is constant.
(A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, and III



26. Two identical stars, a fixed distance D apart, revolve in a circle about their mutual center of mass, as shown above. Each star has mass M and speed v. G is the universal gravitational constant. Which of the following is a correct relationship among these quantities?
(A) v^2 = GM/D (B) v^2 = GM/2D (C) v^2 = GM/D^2 (D) v^2 = MGD (E) v^2 = 2GM^2/D



*27. A spacecraft orbits Earth in a circular orbit of radius R, as shown above. When the spacecraft is at position P shown, a short burst of the ship's engines results in a small increase in its speed. The new orbit is best shown by the solid curve in which of the following diagrams?



*28. The escape speed for a rocket at Earth's surface is v_e . What would be the rocket's escape speed from the surface of a planet with twice Earth's mass and the same radius as Earth?

(A) $2v_e$ (B) $\sqrt{2}v_e$ (C) v_e (D) $\frac{v_e}{\sqrt{2}}$ (E) ½ v_e

29. A hypothetical planet orbits a star with mass one-half the mass of our sun. The planet's orbital radius is the same as the Earth's. Approximately how many Earth years does it take for the planet to complete one orbit?
(A) ½ (B) $\frac{1}{\sqrt{2}}$ (C) 1 (D) $\sqrt{2}$ (E) 2

30. A hypothetical planet has seven times the mass of Earth and twice the radius of Earth. The magnitude of the gravitational acceleration at the surface of this planet is most nearly
(A) 2.9 m/s^2 (B) 5.7 m/s^2 (C) 17.5 m/s^2 (D) 35 m/s^2 (E) 122 m/s^2

31. Two artificial satellites, 1 and 2, are put into circular orbit at the same altitude above Earth's surface. The mass of satellite 2 is twice the mass of satellite 1. If the period of satellite 1 is T, what is the period of satellite 2?
(A) $T/4$ (B) $T/2$ (C) T (D) $2T$ (E) $4T$

32. A planet has a radius one-half that of Earth and a mass one-fifth the Earth's mass. The gravitational acceleration at the surface of the planet is most nearly
(A) 4.0 m/s^2 (B) 8.0 m/s^2 (C) 12.5 m/s^2 (D) 25 m/s^2 (E) 62.5 m/s^2

33. In the following problem, the word "weight" refers to the force a scale registers. If the Earth were to stop rotating, but not change shape,
(A) the weight of an object at the equator would increase.
(B) the weight of an object at the equator would decrease.
(C) the weight of an object at the north pole would increase.
(D) the weight of an object at the north pole would decrease.
(E) all objects on Earth would become weightless.

*34. Assume that the Earth attracts John Glenn with a gravitational force F at the surface of the Earth. When he made his famous second flight in orbit, the gravitational force on John Glenn while he was in orbit was closest to which of the following?
(A) $0.95F$ (B) $0.50F$ (C) $0.25F$ (D) $0.10F$ (E) zero

35. What happens to the force of gravitational attraction between two small objects if the mass of each object is doubled and the distance between their centers is doubled?
(A) It is doubled (B) It is quadrupled (C) It is halved (D) It is reduced fourfold (E) It remains the same

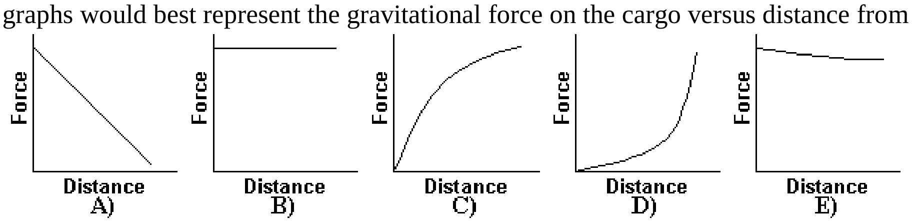
36. One object at the surface of the Moon experiences the same gravitational force as a second object at the surface of the Earth. Which of the following would be a reasonable conclusion?
(A) both objects would fall at the same acceleration
(B) the object on the Moon has the greater mass
(C) the object on the Earth has the greater mass
(D) both objects have identical masses
(E) the object on Earth has a greater mass but the Earth has a greater rate of rotation.

37. Astronauts in an orbiting space shuttle are "weightless" because
(A) of their extreme distance from the earth
(B) the net gravitational force on them is zero
(C) there is no atmosphere in space
D) the space shuttle does not rotate
E) they are in a state of free fall

38. Consider an object that has a mass, *m*, and a weight, *W*, at the surface of the moon. If we assume the moon has a nearly uniform density, which of the following would be closest to the object's mass and weight at a distance halfway between Moon's center and its surface?
(A) ½ *m* & ½ *W* (B) ¼ *m* & ¼ *W* (C) 1 *m* & 1 *W* (D) 1 *m* & ½ *W* (E) 1 *m* & ¼ *W*

39. The mass of a planet can be calculated if it is orbited by a small satellite by setting the gravitational force on the satellite equal to the centripetal force on the satellite. Which of the following would NOT be required in this calculation?
(A) the mass of the satellite
(B) the radius of the satellite's orbit
(C) the period of the satellite's orbit
(D) Newton's universal gravitational constant
(E) all of the above are required for this calculation

40. As a rocket blasts away from the earth with a cargo for the international space station, which of the following graphs would best represent the gravitational force on the cargo versus distance from the surface of the Earth?



41. In chronological order (earliest to latest), place the following events:
(1) Henry Cavendish's experiment
(2) Newton's work leading towards the Law of Universal Gravitation
(3) Tycho Brahe takes astronomical data
(4) Nicolaus Copernicus proposes the heliocentric theory
(5) Johannes Kepler's work on the orbit of Mars.
(A) 43521 (B) 42135 (C) 43251 (D) 35421 (E) 42351

42. A 20 kg boulder rests on the surface of the Earth. Assume the Earth has mass 5.98×10^{24} kg and g = 10 m/s^2. What is the magnitude of the gravitational force that the boulder exerts on the Earth?
(A) 5.98×10^{25} N
(B) 5.98×10^{24} N
(C) 20 N
(D) The boulder exerts no force on the Earth
(E) 200 N

43. An astronaut on the Moon simultaneously drops a feather and a hammer. The fact that they reach the surface at the same instant shows that
(A) no gravity forces act on a body in a vacuum.
(B) the acceleration due to gravity on the Moon is less than the acceleration due to gravity on the Earth.
(C) the gravitational force from the Moon on heavier objects (the hammer) is equal to the gravitational force on lighter objects (the feather).
(D) a hammer and feather have less mass on the Moon than on Earth.
(E) in the absence of air resistance all bodies at a given location fall with the same acceleration.

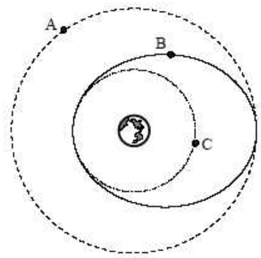
*44. A scientist in the International Space Station experiences "weightlessness" because
(A) there is no gravitational force from the Earth acting on her.
(B) the gravitational pull of the Moon has canceled the pull of the Earth on her.
(C) she is in free fall along with the Space Station and its contents.
(D) at an orbit of 200 miles above the Earth, the gravitational force of the Earth on her is 2% less than on its surface.
(E) in space she has no mass.

*45. A rocket is in a circular orbit with speed v and orbital radius R around a heavy stationary mass. An external impulse is quickly applied to the rocket directly opposite to the velocity and the rocket's speed is slowed to $v/2$, putting the rocket into an elliptical orbit. In terms of R, the size of the semi-major axis a of this new elliptical orbit is

(A) $a = \frac{1}{4}R$ (B) $a = \frac{1}{2}R$ (C) $a = \frac{7}{11}R$ (D) $a = \frac{\sqrt{8}}{3}R$ (E) $a = \frac{4}{7}R$

*46. Kepler's Second Law about "sweeping out equal areas in equal time" can be derived most directly from which conservation law?

(A) energy (B) mechanical energy (C) angular momentum (D) mass (E) linear momentum



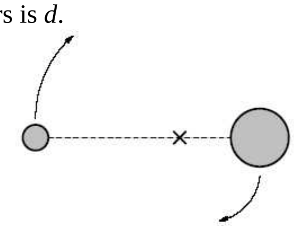
*47. Three equal mass satellites A, B, and C are in coplanar orbits around a planet as shown in the figure. The magnitudes of the angular momenta of the satellites as measured about the planet are L_A, L_B, and L_C. Which of the following statements is correct?

(A) $L_A > L_B > L_C$ (B) $L_C > L_B > L_A$ (C) $L_B > L_C > L_A$ (D) $L_B > L_A > L_C$

(E) The relationship between the magnitudes is different at various instants in time.

<u>Questions 48 – 49</u>

Two stars orbit their common center of mass as shown in the diagram below. The masses of the two stars are $3M$ and M. The distance between the stars is d.



*48. What is the value of the gravitational potential energy of the two star system?

(A) $-\frac{GM^2}{d}$ (B) $\frac{3GM^2}{d}$ (C) $-\frac{GM^2}{d^2}$ (D) $-\frac{3GM^2}{d}$ (E) $-\frac{GM^2}{d^2}$

*49. Determine the period of orbit for the star of mass $3M$.

(A) $\pi\sqrt{\frac{d^3}{GM}}$ (B) $\frac{3\pi}{4}\sqrt{\frac{d^3}{GM}}$ (C) $\pi\sqrt{\frac{d^3}{3GM}}$ (D) $2\pi\sqrt{\frac{d^3}{GM}}$ (E) $\frac{\pi}{4}\sqrt{\frac{d^3}{GM}}$

*50. Two satellites are launched at a distance R from a planet of negligible radius. Both satellites are launched in the tangential direction. The first satellite launches correctly at a speed v_0 and enters a circular orbit. The second satellite, however, is launched at a speed $\frac{1}{2}v_0$. What is the minimum distance between the second satellite and the planet over the course of its orbit?

(A) $\frac{1}{\sqrt{2}}R$ (B) $\frac{1}{2}R$ (C) $\frac{1}{3}R$ (D) $\frac{1}{4}R$ (E) $\frac{1}{7}R$

*51. When two stars are very far apart, their gravitational potential energy is zero; when they are separated by a distance d, the gravitational potential energy of the system is U. What is the gravitational potential energy of the system if the stars are separated by a distance $2d$?

(A) $U/4$ (B) $U/2$ (C) U (D) $2U$ (E) $4U$

52. The gravitational force on a textbook at the top of Pikes Peak (elevation 14,100 ft) is 40 newtons. What would be the approximate gravitational force on the same textbook if it were taken to twice the elevation?
(A) 5 N (B) 10 N (C) 20 N (D) 40 N (E) 80 N

53. Assume the International Space Station has a mass *m* and is in a circular orbit of radius *r* about the center of the Earth. If the Earth has a mass of *M*, what would be the speed of the Space Station around the Earth?
(A) $\sqrt{\frac{GM}{r}}$ (B) $\sqrt{\frac{GM}{r^2}}$ (C) $\sqrt{\frac{GMm}{r^2}}$ (D) $\sqrt{\frac{GMm}{r}}$ (E) $\sqrt{GMmr}$

54. What would be the gravitational force of attraction between the proton in the nucleus and the electron in an orbit of radius 5.3×10^{-11} m in a simple hydrogen atom?
(A) 2.0×10^{-57} N (B) 3.7×10^{-47} N (C) 6.1×10^{-28} N (D) 8.2×10^{-8} N
(E) the only force of attraction would be electrical

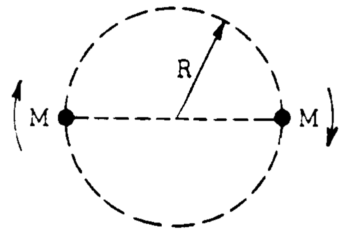
55. Two iron spheres separated by some distance have a minute gravitational attraction, *F*. If the spheres are moved to one half their original separation and allowed to rust so that the mass of each sphere increases 41%, what would be the resulting gravitational force?
(A) $2F$ (B) $4F$ (C) $6F$ (D) $8F$ (E) $10F$

56. A ball which is thrown upward near the surface of the Earth with a velocity of 50 m/s will come to rest about 5 seconds later. If the ball were thrown up with the same velocity on Planet X, after 5 seconds it would be still moving upwards at nearly 31 m/s. The magnitude of the gravitational field near the surface of Planet X is what fraction of the gravitational field near the surface of the Earth?
(A) 0.16 (B) 0.39 (C) 0.53 (D) 0.63 (E) 1.59

57. An object placed on an equal arm balance requires 12 kg to balance it. When placed on a spring scale, the scale reads 120 N. Everything (balance, scale, set of masses, and the object) is now transported to the moon where the gravitational force is one-sixth that on Earth. What are the new readings of the balance and the spring scale (respectively)?
(A) 12 kg, 20 N (B) 1 kg, 120 N (C) 12 kg, 720 N (D) 2 kg, 20 N (E) 2 kg, 120 N

58. Two artificial satellites I and II have circular orbits of radii *R* and 2*R*, respectively, about the same planet. The orbital velocity of satellite I is *v*. What is the orbital velocity of satellite II?
(A) $\frac{v}{2}$ (B) $\frac{v}{\sqrt{2}}$ (C) v (D) $\sqrt{2}v$ (E) $2v$

59. The gravitational acceleration on the surface of the moon is $1.6\ \text{m/s}^2$. The radius of the moon is 1.7×10^6 m. What is the period of a satellite placed in a low circular orbit about the moon?
(A) 1.0×10^3 s (B) 6.5×10^3 s (C) 1.1×10^6 s (D) 5.0×10^6 s (E) 7.1×10^{12} s



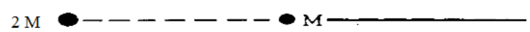
*1977M3. Two stars each of mass M form a binary star system such that both stars move in the same circular orbit of radius R. The universal gravitational constant is G.

a. Use Newton's laws of motion and gravitation to find an expression for the speed v of either star in terms of R, G, and M.

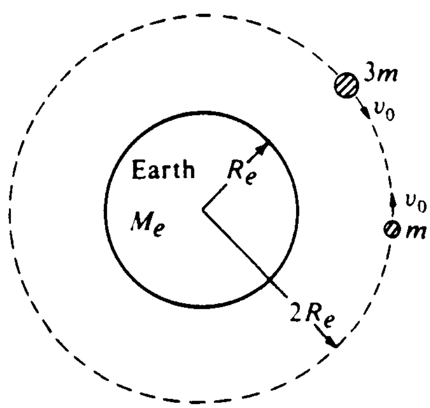
b. Express the total energy E of the binary star system in terms of R, G, and M.

Suppose instead, one of the stars had a mass 2M.

c. On the following diagram, show circular orbits for this star system.



d. Find the ratio of the speeds, $v_{2M}/v_{M.}$

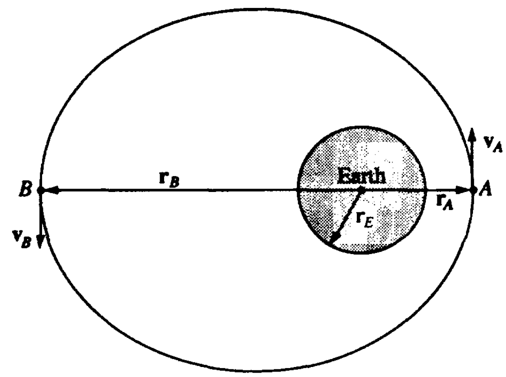


1984M2. Two satellites, of masses *m* and 3*m*, respectively, are in the same circular orbit about the Earth's center, as shown in the diagram above. The Earth has mass M_e and radius R_e. In this orbit, which has a radius of $2R_e$, the satellites initially move with the same orbital speed v_o but in opposite directions.

a. Calculate the orbital speed v_o of the satellites in terms of G, M_e, and R_e.

b. Assume that the satellites collide head-on and stick together. In terms of v_o find the speed v of the combination immediately after the collision.

c. Calculate the total mechanical energy of the system immediately after the collision in terms of G, m, M_e, and R_e. Assume that the gravitational potential energy of an object is defined to be zero at an infinite distance from the Earth.



*1992M3. A spacecraft of mass 1,000 kilograms is in an elliptical orbit about the Earth, as shown above. At point A the spacecraft is at a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth and its velocity, of magnitude $v_A = 7.1 \times 10^3$ meters per second, is perpendicular to the line connecting the center of the Earth to the spacecraft. The mass and radius of the Earth are $M_E = 6.0 \times 10^{24}$ kilograms and $r_E = 6.4 \times 10^6$ meters, respectively.

Determine each of the following for the spacecraft when it is at point A .

a. The total mechanical energy of the spacecraft, assuming that the gravitational potential energy is zero at an infinite distance from the Earth.

b. The magnitude of the angular momentum of the spacecraft about the center of the Earth.

Later the spacecraft is at point B on the exact opposite side of the orbit at a distance $r_B = 3.6 \times 10^7$ meters from the center of the Earth.

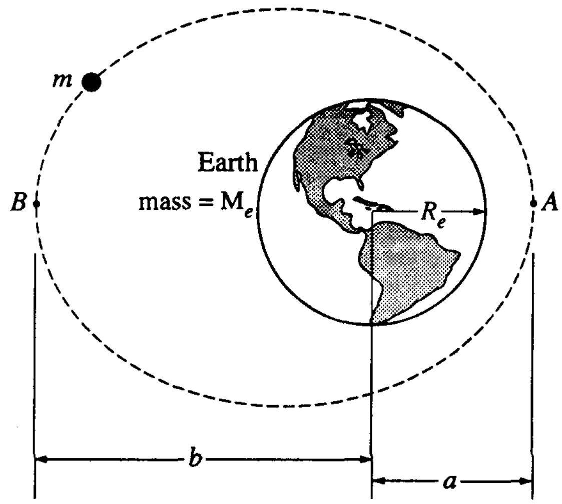
c. Determine the speed v_B of the spacecraft at point B.

Suppose that a different spacecraft is at point A, a distance $r_A = 1.2 \times 10^7$ meters from the center of the Earth. Determine each of the following.

d. The speed of the spacecraft if it is in a <u>circular</u> orbit around the Earth

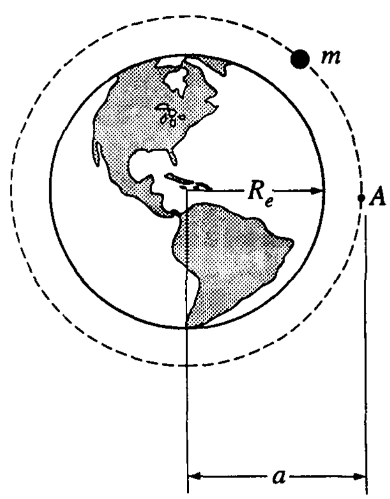
e. The minimum speed of the spacecraft at point A if it is to escape completely from the Earth

---



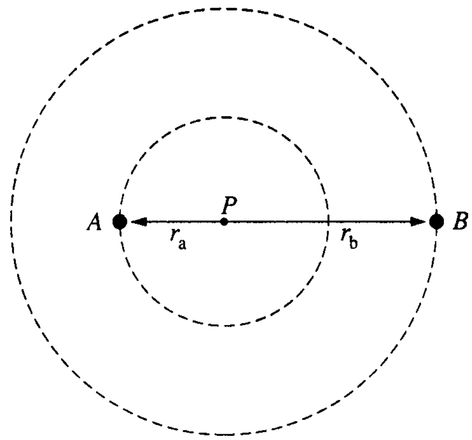
*1994M3 (modified) A satellite of mass *m* is in an elliptical orbit around the Earth, which has mass M_e and radius R_e. The orbit varies from a closest approach of distance *a* at point A to maximum distance of b from the center of the Earth at point B. At point A, the speed of the satellite is v_o. Assume that the gravitational potential energy $U_g = 0$ when masses are an infinite distance apart. Express your answers in terms of *a*, b, *m*, M_e, R_e, v_o, and G.

a. Determine the total energy of the satellite when it is at A.
b. What is the magnitude of the angular momentum of the satellite about the center of the Earth when it is at A?
c. Determine the velocity of the satellite as it passes point B in its orbit.



As the satellite passes point A, a rocket engine on the satellite is fired so that its orbit is changed to a circular orbit of radius a about the center of the Earth.

d. Determine the speed of the satellite for this circular orbit.
e. Determine the work done by the rocket engine to effect this change.



*1995M3 (modified) Two stars, A and B. are in circular orbits of radii r_a and r_b, respectively, about their common center of mass at point P, as shown above. Each star has the same period of revolution T.

Determine expressions for the following three quantities in terms of r_a, r_b, T, and fundamental constants.

a. The centripetal acceleration of star A

b. The mass M_b of star B

c. The mass M_a of star A

d. Determine an expressions for the angular momentum of the system about the center of mass in terms of M_a, M_b, r_a, r_b, T, and fundamental constants.

2007M2. In March 1999 the Mars Global Surveyor (GS) entered its final orbit about Mars, sending data back to Earth. Assume a circular orbit with a period of 1.18×10^2 minutes = 7.08×10^3 s and orbital speed of 3.40×10^3 m/s . The mass of the GS is 930 kg and the radius of Mars is 3.43×10^6 m.

a. Calculate the radius of the GS orbit.

b. Calculate the mass of Mars.

c. Calculate the total mechanical energy of the GS in this orbit.

d. If the GS was to be placed in a lower circular orbit (closer to the surface of Mars), would the new orbital period of the GS be greater than or less than the given period?

_________Greater than _________ Less than

Justify your answer.

e. In fact, the orbit the GS entered was slightly elliptical with its closest approach to Mars at 3.71×10^5 m above the surface and its furthest distance at 4.36×10^5 m above the surface. If the speed of the GS at closest approach is 3.40×10^3 m/s, calculate the speed at the furthest point of the orbit.

2001M2. An explorer plans a mission to place a satellite into a circular orbit around the planet Jupiter, which has mass $M_J = 1.90 \times 10^{27}$ kg and radius $R_J = 7.14 \times 10^7$ m.

a. If the radius of the planned orbit is R, use Newton's laws to show each of the following.

i. The orbital speed of the planned satellite is given by

$$v = \sqrt{\frac{GM_J}{R}}$$

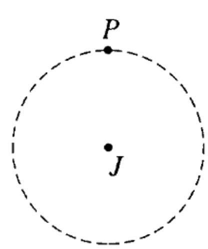
ii. The period of the orbit is given by

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_J}}$$

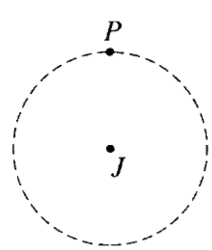
b. The explorer wants the satellite's orbit to be synchronized with Jupiter's rotation. This requires an equatorial orbit whose period equals Jupiter's rotation period of 9 hr 51 min = 3.55×10^4 s. Determine the required orbital radius in meters.

c. Suppose that the injection of the satellite into orbit is less than perfect. For an injection velocity that differs from the desired value in each of the following ways, sketch the resulting orbit on the figure. (J is the center of Jupiter, the dashed circle is the desired orbit, and P is the injection point.) Also, describe the resulting orbit qualitatively but specifically.

i. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>faster</u> than the correct speed for a circular orbit of that radius.



ii. When the satellite is at the desired altitude over the equator, its velocity vector has the correct direction, but the speed is slightly <u>slower</u> than the correct speed for a circular orbit of that radius.



2005M2. A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

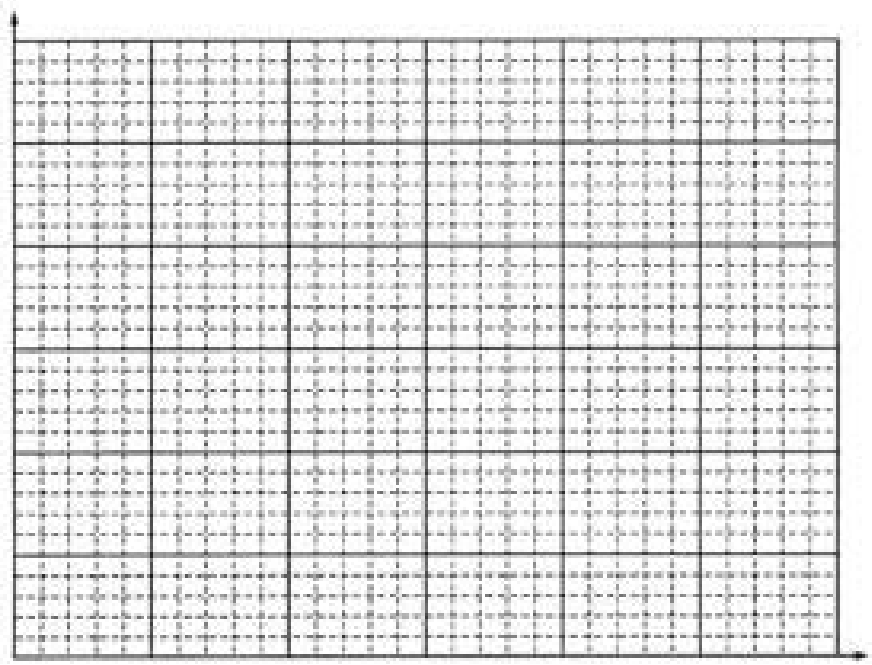
a. Write an algebraic expression for the gravitational force between Saturn and one of its moons.

b. Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

c. Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

d. Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

e. Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



f. Using the graph, calculate a value for the mass of Saturn.

ANSWERS - AP Physics Multiple Choice Practice – Gravitation

	Solution	Answer
1.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed. The smallest radius of orbit will be the fastest satellite.	B
2.	As a satellite moves farther away, it slows down, also decreasing its angular momentum and kinetic energy. The total energy remains the same in the absence of resistive or thrust forces. The potential energy becomes less negative, which is an increase.	E
3.	With different masses, g would have a different value, but the physical characteristics of the objects would not be affected.	D
4.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	A
5.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10	D
6.	Circular orbit = constant r, combined with constant speed gives constant angular momentum (mvr). As it is a circular orbit, the force is centripetal, points toward the center and is always perpendicular to the displacement of the satellite therefore does no work.	E
7.	The gravitational force on an object *is* the weight, and is proportional to the mass. In the same circular orbit, it is only the mass of the body being orbited and the radius of the orbit that contributes to the orbital speed and acceleration.	C
8.	Newton's third law	E
9.	$g = \frac{GM}{r^2}$	D
10.	Force is inversely proportional to distance between the centers squared. R × 4 = F ÷ 16	E
11.	$g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and r × 2 = g ÷ 4, so the net effect is the person's weight is divided by 2	B
12.	A planet of the same size and twice the mass of Earth will have twice the acceleration due to gravity. The period of a mass on a spring has no dependence on g, while the period of a pendulum is inversely proportional to g.	E
13.	Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$	C

14. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 10 = g ÷ 10 and r ÷ 2 = g × 4, so the net effect is g × 4/10 — B

15. Kepler's second law (Law of areas) is based on conservation of angular momentum, which remains constant. In order for angular momentum to remain constant, as the satellite approaches the sun, its speed increases. — D

16. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 4 = g × 4 and if the net effect is $g = g_{Earth}$ then r must be twice that of Earth. — C

17. $g = \frac{GM}{r^2}$. 300 km above the surface of the Earth is only a 5% increase in the distance (1.05 times the distance). This will produce only a small effect on g ($\div 1.05^2$) — D

18. $a = g = \frac{GM}{r^2}$, if $R_2 = 2R_1$ then $a_2 = \frac{1}{4} a_1$ — E

19. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. If r is doubled, v decreases by $\sqrt{2}$ — B

20. Newton's third law — C

21. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 2 = g × 2 and if the net effect is $g = g_{Earth}$ then r must be $\sqrt{2}$ times that of Earth — C

22. From conservation of angular momentum $v_1r_1 = v_2r_2$ — A

23. As the ball moves away, the force of gravity decreases due to the increasing distance. — A

24. $g = \frac{GM}{r^2}$ At the top of its path, it has doubled its original distance from the center of the asteroid. — D

25. Angular speed (in radians per second) is v/R. Since the satellite is not changing speed, there is no tangential acceleration and v^2/r is constant. — E

26. The radius of each orbit is ½ D, while the distance between them is D. This gives — B

$$\frac{GMM}{D^2} = \frac{Mv^2}{D/2}$$

27. An burst of the ships engine produces an increase in the satellite's energy. Now the satellite is moving at too large a speed for a circular orbit. The point at which the burst occurs must remain part of the ship's orbit, eliminating choices A and B. The Earth is no longer at the focus of the ellipse in choice E. — D

28. Escape speed with the speed at which the kinetic energy of the satellite is exactly equal to the negative amount of potential energy within the satellite/mass system. That is $\frac{1}{2}mv^2 = \left|-\frac{GMm}{r}\right|$ which gives the escape speed $v_e = \sqrt{\frac{2GM}{r}}$ B

29. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Also, $T = \frac{2\pi r}{v}$. Since the mass is divided by 2, v is divided by $\sqrt{2}$ D

30. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M × 7 = g × 7 and r × 2 = g ÷ 4, so the net effect is g × 7/4 C

31. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. Notice that satellite mass does not affect orbital speed or period. C

32. $g = \frac{GM}{r^2}$ so the acceleration due to gravity (and the weight of an object) is proportional to the mass of the planet and inversely proportional to the distance from the center of the planet squared. M ÷ 5 = g ÷ 5 and r ÷ 2 = g × 4, so the net effect is g × 4/5 B

33. Part of the gravitational force acting on an object at the equator is providing the necessary centripetal force to move the object in a circle. If the rotation of the earth were to stop, this part of the gravitational force is no longer required and the "full" value of this force will hold the object to the Earth. A

34. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration due to gravity is only slightly smaller in orbit compared to the surface of the Earth. A

35. $F = \frac{GMm}{r^2}$. F is proportional to each mass and inversely proportional to the distance between their centers squared. If each mass is doubled, F is quadrupled. If r is doubled F is quartered. E

36. Since the acceleration due to gravity is less on the surface of the moon, to have the same gravitational force as a second object on the Earth requires the object on the Moon to have a larger mass. B

37. Satellites in orbit are freely falling objects with enough horizontal speed to keep from felling closer to the planet. E

38. The mass of an object will not change based on its location. As one digs into a sphere of uniform density, the acceleration due to gravity (and the weight of the object) varies directly with distance from the center of the sphere. D

39. Combining $\frac{GMm}{r^2} = \frac{mv^2}{r}$ with $T = \frac{2\pi r}{v}$ gives the equation corresponding to Kepler's second law. The mass of the satellite cancels in these equations. A

40. $F = \frac{GMm}{r^2}$ so F is proportional to $1/r^2$. Standard orbital altitudes are not a large percentage of the radius of the Earth. The acceleration due to gravity is only slightly smaller in orbit compared to the surface of the Earth. E

41. NO, this is not part of the curriculum, but interesting to know (and a bit of common sense if you follow the changing of the theories) A

42. Gravitational force is also the weight. mg. E

43. g is the same for all bodies in the absence of air resistance E

44. Satellites in orbit are freely falling objects with enough horizontal speed to keep from felling closer to the planet. C

45. The energy of a circular orbit is E

$$K + U = \frac{1}{2}mv^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{GMm}{2r}$$

The energy of an elliptical orbit is $-\frac{GMm}{2a}$ where a is the semimajor axis. If the speed is cut in half we have $K + U = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{7GMm}{8r}$

Setting $-\frac{7GMm}{8r} = -\frac{GMm}{2a}$ gives $a = (4/7)r$

46. Sweeping out equal areas in based on the satellite moving faster as it moves closer to the body it is orbiting. This is a result of conservation of angular momentum. C

47. The angular momentum of each satellite is conserved independently so we can compare the orbits at any location. Looking at the common point between orbit A and B shows that satellite A is moving faster at that point than satellite B, showing $L_A > L_B$. A similar analysis at the common point between B and C shows $L_B > L_C$ A

48. $U = -\frac{GMm}{r}$ D

49. Since they are orbiting their center of mass, the larger mass has a radius of orbit of $\frac{1}{4}d$. The speed can be found from $\frac{G(3M)M}{d^2} = \frac{(3M)v^2}{d/4}$ which gives $v = \sqrt{\frac{GM}{4d}} = \frac{2\pi(d/4)}{T}$ A

50. The energy of a circular orbit is E

$$K + U = \frac{1}{2}mv^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{GMm}{2r}$$

The energy of an elliptical orbit is $-\frac{GMm}{2a}$ where a is the semimajor axis. If the speed is cut in half we have $K + U = \frac{1}{2}m\left(\frac{v}{2}\right)^2 + (-\frac{Gmm}{r}) = \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{GM}{r}}\right)^2 + (-\frac{GMm}{r}) = -\frac{7GMm}{8r}$

Setting $-\frac{7GMm}{8r} = -\frac{GMm}{2a}$ gives a = (4/7)r

The distance to the planet from this point is r (the radius of the circular orbit and aphelion for the elliptical orbit). The opposite side of the ellipse is 2a away, or 8r/7, making the distance to the planet at perihelion 8r/7 – r = r/7

51. $U = -\frac{GMm}{r}$ B

52. The top of Pikes Peak is a very small fraction of the radius of the Earth. Moving to twice this elevation will barely change the value of g. D

53. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. A

54. $F = \frac{GMm}{r^2}$. The masses of the proton and electron can be found in the table of constants (these masses do not need to be memorized) B

55. $F = \frac{GMm}{r^2}$; If r ÷ 2, F × 4. If each mass is multiplied by 1.41, F is doubled (1.41×1.41) D

56. g = Δv/t = (31 m/s – 50 m/s)/(5 s) = –3.8 m/s^2 B

57. mass is unchanged, weight is changed due to a change in the acceleration due to gravity A

58. Orbital speed is found from setting $\frac{GMm}{r^2} = \frac{mv^2}{r}$ which gives $v = \sqrt{\frac{GM}{r}}$ where M is the object being orbited. B

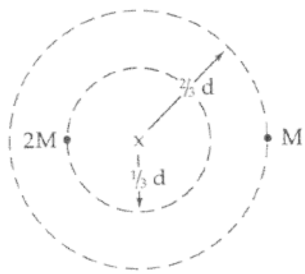
59. g = v^2/r and v = 2πr/T B

1977M3

a. $F_g = F_c$ gives $\frac{GMM}{(2R)^2} = \frac{Mv^2}{R}$. Solving for v gives $v = \frac{1}{2}\sqrt{\frac{GM}{R}}$

b. E = PE + KE = $-\frac{GMM}{2R} + 2\left(\frac{1}{2}Mv^2\right) = -\frac{GMM}{2R} + 2\left(\frac{1}{2}M\left(\frac{1}{2}\sqrt{\frac{GM}{R}}\right)^2\right) = -\frac{GM^2}{4R}$

c.



d. $F_{g2} = F_{g1} = F_c$

$\frac{(2M)v_2{}^2}{{}^1/_3 d} = \frac{Mv_1{}^2}{{}^2/_3 d}$ gives $v_2/v_1 = ½$

1984M2

a. $F_g = F_c$ gives $\frac{GM_e m}{(2R_e)^2} = \frac{mv^2}{2R_e}$ giving $v = \sqrt{\frac{GM_e}{2R_e}}$

b. conservation of momentum gives $(3m)v_0 - mv_0 = (4m)v'$ giving $v' = ½\, v_0$

c. E = PE + KE = $-\frac{GM_e(4m)}{2R_e} + \left(\frac{1}{2}(4m)v^2\right) = -\frac{2GM_e m}{R_e} + 2m\left(\frac{1}{2}\sqrt{\frac{GM_e}{2R_e}}\right)^2 = -\frac{7GM_e m}{4R_e}$

1992M3

a. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2$ = -8.1×10^9 J

b. L = mvr = 8.5×10^{13} kg-m^2/s

c. Angular momentum is conserved so $mv_a r_a = mv_b r_b$ giving $v_b = 2.4 \times 10^3$ m/s

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM/R}$ = 5.8×10^3 m/s

e. Escape occurs when E = PE + KE = 0 giving $-\frac{GMm}{R} + \frac{1}{2}mv^2 = 0\ and\ v_{esc} = \sqrt{2GM/R}$ = 8.2×10^3 m/s

1994M3

a. E = PE + KE = $-\frac{GM_e m}{a} + \frac{1}{2}mv_0{}^2$

b. L = mvr = $mv_0 a$

c. Conservation of angular momentum gives $mv_0 a = mv_b b$, or $v_b = v_0 a/b$

d. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_e/a}$

e. The work done is the change in energy of the satellite. Since the potential energy of the satellite is constant, the change in energy is the change in kinetic energy, or $W = \Delta KE = \frac{1}{2}m\left(\frac{GM_e}{a} - v_0{}^2\right)$

1995M3

a. $v = \frac{2\pi r}{T}$ $and\ a = \frac{v^2}{r} = \frac{4\pi^2 r_a}{T^2}$

b. The centripetal force on star A is due to the gravitational force exerted by star B.
$M_a a_a = \frac{GM_aM_b}{(r_a+r_b)^2}$ and substituting part (a) gives $M_b = \frac{4\pi^2 r_a(r_a+r_b)^2}{GT^2}$

c. The same calculations can be performed with the roles of star A and star B switched.
$M_a = \frac{4\pi^2 r_b(r_a+r_b)^2}{GT^2}$

d. $L_{total} = M_av_ar_a + M_bv_br_b = M_a\frac{2\pi r_a}{T}r_a + M_b\frac{2\pi r_b}{T}r_b = \frac{2\pi}{T}(M_ar_a{}^2 + M_br_b{}^2)$

2007M2

a. v = 2πR/T gives R = 3.83 × 10^6 m

b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ M = \frac{v^2R}{G}$ = 6.64 × 10^{23} kg

c. E = PE + KE = $-\frac{GMm}{R} + \frac{1}{2}mv^2$ = –5.38 × 10^9 J

d. From Kepler's third law r^3/T^2 = constant so if r decreases, then T must also.

e. Conservation of angular momentum gives $mv_1r_1 = mv_2r_2$ so $v_2 = r_1v_1/r_2$, but the distances *above the surface* are given so the radius of Mars must be added to the given distances before plugging them in for each r. This gives v_2 = 3.34 × 10^3 m/s.

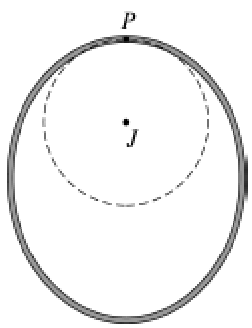
2001M2

a. i. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{GM_J/R}$

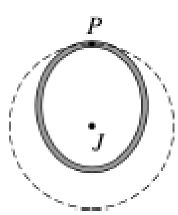
ii. v = d/T = 2πR/T giving $T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{GM_J/R}} = \sqrt{\frac{4\pi^2R^3}{GM_J}}$

b. Plugging numerical values into a.ii. above gives R = 1.59 × 10^8 m

c. i.



ii.



2005M2

a. $F = \frac{GM_S m}{R^2}$

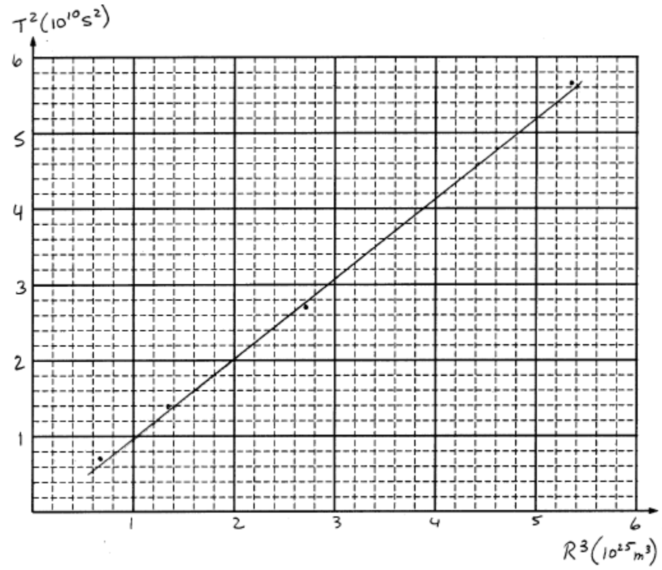
b. $F_g = F_c$ gives $\frac{GMm}{R^2} = \frac{mv^2}{R}$ $and\ v = \sqrt{\frac{GM}{R}} = \frac{2\pi R}{T}$ gives the desired equation $T = \sqrt{\frac{4\pi^2 R^3}{GM}}$

c. T^2 vs. R^3 will yield a straight line (let $y = T^2$ and $x = R^3$, we have the answer to b. as $y = \left(\frac{4\pi^2}{GM}\right)x$ where the quantity in parentheses is the slope of the line.

d.

Orbital Period, T (seconds)	Orbital Radius, R (meters)	T^2 (s^2)	R^3 (m^3)
8.14×10^4	1.85×10^8	0.663×10^{10}	0.633×10^{25}
1.18×10^5	2.38×10^8	1.39×10^{10}	1.35×10^{25}
1.63×10^5	2.95×10^8	2.66×10^{10}	2.57×10^{25}
2.37×10^5	3.77×10^8	5.62×10^{10}	5.36×10^{25}

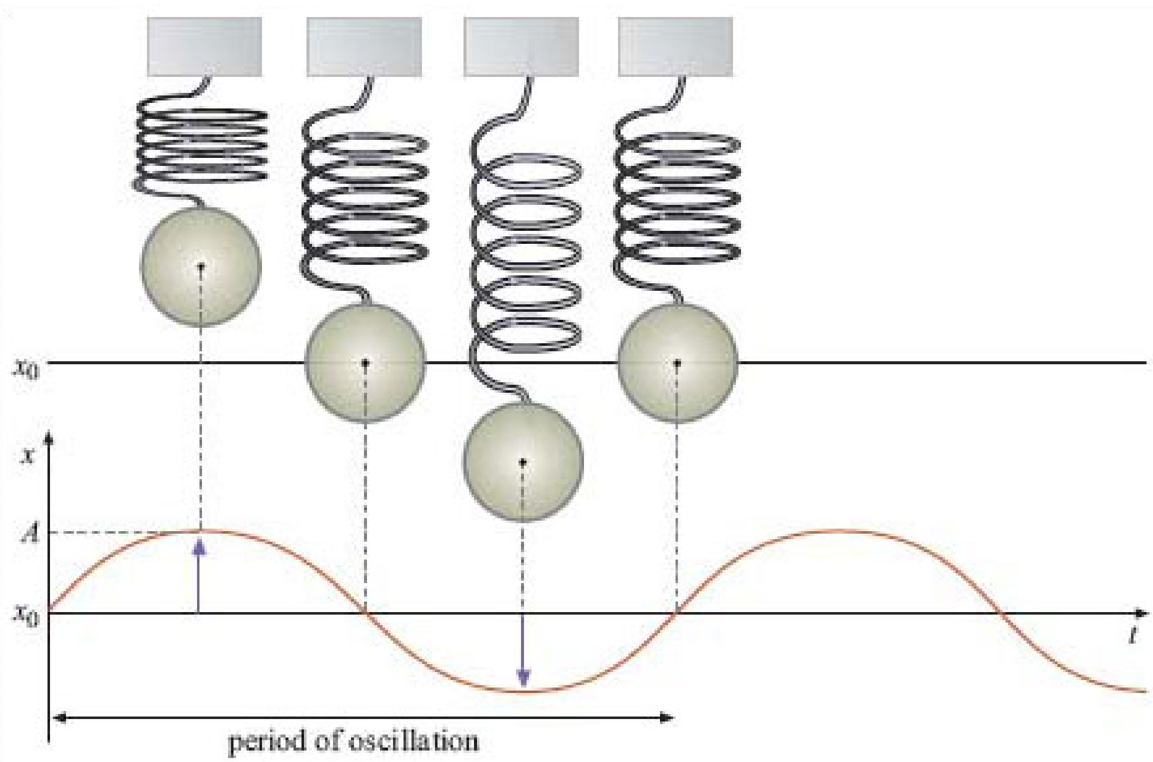
e.



f. From part c. we have an expression for the slope of the line.
Using the slope of the above line gives $M_S = 5.64 \times 10^{26}$ kg

Chapter 7

Oscillations



AP Physics Multiple Choice Practice – Oscillations

1. A mass m, attached to a horizontal massless spring with spring constant k, is set into simple harmonic motion. Its maximum displacement from its equilibrium position is A. What is the mass's speed as it passes through its equilibrium position?

$(A)\ 0 \quad (B)\ A\sqrt{\frac{k}{m}} \quad (C)\ A\sqrt{\frac{m}{k}} \quad (D)\ \frac{1}{A}\sqrt{\frac{k}{m}} \quad (E)\ \frac{1}{A}\sqrt{\frac{m}{k}}$

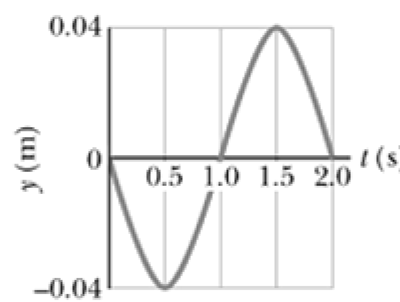
2. The period of a spring-mass system undergoing simple harmonic motion is T. If the amplitude of the spring-mass system's motion is doubled, the period will be:
(A) ¼ T (B) ½ T (C) T (D) 2T (E) 4T

3. A simple pendulum of mass m and length L has a period of oscillation T at angular amplitude θ = 5° measured from its equilibrium position. If the amplitude is changed to 10° and everything else remains constant, the new period of the pendulum would be approximately.
(A) 2T (B) (√2) T (C) T (D) T / (√2) (E) T / 2

4. A mass m is attached to a spring with a spring constant k. If the mass is set into simple harmonic motion by a displacement d from its equilibrium position, what would be the speed, v, of the mass when it returns to the equilibrium position?

$(A)\ v = \sqrt{\frac{kd}{m}} \quad (B)\ v^2 = \frac{kd}{m} \quad (C)\ v = \frac{kd}{mg} \quad (D)\ v^2 = \frac{mgd}{k} \quad (E)\ v = d\sqrt{\frac{k}{m}}$

5. A mass on the end of a spring oscillates with the displacement vs. time graph shown. Which of the following statements about its motion is INCORRECT?
(A) The amplitude of the oscillation is 0.08 m.
(B) The frequency of oscillation is 0.5 Hz.
(C) The mass achieves a maximum in speed at 1 sec.
(D) The period of oscillation is 2 sec.
(E) The mass experiences a maximum in the size of the acceleration at t=1.5 sec



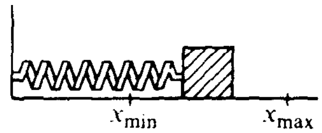
6. What is the period of a simple pendulum if the cord length is 67 cm and the pendulum bob has a mass of 2.4 kg.
(A) 0.259 s (B) 1.63 s (C) 3.86 s (D) 16.3 s (E) 24.3 s

7. If the mass of a simple pendulum is doubled but its length remains constant, its period is multiplied by a factor of

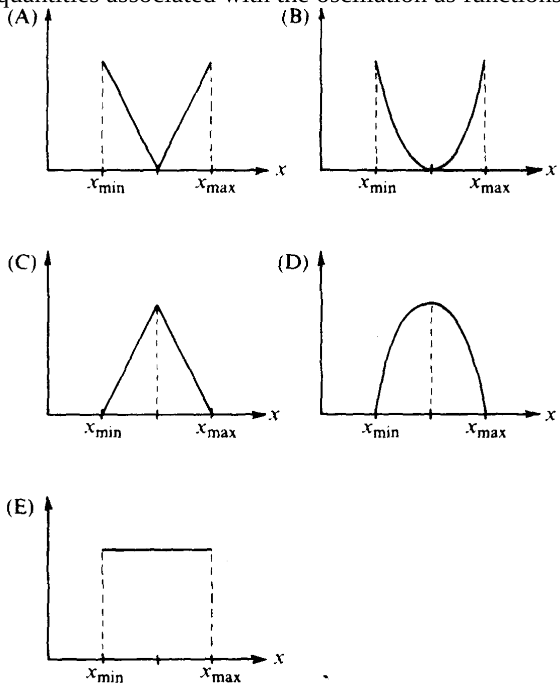
(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 1 (D) $\sqrt{2}$ (E) 2

8. Which of the following is true for a system consisting of a mass oscillating on the end of an ideal spring?
(A) The kinetic and potential energies are equal to each other at all times.
(B) The kinetic and potential energies are both constant.
(C) The maximum potential energy is achieved when the mass passes through its equilibrium position.
(D) The maximum kinetic energy and maximum potential energy are equal, but occur at different times.
(E) The maximum kinetic energy occurs at maximum displacement of the mass from its equilibrium position

9. The length of a simple pendulum with a period on Earth of one second is most nearly
(A) 0.12 m (B) 0.25 m (C) 0.50 m (D) 1.0 m (E) 10.0 m



Questions 10-11: A block oscillates without friction on the end of a spring as shown above. The minimum and maximum lengths of the spring as it oscillates are, respectively, x_{min} and x_{max}. The graphs below can represent quantities associated with the oscillation as functions of the length x of the spring.



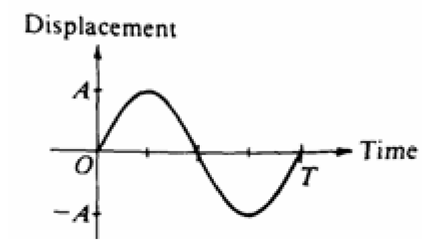
10. Which graph can represent the total mechanical energy of the block-spring system as a function of x ?
    (A) A (B) B (C) C (D) D (E) E

11. Which graph can represent the kinetic energy of the block as a function of x ?
    (A) A (B) B (C) C (D) D (E) E

12. An object swings on the end of a cord as a simple pendulum with period T. Another object oscillates up and down on the end of a vertical spring also with period T. If the masses of both objects are doubled, what are the new values for the Periods?

	Pendulum	Mass on Spring
(A)	$T/\sqrt{2}$	$T\sqrt{2}$
(B)	T	$T\sqrt{2}$
(C)	T	T
(D)	$T\sqrt{2}$	T
(E)	$T\sqrt{2}$	$T/\sqrt{2}$

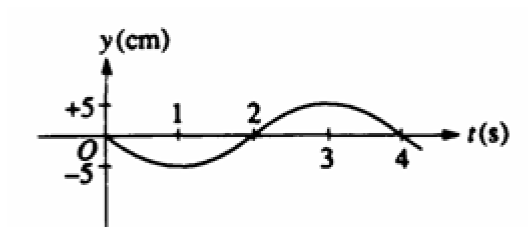
13. An object is attached to a spring and oscillates with amplitude A and period T, as represented on the graph. The nature of the velocity v and acceleration a of the object at time T/4 is best represented by which of the following?
(A) v > 0, a > 0 (B) v > 0, a < 0 (C) v > 0, a = 0
(D) v = 0, a < 0 (E) v = 0, a = 0



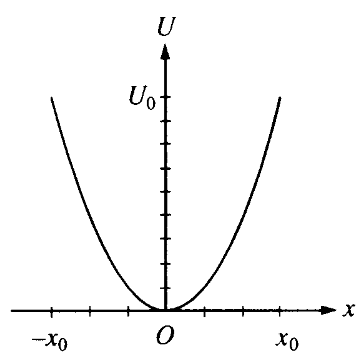
14. When an object oscillating in simple harmonic motion is at its maximum displacement from the equilibrium position. Which of the following is true of the values of its speed and the magnitude of the restoring force?

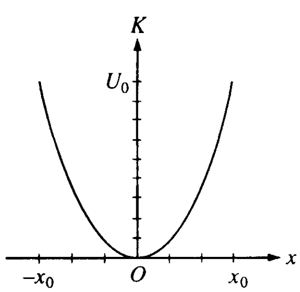
Speed	Restoring Force
(A) Zero	Maximum
(B) Zero	Zero
(C) ½ maximum	½ maximum
(D) Maximum	½ maximum
(E) Maximum	Zero

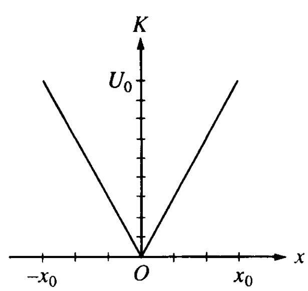
15. A particle oscillates up and down in simple harmonic motion. Its height y as a function of time t is shown in the diagram. At what time t does the particle achieve its maximum positive acceleration?
(A) 1 s (B) 2 s (C) 3 s (D) 4 s
(E) None of the above, because the acceleration is constant

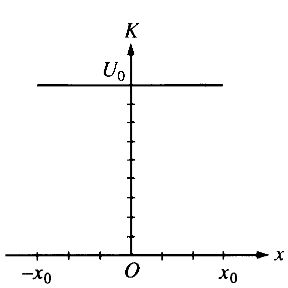


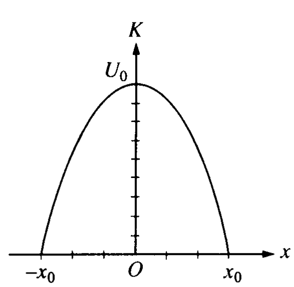
16. The graph shown represents the potential energy U as a function of displacement x for an object on the end of a spring oscillating in simple harmonic motion with amplitude x_o. Which of the following graphs represents the kinetic energy K of the object as a function of displacement x ?

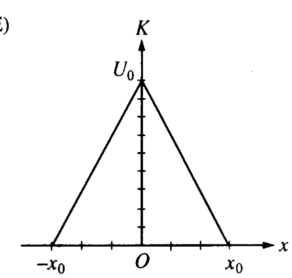


(A)


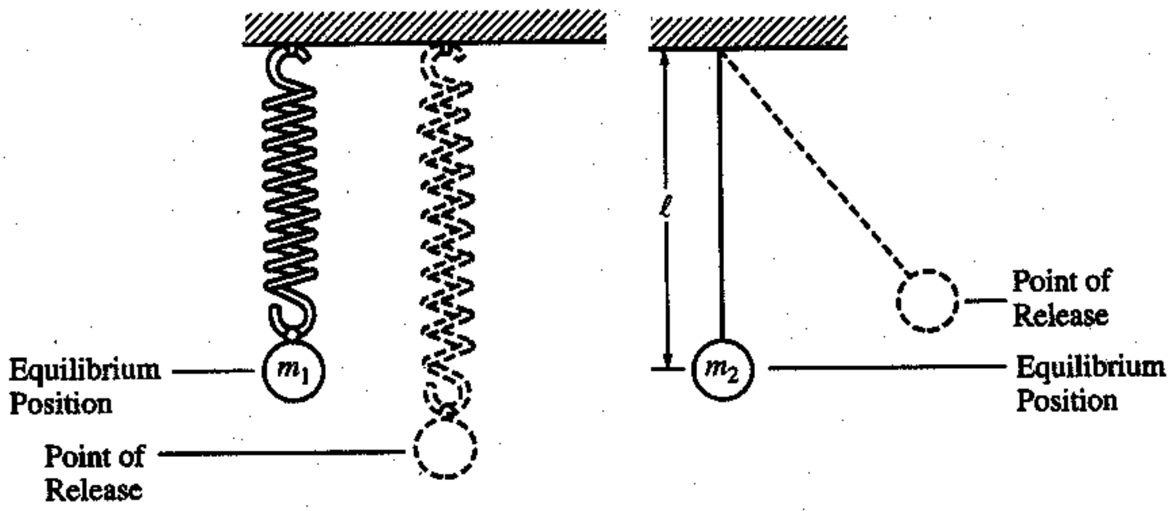
(B)


(C)


(D)


(E)


Questions 17-18



A sphere of mass m_1, which is attached to a spring, is displaced downward from its equilibrium position as shown above left and released from rest. A sphere of mass m_2, which is suspended from a string of length L, is displaced to the right as shown above right and released from rest so that it swings as a simple pendulum with small amplitude. Assume that both spheres undergo simple harmonic motion

17. Which of the following is true for both spheres?
(A) The maximum kinetic energy is attained as the sphere passes through its equilibrium position
(B) The maximum kinetic energy is attained as the sphere reaches its point of release.
(C) The minimum gravitational potential energy is attained as the sphere passes through its equilibrium position.
(D) The maximum gravitational potential energy is attained when the sphere reaches its point of release.
(E) The maximum total energy is attained only as the sphere passes through its equilibrium position.

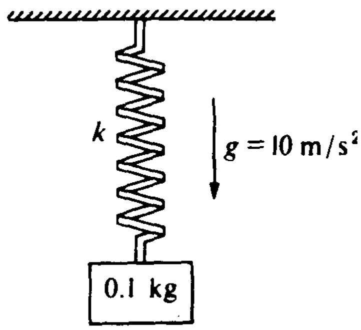
18. If both spheres have the same period of oscillation, which of the following is an expression for the spring constant
(A) $L / m_1 g$
(B) $g / m_2 L$
(C) $m_1 L / g$
(D) $m_2 g / L$
(E) $m_1 g / L$

19. A block attached to the lower end of a vertical spring oscillates up and down. If the spring obeys Hooke's law, the period of oscillation depends on which of the following?
I. Mass of the block
II. Amplitude of the oscillation
III. Force constant of the spring
(A) I only
(B) II only
(C) III only
(D) I and II
(E) I and III

20. A simple pendulum and a mass hanging on a spring both have a period of 1 s when set into small oscillatory motion on Earth. They are taken to Planet X, which has the same diameter as Earth but twice the mass. Which of the following statements is true about the periods of the two objects on Planet X compared to their periods on Earth?
(A) Both are shorter.
(B) Both are the same.
(C) Both are longer.
(D) The period of the mass on the spring is shorter; that of the pendulum is the same.
(E) The period of the pendulum is shorter; that of the mass on the spring is the same

21. A simple pendulum of length *l*, whose bob has mass m, oscillates with a period T. If the bob is replaced by one of mass 4m, the period of oscillation is

(A) $\frac{1}{4}$ T (B) $\frac{1}{2}$ T (C) T (D) 2T (E)4T



Questions 22-23
A 0.l -kilogram block is attached to an initially unstretched spring of force constant k = 40 newtons per meter as shown above. The block is released from rest at time t = 0.

22. What is the amplitude, in meters, of the resulting simple harmonic motion of the block?

(A) $\frac{1}{40}m$ (B) $\frac{1}{20}m$ (C) $\frac{1}{4}m$ (D) $\frac{1}{2}m$ (E) $1m$

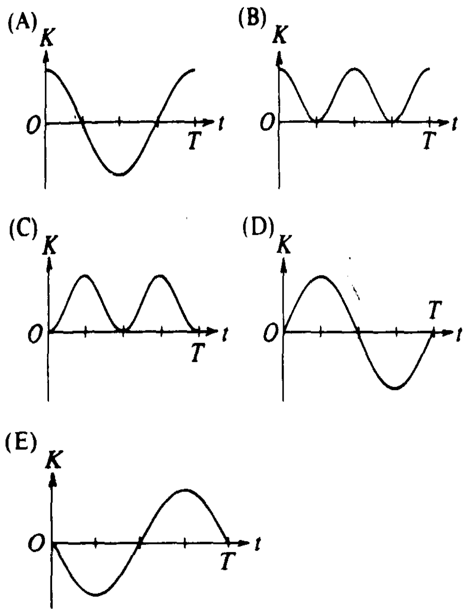
23. What will the resulting period of oscillation be?

(A) $\frac{\pi}{40}s$ (B) $\frac{\pi}{20}s$ (C) $\frac{\pi}{10}s$ (D) $\frac{\pi}{5}s$ (E) $\frac{\pi}{4}s$

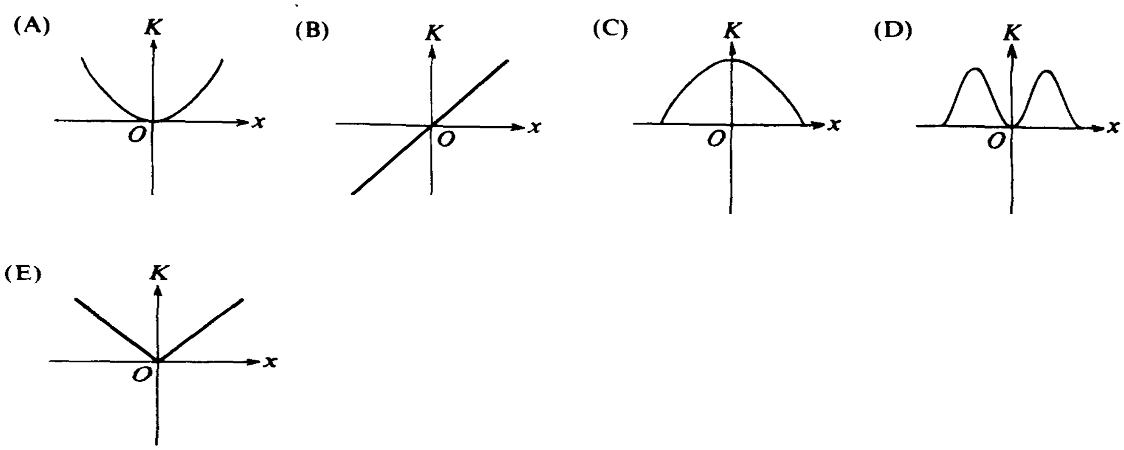
24. A ball is dropped from a height of 10 meters onto a hard surface so that the collision at the surface may be assumed elastic. Under such conditions the motion of the ball is
(A) simple harmonic with a period of about 1.4 s
(B) simple harmonic with a period of about 2.8 s
(C) simple harmonic with an amplitude of 5 m
(D) periodic with a period of about 2.8 s but not simple harmonic
(E) motion with constant momentum

Questions 25-26 refer to the graph below of the displacement x versus time t for a particle in simple harmonic motion.

25. Which of the following graphs shows the kinetic energy K of the particle as a function of time t for one cycle of motion?



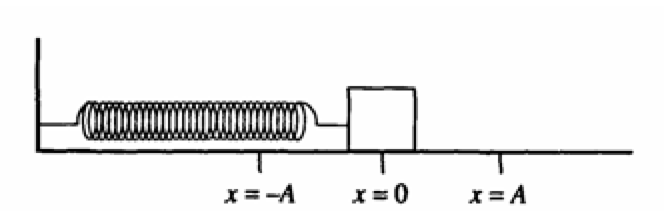
26. Which of the following graphs shows the kinetic energy K of the particle as a function of its displacement x ?



27. A mass m is attached to a vertical spring stretching it distance d. Then, the mass is set oscillating on a spring with an amplitude of A, the period of oscillation is proportional to

(A) $\sqrt{\frac{d}{g}}$ (B) $\sqrt{\frac{g}{d}}$ (C) $\sqrt{\frac{d}{mg}}$ (D) $\sqrt{\frac{m^2 g}{d}}$ (E) $\sqrt{\frac{m}{g}}$

28. Two objects of equal mass hang from independent springs of unequal spring constant and oscillate up and down. The spring of greater spring constant must have the
(A) smaller amplitude of oscillation
(B) larger amplitude of oscillation
(C) shorter period of oscillation
(D) longer period of oscillation
(E) lower frequency of oscillation



Questions 29-30. A block on a horizontal frictionless plane is attached to a spring, as shown above. The block oscillates along the x-axis with simple harmonic motion of amplitude A.

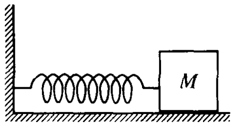
29. Which of the following statements about the block is correct?
(A) At x = 0, its velocity is zero.
(B) At x = 0, its acceleration is at a maximum.
(C) At x = A, its displacement is at a maximum.
(D) At x = A, its velocity is at a maximum.
(E) At x = A, its acceleration is zero.

30. Which of the following statements about energy is correct?
(A) The potential energy of the spring is at a minimum at x = 0.
(B) The potential energy of the spring is at a minimum at x = A.
(C) The kinetic energy of the block is at a minimum at x =0.
(D) The kinetic energy of the block is at a maximum at x = A.
(E) The kinetic energy of the block is always equal to the potential energy of the spring.

31. A simple pendulum consists of a l.0-kilogram brass bob on a string about 1.0 meter long. It has a period of 2.0 seconds. The pendulum would have a period of 1.0 second if the
(A) string were replaced by one about 0.25 meter long
(B) string were replaced by one about 2.0 meters long
(C) bob were replaced by a 0.25-kg brass sphere
(D) bob were replaced by a 4.0-kg brass sphere
(E) amplitude of the motion were increased

32. A pendulum with a period of 1 s on Earth, where the acceleration due to gravity is g, is taken to another planet, where its period is 2 s. The acceleration due to gravity on the other planet is most nearly
(A) g/4 (B) g/2 (C) g (D) 2g (E) 4g

33. A frictionless pendulum of length 3 m swings with an amplitude of 10°. At its maximum displacement, the potential energy of the pendulum is 10 J. What is the kinetic energy of the pendulum when its potential energy is 5 J ?
(A) 3.3 J (B) 5 J (C) 6.7 J (D) 10 J (E) 15 J



34. An ideal massless spring is fixed to the wall at one end, as shown above. A block of mass M attached to the other end of the spring oscillates with amplitude A on a frictionless, horizontal surface. The maximum speed of the block is v_m. The force constant of the spring is

(A) $\frac{Mg}{A}$ (B) $\frac{Mgv_m}{2A}$ (C) $\frac{Mv_m^2}{2A}$ (D) $\frac{Mv_m^2}{A^2}$ (E) $\frac{Mv_m^2}{2A^2}$

35. A simple pendulum has a period of 2 s for small amplitude oscillations. The length of the pendulum is most nearly

(A) 1/6 m (B) 1/4 m (C) 1/2 m (D) 1 m (E) 2 m

36. A mass M suspended by a spring with force constant k has a period T when set into oscillation on Earth. Its period on Mars, whose mass is about 1/9 and radius 1/2 that of Earth, is most nearly

(A) T/3 (B) 2T/3 (C) T (D) 3T/2 (E) 3T

37. A mass M suspended on a string of length L has a period T when set into oscillation on Earth. Its period on Mars, whose mass is about 1/9 and radius 1/2 that of Earth, is most nearly

(A) T/3 (B) 2T/3 (C) T (D) 3T/2 (E) 3T

38. A 1.0 kg mass is attached to the end of a vertical ideal spring with a force constant of 400 N/m. The mass is set in simple harmonic motion with an amplitude of 10 cm. The speed of the 1.0 kg mass at the equilibrium position is

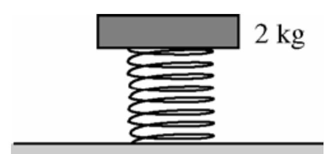
(A) 2 m/s (B) 4 m/s (C) 20 m/s (D) 40 m/s (E) 200 m/s

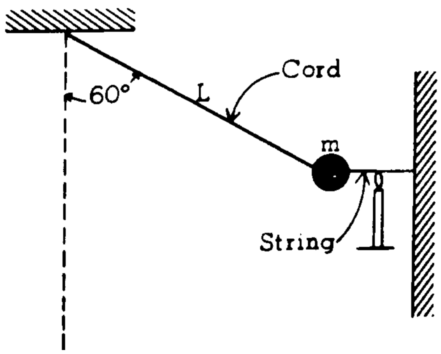
39. An object of mass m hanging from a spring of spring constant k oscillates with a certain frequency. What is the length of a simple pendulum that has the same frequency of oscillation?

(A) mk / g (B) mg / k (C) kg / m (D) k / mg (E) g / mk

40. A platform of mass 2 kg is supported by a spring of negligible mass as shown. The platform oscillates with a period of 3 s when the platform is pushed down and released. What must be the mass of a block that when placed on the platform doubles the period of oscillation to 6 s?

(A) 1 kg (B) 2 kg (C) 4 kg (D) 6 kg (E) 8 kg





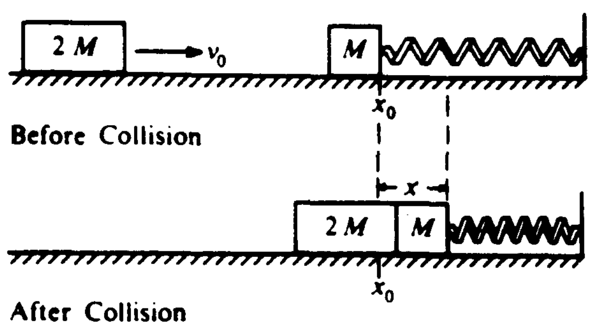
1975B7. A pendulum consists of a small object of mass m fastened to the end of an inextensible cord of length L. Initially, the pendulum is drawn aside through an angle of 60° with the vertical and held by a horizontal string as shown in the diagram above. This string is burned so that the pendulum is released to swing to and fro.

a. In the space below draw a force diagram identifying all of the forces acting on the object while it is held by the string.

b. Determine the tension in the cord before the string is burned.

c. Show that the cord, strong enough to support the object before the string is burned, is also strong enough to support the object as it passes through the bottom of its swing.

d. The motion of the pendulum after the string is burned is periodic. Is it also simple harmonic? Why, or why not?

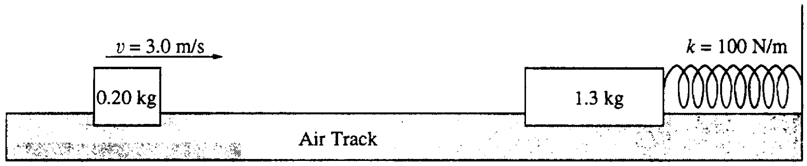


1983B2. A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block Develop expressions for the following quantities in terms of M, k, and v_o

a. v, the speed of the blocks immediately after impact

b. x, the maximum distance the spring is compressed

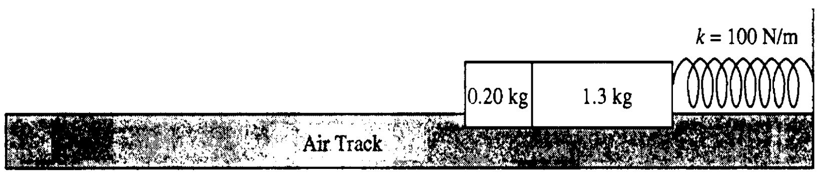
c. T, the period of the subsequent simple harmonic motion



1995B1. As shown above, a 0.20-kilogram mass is sliding on a horizontal, frictionless air track with a speed of 3.0 meters per second when it instantaneously hits and sticks to a 1.3-kilogram mass initially at rest on the track. The 1.3-kilogram mass is connected to one end of a massless spring, which has a spring constant of 100 newtons per meter. The other end of the spring is fixed.

a. Determine the following for the 0.20-kilogram mass immediately before the impact.
   i. Its linear momentum   ii. Its kinetic energy

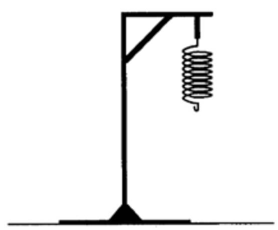
b. Determine the following for the combined masses immediately after the impact.
   i. The linear momentum   ii. The kinetic energy



After the collision, the two masses undergo simple harmonic motion about their position at impact.

c. Determine the amplitude of the harmonic motion.

d. Determine the period of the harmonic motion.



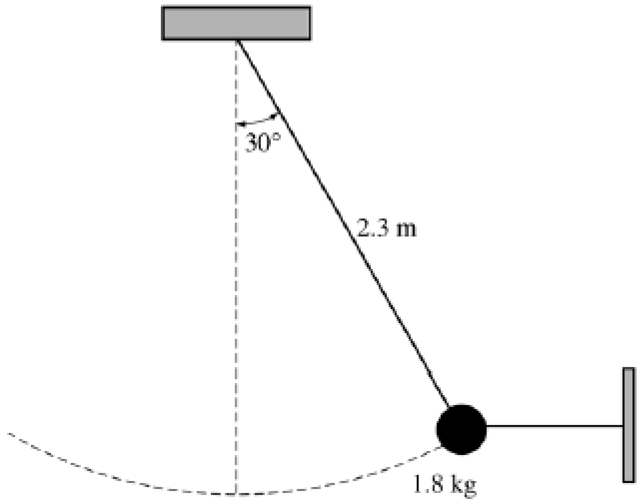
1996B2. A spring that can be assumed to be ideal hangs from a stand, as shown above.

a. You wish to determine experimentally the spring constant k of the spring.
   i. What additional, commonly available equipment would you need?
   ii. What measurements would you make?
   iii. How would k be determined from these measurements?

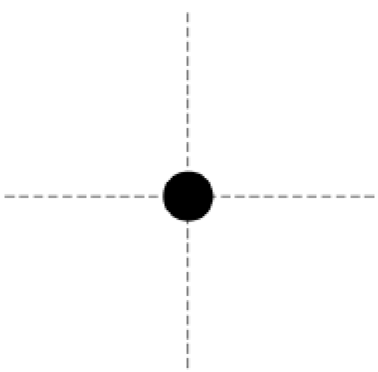
b. Assume that the spring constant is determined to be 500 N/m. A 2.0-kg mass is attached to the lower end of the spring and released from rest. Determine the frequency of oscillation of the mass.

c. Suppose that the spring is now used in a spring scale that is limited to a maximum value of 25 N, but you would like to weigh an object of mass M that weighs more than 25 N. You must use commonly available equipment and the spring scale to determine the weight of the object without breaking the scale.
   i. Draw a clear diagram that shows one way that the equipment you choose could be used with the spring scale to determine the weight of the object,
   ii. Explain how you would make the determination.

2005B2



A simple pendulum consists of a bob of mass 1.8 kg attached to a string of length 2.3 m. The pendulum is held at an angle of 30° from the vertical by a light horizontal string attached to a wall, as shown above.
(a) On the figure below, draw a free-body diagram showing and labeling the forces on the bob in the position shown above.

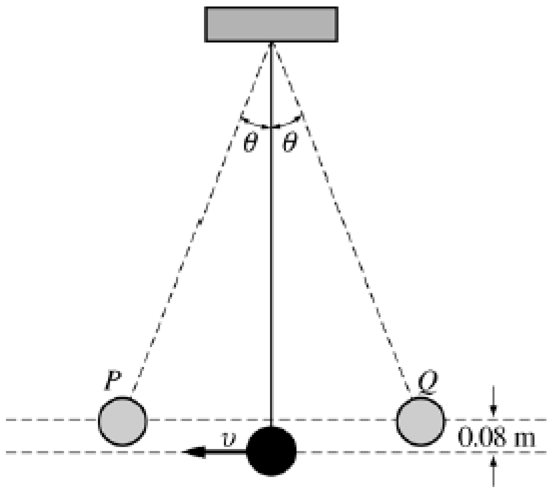


(b) Calculate the tension in the horizontal string.
(c) The horizontal string is now cut close to the bob, and the pendulum swings down. Calculate the speed of the bob at its lowest position.

(d) How long will it take the bob to reach the lowest position for the first time?

2005B2B

A simple pendulum consists of a bob of mass 0.085 kg attached to a string of length 1.5 m. The pendulum is raised to point *Q*, which is 0.08 m above its lowest position, and released so that it oscillates with small amplitude θ between the points *P* and *Q* as shown below.



Note: Figure not drawn to scale.

(a) On the figures below, draw free-body diagrams showing and labeling the forces acting on the bob in each of the situations described.

i. When it is at point P

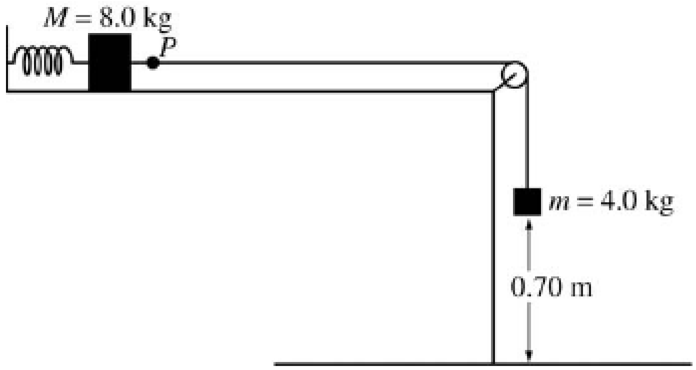
ii. When it is in motion at its lowest position

(b) Calculate the speed v of the bob at its lowest position.

(c) Calculate the tension in the string when the bob is passing through its lowest position.

(d) Describe one modification that could be made to double the period of oscillation.

2006B1



An ideal spring of unstretched length 0.20 m is placed horizontally on a frictionless table as shown above. One end of the spring is fixed and the other end is attached to a block of mass M = 8.0 kg. The 8.0 kg block is also attached to a massless string that passes over a small frictionless pulley. A block of mass m = 4.0 kg hangs from the other end of the string. When this spring-and-blocks system is in equilibrium, the length of the spring is 0.25 m and the 4.0 kg block is 0.70 m above the floor.

(a) On the figures below, draw free-body diagrams showing and labeling the forces on each block when the system is in equilibrium.

M = 8.0 kg m = 4.0 kg

(b) Calculate the tension in the string.

(c) Calculate the force constant of the spring.

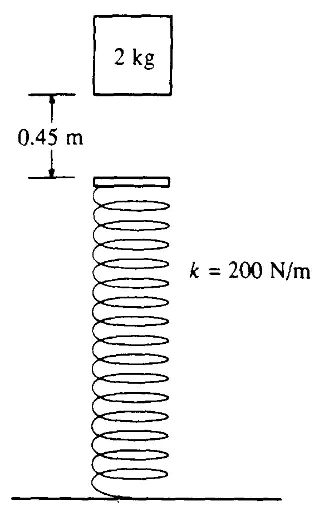
The string is now cut at point P.

(d) Calculate the time taken by the 4.0 kg block to hit the floor.

(e) Calculate the frequency of oscillation of the 8.0 kg block.

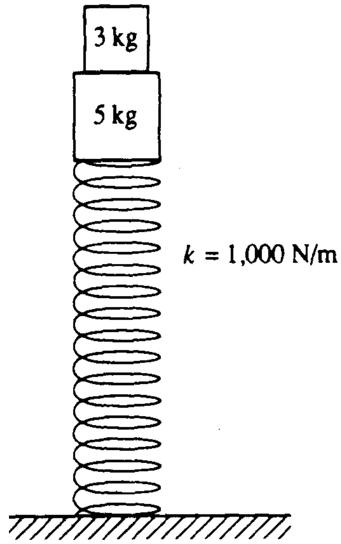
(f) Calculate the maximum speed attained by the 8.0 kg block.

C1989M3. A 2-kilogram block is dropped from a height of 0.45 meter above an uncompressed spring, as shown above. The spring has an elastic constant of 200 newtons per meter and negligible mass. The block strikes the end of the spring and sticks to it.



a. Determine the speed of the block at the instant it hits the end of the spring
b. Determine the force in the spring when the block reaches the equilibrium position
c. Determine the distance that the spring is compressed at the equilibrium position
d. Determine the speed of the block at the equilibrium position
e. Determine the resulting amplitude of the oscillation that ensues
f. Is the speed of the block a maximum at the equilibrium position, explain.
g. Determine the period of the simple harmonic motion that ensues

1990M3. A 5-kilogram block is fastened to an ideal vertical spring that has an unknown spring constant. A 3-kilogram block rests on top of the 5-kilogram block, as shown above.



a. When the blocks are at rest, the spring is compressed to its equilibrium position a distance of Δx_1 = 20 cm, from its original length. Determine the spring constant of the spring

The 3 kg block is then raised 50 cm above the 5 kg block and dropped onto it.

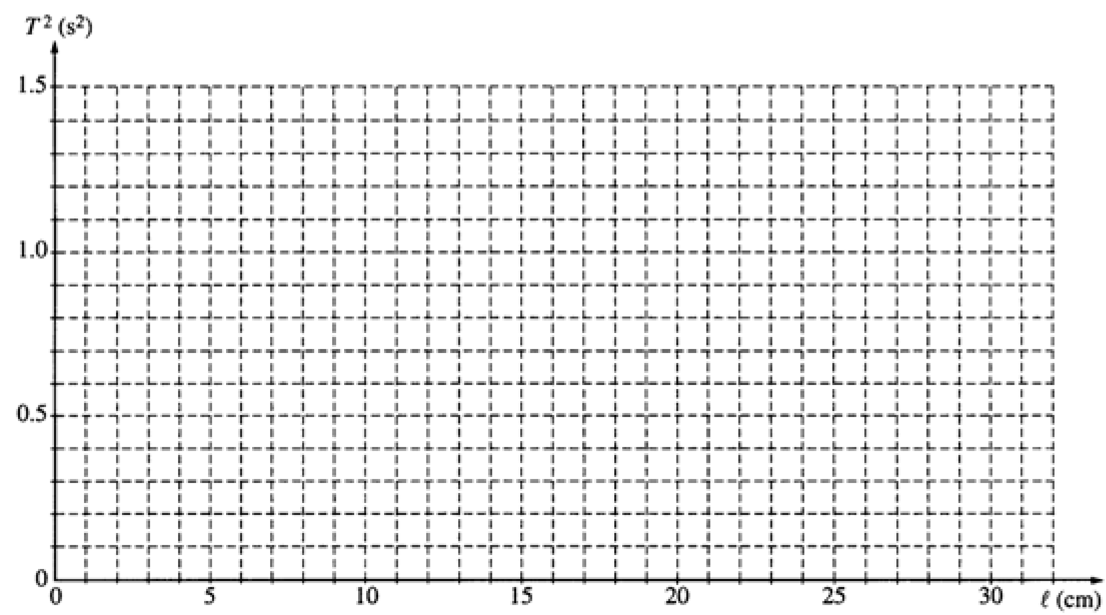
b. Determine the speed of the combined blocks after the collision
c. Setup, plug in known values, but do not solve an equation to determine the amplitude Δx_2 of the resulting oscillation
d. Determine the resulting frequency of this oscillation.
e. Where will the block attain its maximum speed, explain.
f. Is this motion simple harmonic.

(2000 M1) You are conducting an experiment to measure the acceleration due to gravity g_u at an unknown location. In the measurement apparatus, a simple pendulum swings past a photogate located at the pendulum's lowest point, which records the time t_{10} for the pendulum to undergo 10 full oscillations. The pendulum consists of a sphere of mass m at the end of a string and has a length *l*. There are four versions of this apparatus, each with a different length. All four are at the unknown location, and the data shown below are sent to you during the experiment.

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62		
18	8.89		
21	10.09		
32	12.08		

a. For each pendulum, calculate the period T and the square of the period. Use a reasonable number of significant figures. Enter these results in the table above.

b. On the axes below, plot the square of the period versus the length of the pendulum. Draw a best-fit straight line for this data.

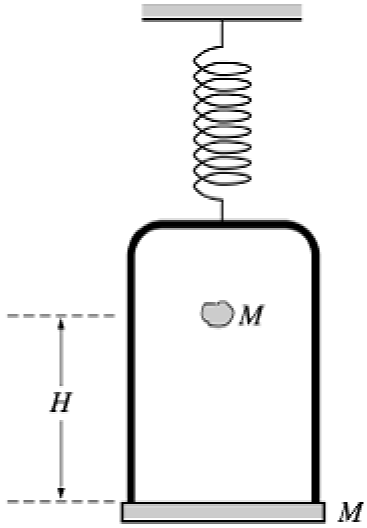


c. Assuming that each pendulum undergoes small amplitude oscillations, from your fit, determine the experimental value g_{exp} of the acceleration due to gravity at this unknown location. Justify your answer.

d. If the measurement apparatus allows a determination of g_u that is accurate to within 4%, is your experimental value in agreement with the value 9.80 m/s^2 ? Justify your answer.

e. Someone informs you that the experimental apparatus is in fact near Earth's surface, but that the experiment has been conducted inside an elevator with a constant acceleration *a*. If the elevator is moving down, determine the direction of the elevator's acceleration, justify your answer.

C2003M2.



An ideal massless spring is hung from the ceiling and a pan suspension of total mass *M* is suspended from the end of the spring. A piece of clay, also of mass *M*, is then dropped from a height *H* onto the pan and sticks to it. Express all algebraic answers in terms of the given quantities and fundamental constants.

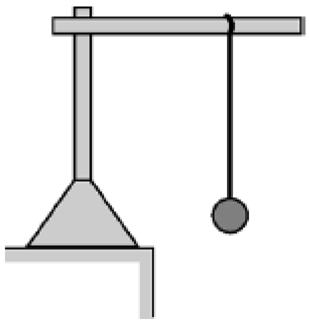
(a) Determine the speed of the clay at the instant it hits the pan.

(b) Determine the speed of the clay and pan just after the clay strikes it.

(c) After the collision, the apparatus comes to rest at a distance H/2 below the current position. Determine the spring constant of the attached spring.

(d) Determine the resulting period of oscillation

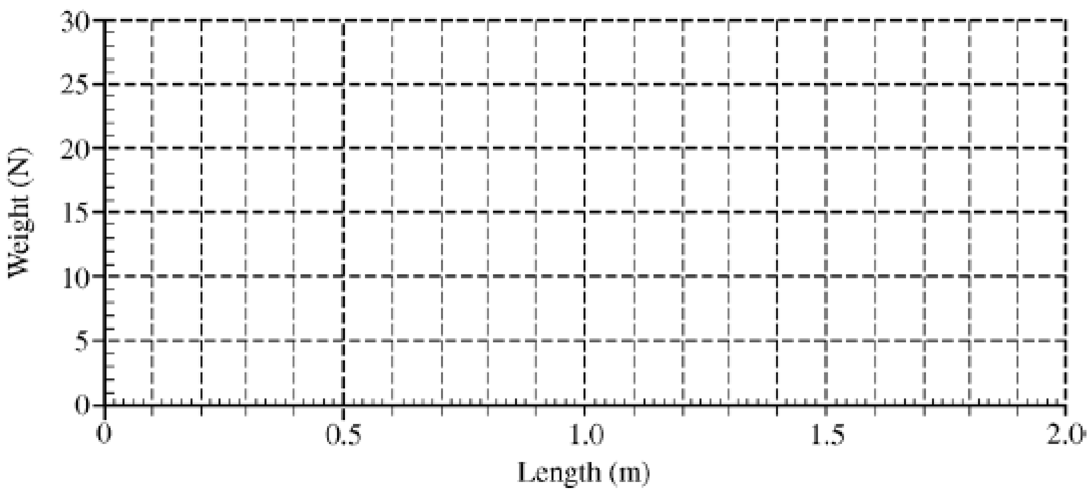
C2008M3



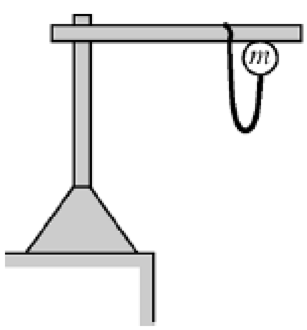
In an experiment to determine the spring constant of an elastic cord of length 0.60 m, a student hangs the cord from a rod as represented above and then attaches a variety of weights to the cord. For each weight, the student allows the weight to hang in equilibrium and then measures the entire length of the cord. The data are recorded in the table below:

Weight (N)	0	10	15	20	25
Length (m)	0.60	0.97	1.24	1.37	1.64

(a) Use the data to plot a graph of weight versus length on the axes below. Sketch a best-fit straight line through the data.



(b) Use the best-fit line you sketched in part (a) to determine an experimental value for the spring constant *k* of the cord.



The student now attaches an object of unknown mass *m* to the cord and holds the object adjacent to the point at which the top of the cord is tied to the rod, as shown. When the object is released from rest, it falls 1.5 m before stopping and turning around. Assume that air resistance is negligible.

(c) Calculate the value of the unknown mass *m* of the object.

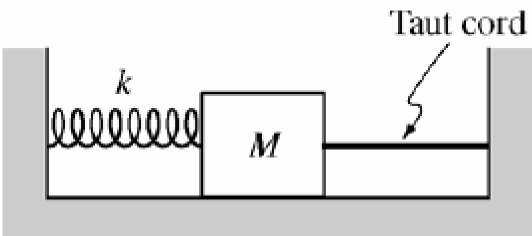
(d) i. Determine the magnitude of the force in the cord when the when the mass reaches the equilibrium position.

ii. Determine the amount the cord has stretched when the mass reaches the equilibrium position.

iii. Calculate the speed of the object at the equilibrium position

iv. Is the speed in part iii above the maximum speed, explain your answer.

Supplemental



One end of a spring of spring constant k is attached to a wall, and the other end is attached to a block of mass M, as shown above. The block is pulled to the right, stretching the spring from its equilibrium position, and is then held in place by a taut cord, the other end of which is attached to the opposite wall. The spring and the cord have negligible mass, and the tension in the cord is F_T . Friction between the block and the surface is negligible. Express all algebraic answers in terms of M, k, F_T , and fundamental constants.

(a) On the dot below that represents the block, draw and label a free-body diagram for the block.

•

(b) Calculate the distance that the spring has been stretched from its equilibrium position.

The cord suddenly breaks so that the block initially moves to the left and then oscillates back and forth.
(c) Calculate the speed of the block when it has moved half the distance from its release point to its equilibrium position.
(d) Calculate the time after the cord breaks until the block first reaches its position furthest to the left.
(e) Suppose instead that friction is not negligible and that the coefficient of kinetic friction between the block and the surface is μ_k . After the cord breaks, the block again initially moves to the left. Calculate the initial acceleration of the block just after the cord breaks.

ANSWERS - AP Physics Multiple Choice Practice – Oscillations

	Solution	Answer
1.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} mv^2$	B
2.	The period of a mass-spring is only affected by mass and k so its stays the same	C
3.	The period of a pendulum is only affected by length and "g" so it stays the same	C
4.	Energy conservation. $U_{sp} = K$ $\frac{1}{2} k d^2 = \frac{1}{2} mv^2$	E
5.	The amplitude from the graph is 0.04 not 0.08, the rest are true	A
6.	The mass is irrelevant, only the length matters. T is found with $2\pi \sqrt{(L/g)}$	B
7.	Mass does not affect the period, only the length matters	C
8.	Energy is conserved here and switches between kinetic and potential which have maximums at different locations	D
9.	Sub into $T = 2\pi \sqrt{(L/g)}$ and solve for L	B
10.	Only conservative forces are acting which means mechanical energy must be conserved so it stays constant as the mass oscillates	E
11.	The box momentarily stops at x(min) and x(max) so must have zero K at these points. The box accelerates the most at the ends of the oscillation since the force is the greatest there. This changing acceleration means that the box gains speed quickly at first but not as quickly as it approaches equilibrium. This means that the KE gain starts of rapidly from the endpoints and gets less rapid as you approach equilibrium where there would be a maximum speed and maximum K, but zero force so less gain in speed. This results in the curved graph.	D
12.	Pendulum is unaffected by mass. Mass-spring system has mass causing the T to change proportional to $\sqrt{m}$ so since the mass is doubled the period is changed by $\sqrt{2}$	B
13.	At T/4 the mass reaches maximum + displacement where the restoring force is at a maximum and pulling in the opposite direction and hence creating a negative acceleration. At maximum displacement the mass stops momentarily and has zero velocity	D
14.	See #13 above	A
15.	+ Acceleration occurs when the mass is at negative displacements since the force will be acting in the opposite direction of the displacement to restore equilibrium. Based on $F=k\Delta x$ the most force, and therefore the most acceleration occurs where the most displacement is	A
16.	As the object oscillates, its total mechanical energy is conserved and transfers from U to K back and forth. The only graph that makes sense to have an equal switch throughout is D	D
17.	For the spring, equilibrium is shown where the maximum transfer of kinetic energy has occurred and likewise for the pendulum the bottom equilibrium position has the maximum transfer of potential energy into spring energy.	A
18.	Set period formulas equal to each other and rearrange for k	E
19.	In a mass-spring system, both mass and spring constant (force constant) affect the period.	E

20. The mass spring system in unaffected because the attached mass and spring constant are the same. Based on $g = GM_p / R^2$ and the given values, g on planet X would be greater. Using the pendulum period formula, larger g means smaller period. E

21. Mass does not affect the period of a pendulum C

22. At the current location all of the energy is gravitational potential. As the spring stretches to its max location all of that gravitational potential will become spring potential when it reaches its lowest position. When the box oscillates back up it will return to its original location converting all of its energy back to gravitational potential and will oscillate back and forth between these two positions. As such the maximum stretch bottom location represents twice the amplitude so simply halving that max Δx will give the amplitude. Finding the max stretch: → The initial height of the box h and the stretch Δx have the same value ($h=\Delta x$) A

$U = U_{sp}$ $mg(\Delta x_1) = ½ k\Delta x_1^2$ $mg = ½ k \Delta x_1$ $\Delta x_1 = .05$ m.
This is 2A, so the amplitude is 0.025 m or 1/40 m.

Alternatively, we could simply find the equilibrium position measured from the initial top position based on the forces at equilibrium, and this equilibrium stretch measured from the top will be the amplitude directly. To do this:
$F_{net} = 0$ $F_{sp} = mg$ $k\Delta x_2 = mg$ $\Delta x_2 = 0.025$ m, which is the amplitude

23. Plug into period for mass-spring system $T = 2\pi \sqrt{(m/k)}$ C

24. Based on free fall, the time to fall down would be 1.4 seconds. Since the collision with the ground is elastic, all of the energy will be returned to the ball and it will rise back up to its initial height completing 1 cycle in a total time of 2.8 seconds. It will continue doing this oscillating up and down. However, this is not simple harmonic because to be simple harmonic the force should vary directly proportional to the displacement but that is not the case in this situation D

25. Energy will never be negative. The max kinetic occurs at zero displacement and the kinetic energy become zero when at the maximum displacement B

26. Same reasoning as above, it must be C C

27. First use the initial stretch to find the spring constant. $F_{sp} = mg = k\Delta x$ $k = mg / d$ A

Then plug that into $T = 2\pi \sqrt{(m/k)}$ $$T = 2\pi\sqrt{\frac{m}{(\frac{mg}{d})}}$$

28. Based on $T = 2\pi \sqrt{(m/k)}$ the larger spring constant makes a smaller period C

29. Basic fact about SHM. Amplitude is max displacement C

30. Basic fact about SHM. Spring potential energy is a min at x=0 with no spring stretch A

31. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ the length equates to ½ the period A

32. Based on $T = 2\pi \sqrt{(L/g)}$, ¼ g would double the period A

33. At max displacement, the total energy is equal to the potential energy of 10J. Energy is conserved so when the K becomes 5J the U would have to be 5J also to conserve E. B

34. Using energy conservation. $U_{sp} = K$ $½ k A^2 = ½ mv_m^2$ solve for k D

35. Plug into $T = 2\pi \sqrt{(L/g)}$ D

36. Since this is a mass spring, only the attached mass and k affect the period and both are the same. C

37. Based on $g = GM_p / R^2$, g of mars is 4/9 that of earth. Then based on $T = 2\pi \sqrt{(L/g)}$, with g changing to "4/9 g" gives a period changing by $\sqrt{9/4}$ or 3/2 T D

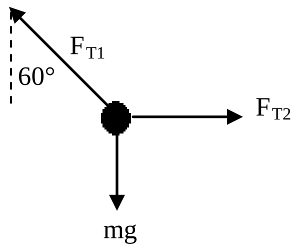
38. Using energy conservation. $U_{sp} = K$ $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$ solve for v A

39. Since the frequencies are the same, the periods are also the same. Set the period for the mass-spring system equal to the period for the string pendulum and rearrange for L. B

40. Based on $T = 2\pi \sqrt{(m/k)}$, in order to double the period, the mass would have to be increased by 4x the original amount. Here is the tricky part …. you are to increase the mass to 4 x its original value by adding mass the to 2kg tray. So to make the total mass have a value of 8 kg, only 6 kg of extra mass would need to be added to the tray. D

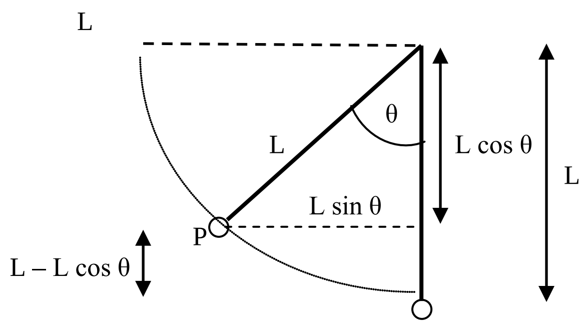
1975B7.

(a)



(b) $F_{NET(Y)} = 0$
$F_{T1} \cos \theta = mg$
$F_{T1} = mg / \cos(60) = 2mg$

(c) When the string is cut it swings from top to bottom, similar to the diagram for 1974B1 from work-energy problems with θ on the opposite side as shown below



$U_{top} = K_{bot}$
$mgh = ½ mv^2$

$$v = \sqrt{2g(L - L\cos 60)}$$

$$v = \sqrt{2g(L - \frac{L}{2})}$$

$$v = \sqrt{gL}$$

Then apply $F_{NET(C)} = mv^2 / r$

$(F_{T1} - mg) = m(gL) / L$

$F_{T1} = 2mg$. Since it's the same force as before, it will be possible.

(d) This motion is not simple harmonic because the restoring force, $(F_{gx}) = mg \sin \theta$, is not directly proportional to the displacement due to the sin function. For small angles of θ the motion is approximately SHM, though not exactly, but in this example the larger value of θ creates and even larger disparity.

1983B2.

a) Apply momentum conservation perfect inelastic. $p_{before} = p_{after}$ $2Mv_o = (3M)v_f$ $v_f = 2/3\ v_o$

b) Apply energy conservation. $K = U_{sp}$ $½\ (3M)(2/3\ v_o)^2 = ½\ k\ \Delta x^2$ $\sqrt{\frac{4Mv_o^{\ 2}}{3k}}$

c) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3m}{k}}$

1995 B1.

a) i) $p = mv$ $= (0.2)(3) = 0.6$ kg m/s
ii) $K = ½\ mv^2$ $= ½\ (0.2)(3)^2 = 0.9$ J

b) i.) Apply momentum conservation $p_{before} = p_{after} = 0.6$ kg m/s
ii) First find the velocity after, using the momentum above
$0.6 = (1.3+0.2)\ v_f$ $v_f = 0.4$ m/s, then find K, $K = ½\ (m_1+m_2)\ v_f^{\ 2} = ½\ (1.3+0.2)(0.4)^2 = 0.12$ J

c) Apply energy conservation $K = U_{sp}$ $0.12\ J = ½\ k\Delta x^2 = ½\ (100)\ \Delta x^2$ $\Delta x = 0.05$ m

d) Period is given by $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.5}{100}} = 0.77s$

1996B2.

a) Use a ruler and known mass. Hang the known mass on the spring and measure the stretch distanceΔx. The force pulling the spring F_{sp} is equal to the weight (mg). Plug into $F_{sp} = k\ \Delta x$ and solve for k

b) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{500}} = 0.4s$

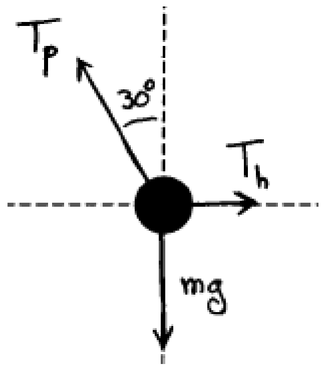
… then the frequency is given by $f = 1/T = 2.5$ Hz

c) Put the spring and mass on an incline and tilt it until it slips and measure the angle. Use this to find the coefficient of static friction on the incline $u_s = \tan\theta$. Then put the spring and mass on a horizontal surface and pull it until it slips. Based on $F_{net} = 0$, we have $F_{spring} - \mu_s$ mg, Giving $mg = F_{spring} / \mu$.
Since μ is most commonly less than 1 this will allow an mg value to be registered larger than the spring force.

A simpler solution would be to put the block and spring in water. The upwards buoyant force will allow for a weight to be larger than the spring force. This will be covered in the fluid mechanics unit.

2005B2.

a) FBD



b) Apply $F_{net(X)} = 0$ $\quad$ $F_{net(Y)} = 0$

$T_P \cos 30 = mg$ $\quad$ $T_P \sin 30 = T_H$

$T_P = 20.37$ N $\quad$ $T_H = 10.18$ N

c) Conservation of energy – Diagram similar to 1975B7.

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$

$g(L – L \cos \theta) = ½\, v^2$

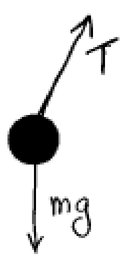
$(10)\,(2.3 – 2.3 \cos 30) = ½\, v^2$ $\quad$ $v_{bottom} = 2.5$ m/s

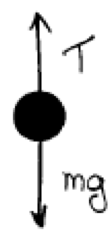
d) The bob will reach the lowest position in ¼ of the period.

$$T = \frac{1}{4}\left(2\pi\sqrt{\frac{L}{g}}\right) = \frac{\pi}{2}\sqrt{\frac{2.3}{9.8}} = 0.76\ s$$

B2005B2.

FBD $\quad$ i) $\quad$ ii)





b) Apply energy conservation?

$U_{top} = K_{bottom}$

$mgh = ½\, m\, v^2$ $\quad$ $(9.8)(.08) – ½\, v^2$ $\quad$ $v = 1.3$ m/s

c) $F_{net(c)} = mv^2/r$

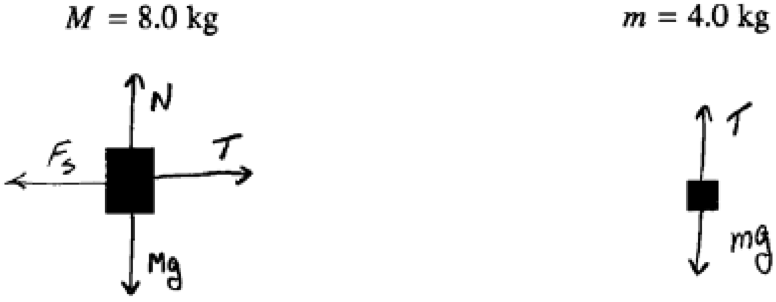
$F_t – mg = mv^2/r$ $\quad$ $F_t = mv^2/r + mg$ $\quad$ $(0.085)(1.3)^2/(1.5) + (0.085)(9.8)$ $\quad$ $F_t = 0.93$ N

d) "g" and "L" are the two factors that determine the pendulum period based on $\left(T = 2\pi\sqrt{\frac{L}{g}}\right)$

To double the value of T, L should be increased by 4x or g should be decreased by ¼. The easiest modification would be simply to increase the length by 4 x

2006B1.

a) FBD



b) Simply isolating the 4 kg mass at rest. $F_{net} = 0$ $F_t - mg = 0$ $F_t = 39$ N

c) Tension in the string is uniform throughout, now looking at the 8 kg mass,
$F_{sp} = F_t = k\Delta x$ $39 = k\,(0.05)$ $k = 780$ N/m

d) 4 kg mass is in free fall. $D = v_i t + \frac{1}{2} g t^2$ $-0.7 = 0 + \frac{1}{2}(-9.8)t^2$ $t = 0.38$ sec

e) First find the period. $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{780}} = 0.63s$

… then the frequency is given by $f = 1/T = 1.6$ Hz

f) The 8 kg block will be pulled towards the wall and will reach a maximum speed when it passes the relaxed length of the spring. At this point all of the initial stored potential energy is converted to kinetic energy
$U_{sp} = K$ $\frac{1}{2} k \Delta x^2 = \frac{1}{2} mv^2$ $\frac{1}{2}(780)(0.05)^2 = \frac{1}{2}(8)v^2$ $v = 0.49$ m/s

---

C1989M3.

a) Apply energy conservation from top to end of spring using h=0 as end of spring.
$U = K$ $mgh = \frac{1}{2} m v^2$ $(9.8)(0.45) = \frac{1}{2} v^2$ $v = 3$ m/s

b) At equilibrium the forces are balanced $F_{net} = 0$ $F_{sp} = mg = (2)(9.8) = 19.6$ N

c) Using the force from part b, $F_{sp} = k\,\Delta x$ $19.6 = 200\,\Delta x$ $\Delta x = 0.098$ m

d) Apply energy conservation using the equilibrium position as h = 0. (Note that the height at the top position is now increased by the amount of Δx found in part c $h_{new} = h + \Delta x = 0.45 + 0.098 = 0.548$ m
$U_{top} = U_{sp} + K_{\text{(at equil)}}$
$mgh_{new} = \frac{1}{2} k \Delta x^2 + \frac{1}{2} mv^2$ $(2)(9.8)(0.548) = \frac{1}{2}(200)(0.098)^2 + \frac{1}{2}(2)(v^2)$ $v = 3.13$ m/s

e) Use the turn horizontal trick. Set equilibrium position as zero spring energy then solve it as a horizontal problem where $K_{equil} = U_{sp\text{(at max amp.)}}$ $\frac{1}{2} mv^2 = \frac{1}{2} k\Delta x^2$ $\frac{1}{2}(2)(3.13)^2 = \frac{1}{2}(200)(A^2)$ $A = 0.313$ m

f) This is the maximum speed because this was the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the box down until it reaches its maximum compression and stops momentarily.

g) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = 0.63s$

C1990M3.

a) Equilibrium so $F_{net} = 0$, $F_{sp} = mg$ $k\Delta x = mg$ $k(0.20) = (8)(9.8)$ $k = 392$ N/m

b) First determine the speed of the 3 kg block prior to impact using energy conservation

$U = K$ $mgh = ½ m v^2$ $(9.8)(0.50) = ½ v^2$ $v = 3.13$ m/s

Then solve perfect inelastic collision. $p_{before} = p_{after}$ $m_1 v_{1i} = (m_1+m_2) v_f$ $(3)(3.13)=(8)v_f$ $v_f = 1.17$ m/s

c) Since we do not know the speed at equilibrium nor do we know the amplitude Δx_2 the turn horizontal trick would not work initially. If you first solve for the speed at equilibrium as was done in 1989M3 first, you could then use the turn horizontal trick. However, since this question is simply looking for an equation to be solved, we will use energy conservation from the top position to the lowest position where the max amplitude is reached. For these two positions, the total distance traveled is equal to the distance traveled to equilibrium + the distance traveled to the max compression $(\Delta x_1 + \Delta x_2) = (0.20 + \Delta x_2)$ which will serve as both the initial height as well as the total compression distance. We separate it this way because the distance traveled to the maximum compression from equilibrium is the resulting amplitude Δx_2 that the question is asking for.

Apply energy conservation

$U_{top} + K_{top} = U_{sp(max\text{-}comp)}$

$mgh + ½ m v^2 = ½ k \Delta x_2^2$ $(8)(9.8)(0.20 + \Delta x_2) + ½ (8)(1.17)^2 = ½ (392)(0.20 + \Delta x_2)^2$

The solution of this quadratic would lead to the answer for Δx_2 which is the amplitude.

d) First find period $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{8}{392}} = 0.90s$ Then find frequency f = 1/T = 1.11 Hz

e) The maximum speed will occur at equilibrium because the net force is zero here and the blocks stop accelerating in the direction of motion momentarily. Past this point, an upwards net force begins to exist which will slow the blocks down as they approach maximum compressions and begin to oscillate.

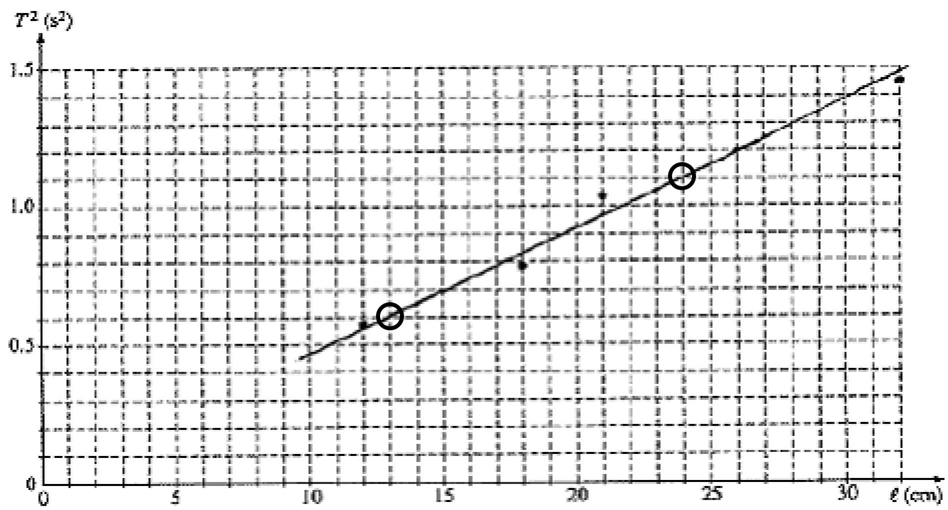
f) This motion is simple harmonic because the force acting on the masses is given by $F=k\Delta x$ and is therefore directly proportional to the displacement meeting the definition of simple harmonic motion

C2000M1.

a)

ℓ (cm)	t_{10} (s)	T (s)	T^2 (s^2)
12	7.62	0.762	0.581
18	8.89	0.889	0.790
21	10.09	1.009	1.018
32	12.08	1.208	1.459

b)



c) We want a linear equation of the form y = mx.

Based on $T = 2\pi\sqrt{\frac{L}{g}}$ $\quad T^2 = 2^2\pi^2\frac{L}{g}$ $\quad T^2 = \frac{4\pi^2}{g} L$

y = m x

This fits our graph with y being T^2 and x being L. Finding the slope of the line will give us a value that we can equate to the slope term above and solve it for g. Since the points don't fall on the line we pick random points as shown circled on the graph and find the slope to be = 4.55. Set this = to $4\pi^2$ / g and solve for g = 8.69 m/s^2

d) A +/- 4% deviation of the answer (8.69) puts its possible range in between 8.944 – 8.34 so this result does not agree with the given value 9.8

e) Since the value of g is less than it would normally be (you feel lighter) the elevator moving down would also need to be **accelerating down** to create a lighter feeling and smaller F_n. Using down as the positive direction we have the following relationship, $F_{net} = ma$ $\quad mg - F_n = ma$ $\quad F_n = mg - ma$
For F_n the be smaller than usual, a would have to be + which we defined as down.

C2003M2.

a) Apply energy conservation $U_{top} = K_{bot}$ mgh = ½ mv^2 $v = \sqrt{2gH}$

b) Apply momentum conservation perfect inelastic $p_{before} = p_{after}$

$Mv_{ai} = (M+M)\, v_f$ $M(\sqrt{2gH}) = 2Mv_f$ $v_f = \frac{1}{2}\sqrt{2gH}$

c) Again we cannot use the turn horizontal trick because we do not know information at the equilibrium position. While the tray was initially at its equilibrium position, its collision with the clay changed where this location would be.

Even though the initial current rest position immediately after the collision has an unknown initial stretch to begin with due to the weight of the tray and contains spring energy, we can set this as the zero spring energy position and use the additional stretch distance H/2 given to equate the conversion of kinetic and gravitational energy after the collision into the additional spring energy gained at the end of stretch.

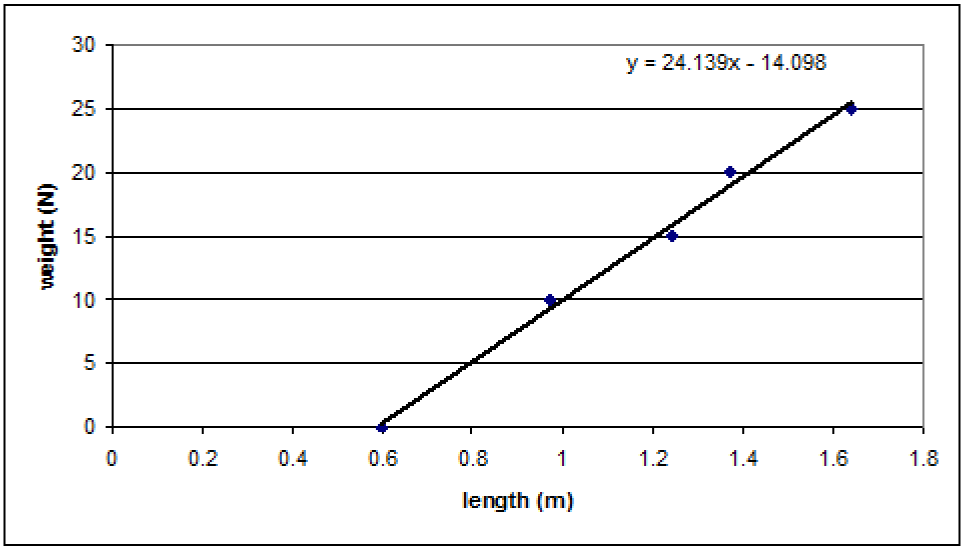
Apply energy conservation $K + U = U_{sp\ (gained)}$ ½ mv^2 + mgh = ½ k Δx^2
Plug in mass (2m), h = H/2 and Δx = H/2 ➔ ½ $(2m)v^2$ + (2m)g(H/2) = ½ $k(H/2)^2$
plug in v_f from part b m(2gH/4) + mgH = $kH^2/8$ ….

Both sides * (1/H) ➔ mg/2 + mg = kH/8 ➔ 3/2 mg = kH/8 k = 12mg / H

d) Based on $T = 2\pi\sqrt{\frac{2M}{\frac{12Mg}{H}}} = 2\pi\sqrt{\frac{H}{6g}}$

C2008M3

(a)



(b) The slope of the line is F / Δx which is the spring constant. Slope = 24 N/m

(c) Apply energy conservation. $U_{top} = U_{sp}$(bottom).
Note that the spring stretch is the final distance – the initial length of the spring. 1.5 – 0.6 = 0.90 m

$mgh = \frac{1}{2} k \Delta x^2$ $m(9.8)(1.5) = \frac{1}{2}(24)(0.9)^2$ m = 0.66 kg

(d) i) At equilibrium, the net force on the mass is zero so $F_{sp} = mg$ $F_{sp} = (0.66)(9.8)$ $F_{sp} = 6.5$ N

ii) $F_{sp} = k \Delta x$ $6.5 = (24) \Delta x$ $\Delta x = 0.27$ m

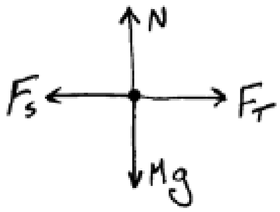
iii) Measured from the starting position of the mass, the equilibrium position would be located at the location marked by the unstretched cord length + the stretch found above. 0.6+0.27 = 0.87m. Set this as the h=0 location and equate the U_{top} to the U_{sp} + K here.

$mgh = \frac{1}{2} k \Delta x^2 + 1/2\, mv^2$ $(0.66)(9.8)(0.87) = \frac{1}{2}(24)(0.27)^2 + \frac{1}{2}(0.66) v^2$ v = 3.8 m/s

iv) This is the maximum speed because this is the point when the spring force and weight were equal to each other and the acceleration was zero. Past this point, the spring force will increase above the value of gravity causing an upwards acceleration which will slow the mass down until it reaches its maximum compression and stops momentarily.

Supplemental.

(a)



(b) $F_{net} = 0$ $F_t = F_{sp} = k\Delta x$ $\Delta x = F_t / k$

(c) Using energy conservation $U_{sp} = U_{sp} + K$ note that the second postion has both K and U_{sp} since the spring still has stretch to it.

$\frac{1}{2} k \Delta x^2 = \frac{1}{2} k\Delta x_2^{\ 2} + \frac{1}{2} mv^2$

$k (\Delta x)^2 = k(\Delta x/2)^2 + Mv^2$

$\frac{3}{4} k(\Delta x)^2 = Mv^2$, plug in Δx from (b) … $\frac{3}{4} k(F_t/k)^2 = Mv^2$ $$v = \frac{F_t}{2}\sqrt{\frac{3}{kM}}$$

(d) To reach the position from the far left will take ½ of a period of oscillation.

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad t = \frac{1}{2}2\pi\sqrt{\frac{M}{k}} \qquad = \pi\sqrt{\frac{M}{k}}$$

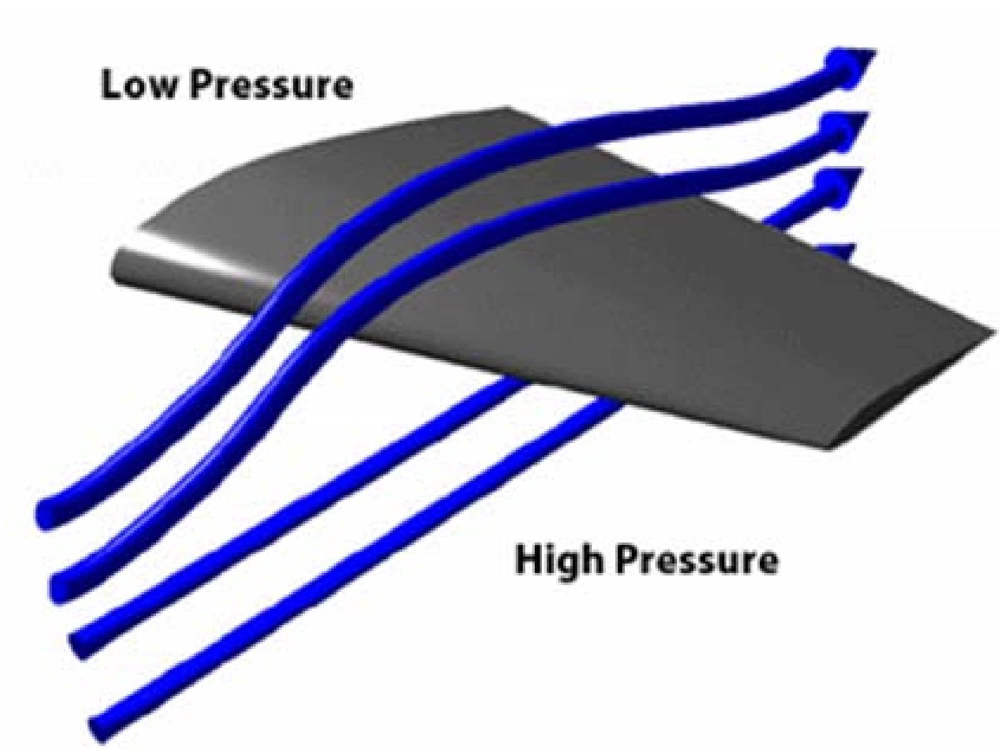
(e) The forces acting on the block in the x direction are the spring force and the friction force. Using left as + we get

$F_{net} = ma$ $F_{sp} - f_k = ma$

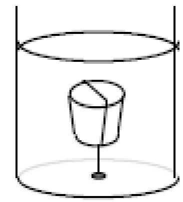
From (b) we know that the initial value of F_{sp} is equal to F_t which is an acceptable variable so we simply plug in F_t for F_{sp} to get $F_t - \mu_k mg = ma$ ➔ $a = F_t / m - \mu_k g$

Chapter 8

Fluid Mechanics



AP Physics Multiple Choice Practice – Fluid Mechanics



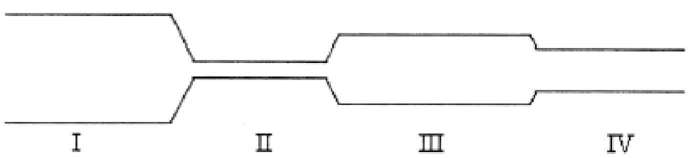
1. A cork has weight mg and density 25% of water density. A string is tied around the cork and attached to the bottom of a water-filled container. The cork is totally immersed. Express in terms of the cork weight mg, the tension in the string
   A) 0
   B) mg
   C) 2mg
   D) 3mg
   E) 4mg

2. Which of the following is the best statement of Pascals Law?
   A) pressure on a confined liquid is transmitted equally in all directions
   B) a numerical arrangement where each number is the sum of the two numbers above
   C) two electrons cannot occupy the same quantum state at the same time
   D) the volume of a gas is directly related to its temperature
   E) the farther away a galaxy is the faster it is receding

3. When submerged under water, the apparent mass of one cubic meter of pure gold is 18300 kg. What would be its mass in air?
   A) 16300 kg B) 17300 kg C) 18300 kg D) 19300 kg E) 20300 kg

4. An ideal fluid flows through a long horizontal circular pipe. In one region of the pipe, it has radius R. The pipe then widens to radius 2R. What is the ratio of the fluids speed in the region of radius R to the speed of the fluid in region with radius 2R
   A) ¼ B) ½ C) 1 D) 2 E) 4

5. A fluid is forced through a pipe of changing cross section as shown. In which section would the pressure of the fluid be a minimum



   A) I B) II C) III D) IV E) all section have the same pressure.

6. Three fishing bobbers all float on top of water. They have the following relationships:
   -A,B: same mass, same density, different shapes
   -B,C: same size, same shape,
   mass & density C < mass & density B
   Three identical weights are tied to each bob, and each is pulled completely beneath the water. Which bob will displace the greatest amount of water
   A) A
   B) B
   C) C
   D) A and B
   E) All displace the same amount of water.

7. A hydraulic press allows large masses to be lifted with small forces as a result of which principle?
   A) Pascal's
   B) Bernoulli's
   C) Archimedes'
   D) Huygens'
   E) Newton's

8. A 500 N weight sits on the small piston of a hydraulic machine. The small piston has an area of 2 cm^2 . If the large piston has an area of 40 cm^2, how much weight can the large piston support?
A) 25 N
B) 500 N
C) 10000 N
D) 40000 N

9. As a rock sinks deeper and deeper into water of constant density, what happens to the buoyant force on it?
A) It increases.
B) It remains constant.
C) It decreases.
D) It may increase or decrease, depending on the shape of the rock.

10. 50 cm^3 of wood is floating on water, and 50 cm^3 of iron is totally submerged. Which has the greater buoyant force on it?
A) The wood.
B) The iron.
C) Both have the same buoyant force.
D) Cannot be determined without knowing their densities.

11. Salt water is more dense than fresh water. A ship floats in both fresh water and salt water. Compared to the fresh water, the amount of water displaced in the salt water is
A) more.
B) less.
C) the same.
D) Cannot be determined from the information given.

12. A liquid has a specific gravity of 0.357. What is its density?
A) 357 kg/m^3 B) 643 kg/m^3 C) 1000 kg/m^3 D) 3570 kg/m^3

13. Water flows through a pipe. The diameter of the pipe at point B is larger than at point A. Where is the water pressure greater?
A) Point A
B) Point B
C) Same at both A and B
D) Cannot be determined from the information given.

14. Liquid flows through a 4 cm diameter pipe at 1.0 m/s. There is a 2 cm diameter restriction in the line. What is the velocity in this restriction?
A) 0.25 m/s B) 0.50 m/s C) 2 m/s D) 4 m/s

15. A copper block is connected to a string and submerged in a container of water.
Position 1: The copper is completely submerged, but just under the surface of the water.
Position 2: The copper is completely submerged, mid-way between the water surface and the bottom of the container.
Position 3: The copper is completely submerged, but just above the bottom surface of the container.

Assume that the water is incompressible. What is the ranking of the buoyant forces (*B*) acting on the copper blocks for these positions, from least to greater?
(A) $B_1 < B_2 < B_3$
(B) $B_3 < B_2 < B_1$
(C) $B_1 = B_2 = B_3$
(D) $B_1 < B_2 = B_3$
(E) $B_3 < B_1 = B_2$

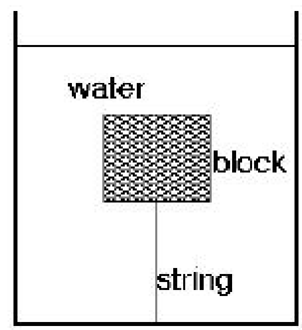
16. Two objects labeled K and L have equal mass but densities $0.95D_o$ and D_o, respectively. Each of these objects floats after being thrown into a deep swimming pool. Which is true about the buoyant forces acting on these objects?
(A) The buoyant force is greater on Object K since it has a lower density and displaces more water.
(B) The buoyant force is greater on Object K since it has lower density and lower density objects always float "higher" in the fluid.
(C) The buoyant force is greater on Object L since it is denser than K and therefore "heavier."
(D) The buoyant forces are equal on the objects since they have equal mass.
(E) Without knowing the specific gravity of the objects, nothing can be determined.

17. A driveway is 22.0 m long and 5.0 m wide. If the atmospheric pressure is 1.0×10^5 Pa, How much force does the atmosphere exert on the driveway?
(A) 9.09×10^{-8} N
(B) 1.1×10^{-3} N
(C) 909 N
(D) 4545 N
(E) 1.1×10^7 N

18. Which of the following could be a correct unit for pressure?

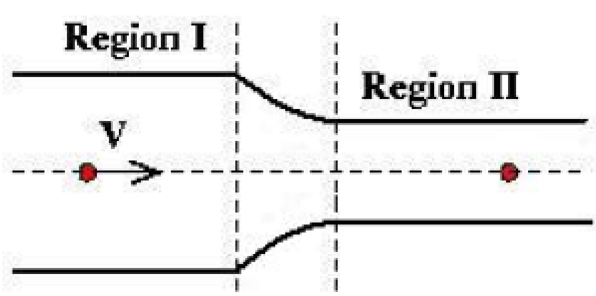
$(A)\ \frac{kg}{m^2} \qquad (B)\ \frac{kg}{m \cdot s} \qquad (C)\ \frac{kg}{s^2} \qquad (D)\ \frac{kg}{m \cdot s^2} \qquad (E)\ \frac{m \cdot s}{kg}$

19. A block is connected to a light string attached to the bottom of a large container of water. The tension in the string is 3.0 N. The gravitational force from the earth on the block is 5.0 N. What is the block's volume?
(A) $2.0 \times 10^{-4}\ m^3$
(B) $3.0 \times 10^{-4}\ m^3$
(C) $5.0 \times 10^{-4}\ m^3$
(D) $8.0 \times 10^{-4}\ m^3$
(E) $1.0 \times 10^{-3}\ m^3$



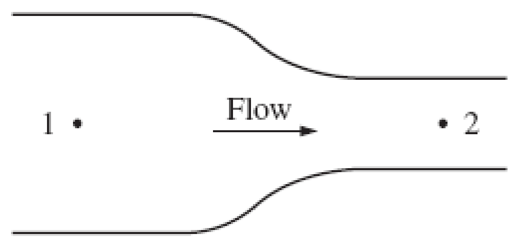
20. A cube of unknown material and uniform density floats in a container of water with 60% of its volume submerged. If this same cube were placed in a container of oil with density 800 kg/m^3, what portion of the cube's volume would be submerged while floating?
(A) 33% (B) 50% (C) 58% (D) 67% (E) 75%

21. A piece of an ideal fluid is marked as it moves along a horizontal streamline through a pipe, as shown in the figure. In Region I, the speed of the fluid on the streamline is V. The cylindrical, horizontal pipe narrows so that the radius of the pipe in Region II is half of what it was in Region I. What is the speed of the marked fluid when it is in Region II?



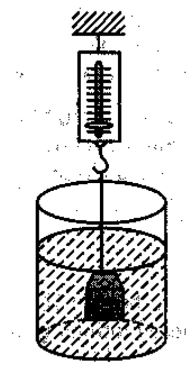
(A) 4V (B) 2V (C) V (D) V/2 (E) V/4

22. A fluid flows steadily from left to right in the pipe shown. The diameter of the pipe is less at point 2 then at point 1, and the fluid density is constant throughout the pipe. How do the velocity of flow and the pressure at points 1 and 2 compare?



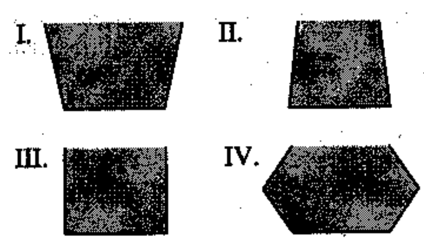
Velocity	Pressure
(A) $v1 < v2$	$p1 = p2$
(B) $v1 < v2$	$p1 > p2$
(C) $v1 = v2$	$p1 < p2$
(D) $v1 > v2$	$p1 = p2$
(E) $v1 > v2$	$p1 > p2$

23. The figure shows an object of mass 0.4 kg that is suspended from a scale and submerged in a liquid. If the reading on the scale is 3 N, then the buoyant force that the fluid exerts on the object is most nearly



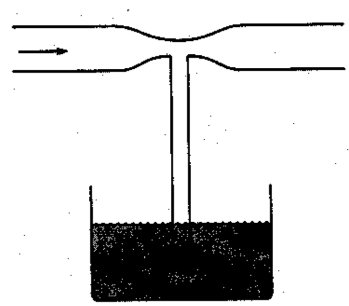
(A) 1.3 N
(B) 1.0N
(C) 0.75 N
(D) 0.33 N
(E) 0.25 N

24. Each of the beakers shown is filled to the same depth h with liquid of density ρ. The area A of the flat bottom is the same for each beaker. Which of the following ranks the beakers according to the net downward force exerted by the liquid on the flat bottom, from greatest to least force?



(A) I, III, II, IV
(B) I, IV, III, II
(C) II, III, IV, I
(D) IV, III, I, II
(E) None of the above, the force on each is the same.

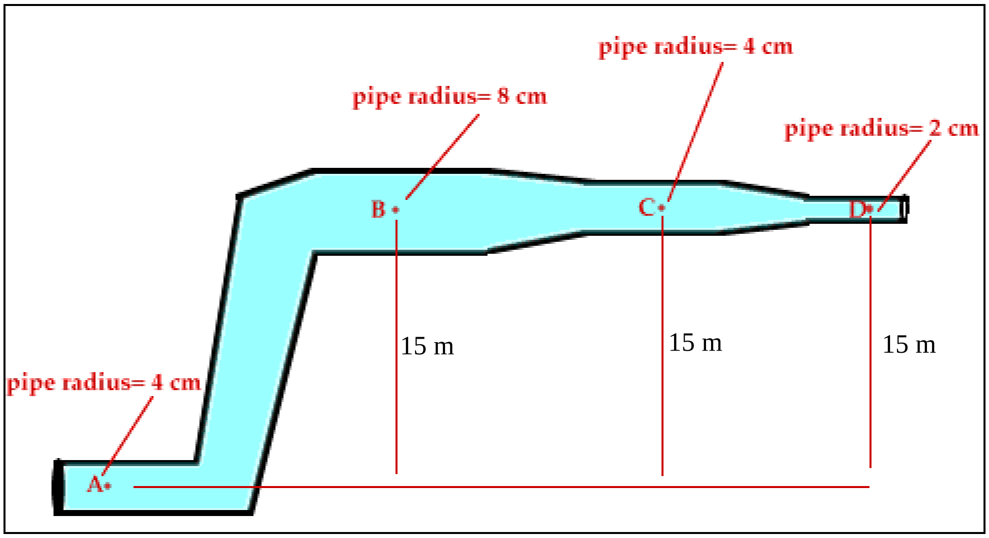
25. A T-shaped tube with a constriction is inserted in a vessel containing a liquid, as shown. What happens if air is blown through the tube from the left, as shown by the arrow in the diagram?



(A) The liquid level in the tube rises to a level above the surface of the liquid in the surrounding tube
(B) The liquid level in the tube falls below the level of the surrounding liquid
(C) The liquid level in the tube remains where it is
(D) The air bubbles out at the bottom of the tube.
(E) Any of the above depending on how hard the air flows.

26. A spring scale calibrated in kilograms is used to determine the density of a rock specimen. The reading on the spring scale is 0.45 kg when the specimen is suspended in air and 0.36 kg when the specimen is fully submerged in water. If the density of water is 1000 kg/m^3, the density of the rock specimen is
(A) 2.0×10^2 kg/m^3 (B) 8.0×10^2 kg/m^3 (C) 1.25×10^3 kg/m^3 (D) 4.0×10^3 kg/m^3 (E) 5.0×10^3 kg/m^3

Questions 27-28: Refer to the diagram below and use 10 m/s^2 for *g* and 100,000 N/m^2 for 1 atm.



27. The pressure at A is 9.5 atm and the water velocity is 10 m/s. What is the water velocity at point C?
(a) 2.5 m/s (b) 5 m/s (c) 10 m/s (d) 20 m/s (e) 40 m/s

28. The pressure at C is
(a) 0 N/m^2 (b) 100,000 N/m^2 (c) 150,000 N/m^2 (d) 800,000 N/m^2 (e) 1,100,000 N/m^2

29. One cubic centimeter of iron (density ~7.8 g/cm^3) and 1 cubic centimeter of aluminum (density ~2.7 g/cm^3) are dropped into a pool. Which has the largest buoyant force on it?
(a) iron (b) aluminum (c) both are the same. (d) neither has a buoyant force on it.

30. One kilogram of iron (density ~7.8 g/cm^3) and 1 kilogram of aluminum (density ~2.7 g/cm^3) are dropped into a pool. Which has the largest buoyant force on it?
(a) iron (b) aluminum (c) both are the same. (d) neither has a buoyant force on it.

31. Find the approximate minimum mass needed for a spherical ball with a 40 cm radius to sink in a liquid of density 1.4×10^3 kg/m^3
(a) 37.5 kg (b) 375 kg (c) 3750 kg (d) 37500 kg (e) 375000 kg

32. What vertical percentage of a 0.25 m deep sheet of ice, whose density is 0.95×10^3 kg/m^3, will be visible in an ocean whose density is 1.1×10^3 kg/m^3
(a) 14% (b) 34% (c) 58% (d) 71% (e) 87%

33. The idea that the velocity of a fluid is high when pressure is low and that the velocity of a fluid is low when the pressure is high embodies a principle attributed to
(a) Torricelli (b) Pascal (c) Galileo (d) Archimedes (e) Bernoulli

34. The mass of a 1.3 m^3 object with a specific gravity of 0.82 is
(a) 630 kg (b) 730 kg (c) 820 kg (d) 1100 kg (e) 1600 kg

35. The apparent weight of a 600 kg object of volume 0.375 m^3 submerged in a liquid of density 1.25×10^3 kg/m^3 is
(a) 180 N (b) 250 N (c) 480 N (d) 1300 N (e) 4700 N

36. A conduit of radius 7R carries a uniformly dense liquid to a spigot of radius R at the same height, where it has a velocity of V. What is its initial velocity
(a) 0.02V (b) 0.11V (c) V (d) 7V (e) 49V

37. The pressure in a pipe carrying a liquid with a density of ρ and an initial velocity v at the inlet is P, which is y meters lower than its outlet, which has a velocity of 2v. In these terms, what is the final pressure?

$(A)\ \frac{P}{2}\rho\left(3v^2+2gy\right)$

$(B)\ P-\frac{1}{2}\rho\left(3v^2+2gy\right)$

$(C)\ P+\frac{1}{2}\rho\left(-3v^2+\rho gy\right)$

$(D)\ \frac{\frac{1}{2}\rho\left(v^2-4v^2\right)-\rho gy}{P}$

$(E)\ P\left[\frac{1}{2}\rho\left(v^2-4v^2\right)-\rho gy\right]$

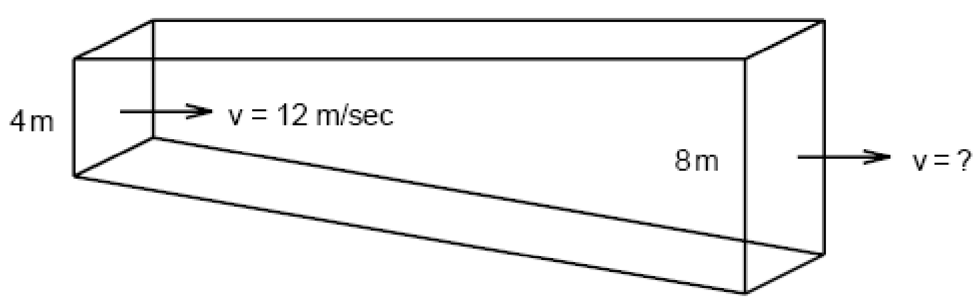
38. The units of specific gravity are
(a) kg/m^3 (b) g/m^3 (c) m/s^2 (d) N/m (e) none of the above

39. The buoyant force on an object is equal to the weight of the water displaced by a submerged object. This is a principle attributed to
(a) Torricelli (b) Pascal (c) Galileo (d) Archimedes (e) Bernoulli

40. If the gauge pressure of a device reads $2.026\times10^5\ N/m^2$, the absolute pressure it is measuring is
(a) $1.013 \times 10^5\ N/m^2$
(b) $2.052 \times 10^5\ N/m^2$
(c) $2.026 \times 10^5\ N/m^2$
(d) $3.039 \times 10^5\ N/m^2$
(e) $6.078 \times 10^5\ N/m^2$

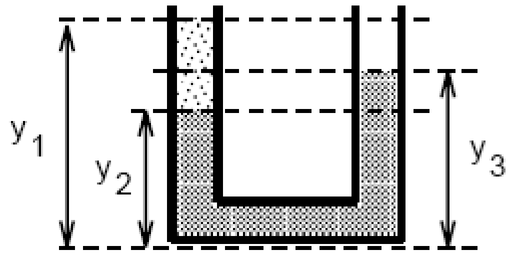
41. A block of mass m, density ρ_B, and volume V is completely submerged in a liquid of density ρ_L. The density of the block is greater than the density of the liquid. The block
(a) floats, because $\rho_B > \rho_L$
(b) experiences a buoyant force equal to ρ_B gV.
(c) experiences a buoyant force equal to ρ_L gV.
(d) experiences a buoyant force equal to m_B g
(e) does not experience any buoyant force, because $\rho_B > \rho_L$.

42. A river gradually deepens, from a depth of 4 m to a depth of 8 m as shown. The width, W, of the river does not change. At the depth of 4 m, the river's speed is 12 m/sec. Its velocity at the 8 m depth is



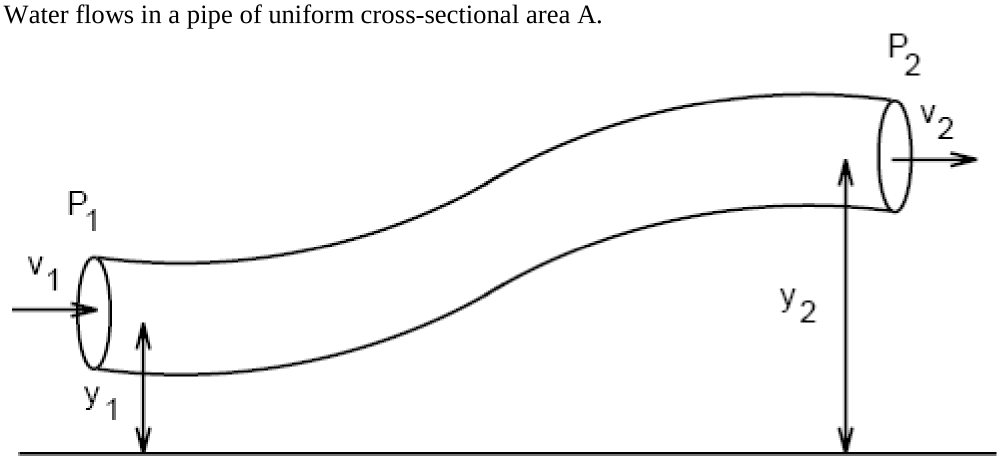
(a) 12 m/sec (b) 24 m/sec (c) 6 m/sec (d) 8 m/sec (e) 16 m/sec

43. In the open manometer shown, water occupies a part of the left arm, from a height of y_1 to a height of y_2. The remainder of the left arm, the bottom of the tube, and the right arm to a height of y_3 are filled with mercury.



Which of the following is correct?
(a) the pressure at a height y_3 is the same in both arms.
(b) the pressure at a height y_2 is the same in both arms.
(c) the pressure at the bottom of the right arm is greater than at the bottom of the left arm.
(d) the pressure at a height y_3 is less in the left arm than in the right arm.
(e) the pressure at a height y_1 is greater in the left arm than in the right arm.

44. Water flows in a pipe of uniform cross-sectional area A.

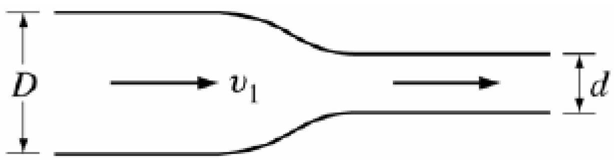


The pipe changes height from y_1 = 2 meters to y_2 = 3 meters. Since the areas are the same, we can say $v_1 = v_2$. Which of the following is true?
(a) $P_1 = P_2 + \rho g(y_2 - y_1)$
(b) $P_1 = P_2$
(c) $P_1 = 0$
(d) $P_2 = 0$
(e) $\rho_1 > \rho_2$

45. A vertical force of 30 N is applied uniformly to a flat button with a radius of 1 cm that is lying on a table. Which of the following is the best order of magnitude estimate for the pressure applied to the button?
(A) 10 Pa
(B) 10^2 Pa
(C) 10^3 Pa
(D) 10^4 Pa
(E) 10^5 Pa

46. A ball that can float on water has mass 5.00 kg and volume 2.50×10^{-2} m^3. What is the magnitude of the downward force that must be applied to the ball to hold it motionless and completely submerged in freshwater of density 1.00×10^3 kg/m^3?
(A) 20.0 N
(B) 25.0 N
(C) 30.0 N
(D) 200 N
(E) 250 N

47. Water flows through the pipe shown. At the larger end, the pipe has diameter D and the speed of the water is v_1 .



What is the speed of the water at the smaller end, where the pipe has diameter d ?

$(A)\ v_1$ $\quad (B)\ \frac{d}{D}v_1$ $\quad (C)\ \frac{D}{d}v_1$ $\quad (D)\ \frac{d^2}{D^2}v_1$ $\quad (E)\ \frac{D^2}{d^2}v_1$

AP Physics Free Response Practice – Fluids

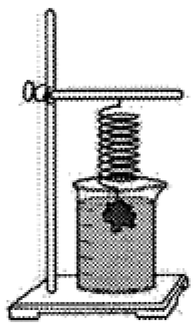
2002B6.

In the laboratory, you are given a cylindrical beaker containing a fluid and you are asked to determine the density ρ of the fluid. You are to use a spring of negligible mass and unknown spring constant k attached to a stand. An irregularly shaped object of known mass *m* and density *D* (D >> ρ) hangs from the spring. You may also choose from among the following items to complete the task.

- A metric ruler
- A stopwatch
- String

(a) Explain how you could experimentally determine the spring constant *k*.

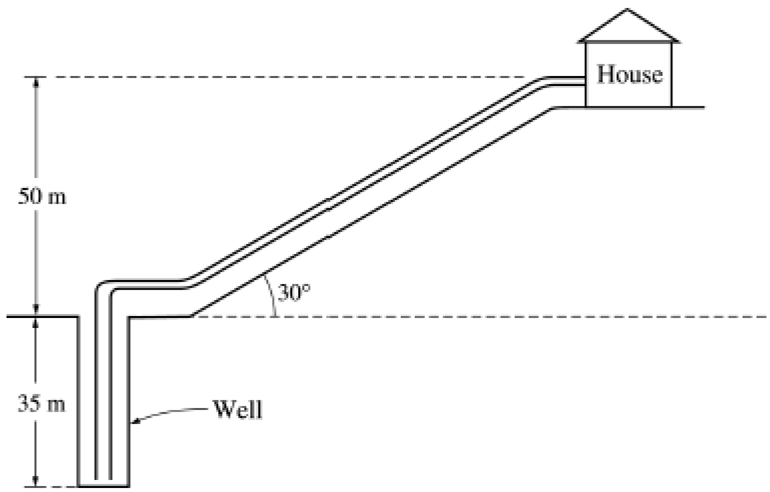
(b) The spring-object system is now arranged so that the object (but none of the spring) is immersed in the unknown fluid, as shown. Describe any changes that are observed in the spring-object system and explain why they occur.



(c) Explain how you could experimentally determine the density of the fluid.

(d) Show explicitly, using equations, how you will use your measurements to calculate the fluid density ρ. Start by identifying any symbols you use in your equations.

Symbol	Physical quantity



B2003B6.

A pump, submerged at the bottom of a well that is 35 m deep, is used to pump water uphill to a house that is 50 m above the top of the well, as shown above. The density of water is 1,000 kg/m^3. Neglect the effects of friction, turbulence, and viscosity.

(a) Residents of the house use 0.35 m^3 of water per day. The day's pumping is completed in 2 hours during the day.

i. Calculate the minimum work required to pump the water used per day

ii. Calculate the minimum power rating of the pump.

(b) In the well, the water flows at 0.50 m/s and the pipe has a diameter of 3.0 cm. At the house the diameter of the pipe is 1.25 cm.

i. Calculate the flow velocity at the house when a faucet in the house is open.

ii. Calculate the pressure at the well when the faucet in the house is open.

2003B6.
A diver descends from a salvage ship to the ocean floor at a depth of 35 m below the surface. The density of ocean water is 1.025×10^3 kg/m^3.

(a) Calculate the gauge pressure on the diver on the ocean floor.

(b) Calculate the absolute pressure on the diver on the ocean floor.

The diver finds a rectangular aluminum plate having dimensions 1.0 m x 2.0 m x 0.03 m. A hoisting cable is lowered from the ship and the diver connects it to the plate. The density of aluminum is 2.7×10^3 kg/m^3. Ignore the effects of viscosity.

(c) Calculate the tension in the cable if it lifts the plate upward at a slow, constant velocity.
(d) Will the tension in the hoisting cable increase, decrease, or remain the same if the plate accelerates upward at 0.05 m/s^2?

____ increase ____ decrease ____ remain the same

Explain your reasoning.

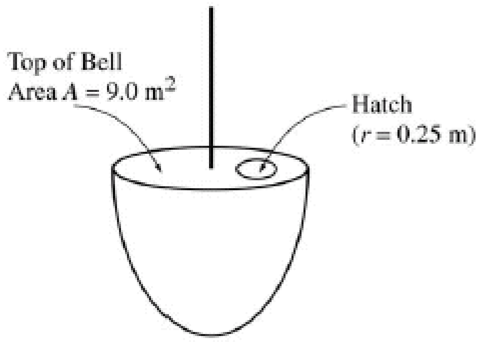
2004B2.
While exploring a sunken ocean liner, the principal researcher found the absolute pressure on the robot observation submarine at the level of the ship to be about 413 atmospheres. The inside of the submarine is kept at atmospheric pressure. The density of seawater is 1025 kg/m^3 .
(a) Calculate the gauge pressure on the sunken ocean liner.
(b) Calculate the depth of the sunken ocean liner.
(c) Calculate the magnitude of the net force due to the fluid pressures only on a viewing port of the submarine at this depth if the viewing port has a surface area of 0.0100 m^2 .
(d) What prevents the 'net force' found in part c from accelerating and moving the viewing port.

Suppose that the ocean liner came to rest at the surface of the ocean before it started to sink. Due to the resistance of the seawater, the sinking ocean liner then reached a terminal velocity of 10.0 m/s after falling for 30.0 s.

(e) Determine the magnitude of the average acceleration of the ocean liner during this period of time.

(f) Assuming the acceleration was constant, calculate the distance *d* below the surface at which the ocean liner reached this terminal velocity.

(g) Calculate the time *t* it took the ocean liner to sink from the surface to the bottom of the ocean.



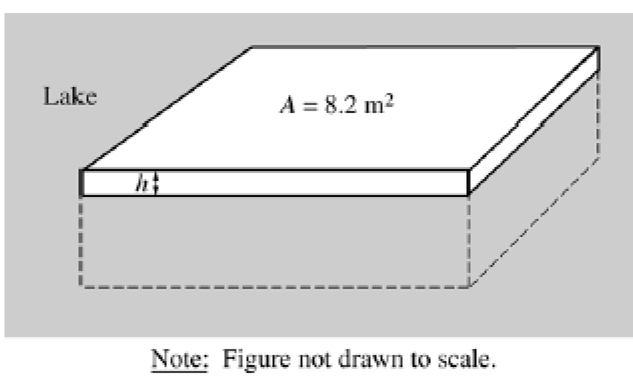
B2004B2.

The experimental diving bell shown above is lowered from rest at the ocean's surface and reaches a maximum depth of 80 m. Initially it accelerates downward at a rate of 0.10 m/s^2 until it reaches a speed of 2.0 m/s, which then remains constant. During the descent, the pressure inside the bell remains constant at 1 atmosphere. The top of the bell has a cross-sectional area A = 9.0 m^2. The density of seawater is 1025 kg/m^3.

(a) Calculate the total time it takes the bell to reach the maximum depth of 80 m.
(b) Calculate the weight of the water on the top of the bell when it is at the maximum depth.
(c) Calculate the absolute pressure on the top of the bell at the maximum depth.

On the top of the bell there is a circular hatch of radius r = 0.25 m.
(d) Calculate the minimum force necessary to lift open the hatch of the bell at the maximum depth.
(e) What could you do to reduce the force necessary to open the hatch at this depth? Justify your answer.

2005B5.

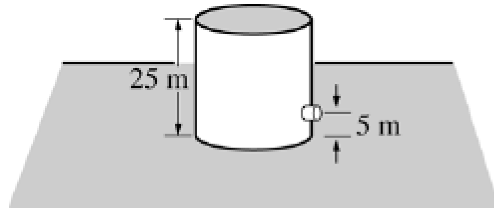


Note: Figure not drawn to scale.

A large rectangular raft (density 650 kg/m^3) is floating on a lake. The surface area of the top of the raft is 8.2 m^2 and its volume is 1.80 m^3. The density of the lake water is 1000 kg/m^3.
(a) Calculate the height h of the portion of the raft that is above the surrounding water.
(b) Calculate the magnitude of the buoyant force on the raft and state its direction.
(c) If the average mass of a person is 75 kg, calculate the maximum number of people that can be on the raft without the top of the raft sinking below the surface of the water. (Assume that the people are evenly distributed on the raft.)

B2005B5.

A large tank, 25 m in height and open at the top, is completely filled with saltwater (density 1025 kg/m^3). A small drain plug with a cross-sectional area of 4.0×10^{-5} m^2 is located 5.0 m from the bottom of the tank.



The plug breaks loose from the tank, and water flows from the drain.

(a) Calculate the force exerted by the water on the plug before the plug breaks free.

(b) Calculate the speed of the water as it leaves the hole in the side of the tank.

(c) Calculate the volume flow rate of the water from the hole.

2007B4.

The large container shown in the cross section is filled with a liquid of density 1.1×10^3 kg/m^3. A small hole of area 2.5×10^{-6} m^2 is opened in the side of the container a distance *h* below the liquid surface, which allows a stream of liquid to flow through the hole and into a beaker placed to the right of the container. At the same time, liquid is also added to the container at an appropriate rate so that *h* remains constant. The amount of liquid collected in the beaker in 2.0 minutes is 7.2×10^{-4} m^3.

(a) Calculate the volume rate of flow of liquid from the hole in m3 s.

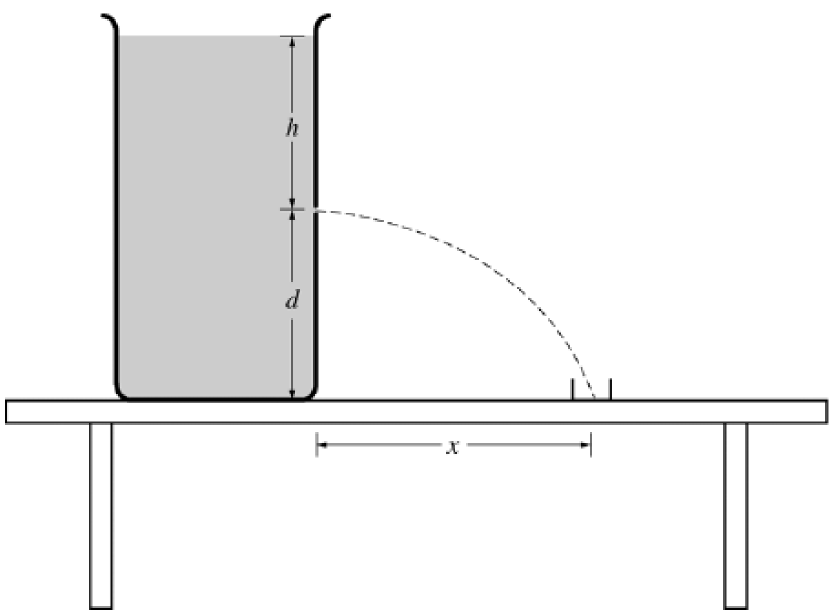
(b) Calculate the speed of the liquid as it exits from the hole.

(c) Calculate the height *h* of liquid needed above the hole to cause the speed you determined in part (b).

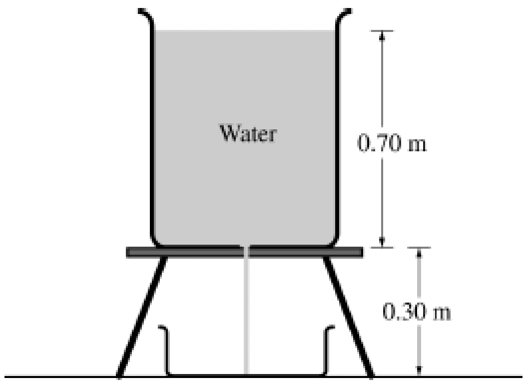
(d) Suppose that there is now less liquid in the container so that the height *h* is reduced to *h*/2. In relation to the collection beaker, where will the liquid hit the tabletop?

____ Left of the beaker ____ In the beaker ____ Right of the beaker

Justify your answer.



B2007B4.



A cylindrical tank containing water of density 1000 kg/m^3 is filled to a height of 0.70 m and placed on a stand as shown in the cross section above. A hole of radius 0.0010 m in the bottom of the tank is opened. Water then flows through the hole and through an opening in the stand and is collected in a tray 0.30 m below the hole. At the same time, water is added to the tank at an appropriate rate so that the water level in the tank remains constant.

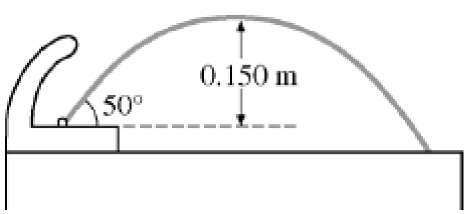
(a) Calculate the speed at which the water flows out from the hole.

(b) Calculate the volume rate at which water flows out from the hole.

(c) Calculate the volume of water collected in the tray in t = 2.0 minutes.

(d) Calculate the time it takes for a given droplet of water to fall 0.25 m from the hole.

2008B4.



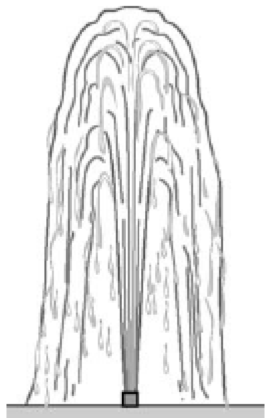
A drinking fountain projects water at an initial angle of 50° above the horizontal, and the water reaches a maximum height of 0.150 m above the point of exit. Assume air resistance is negligible.

(a) Calculate the speed at which the water leaves the fountain.

(b) The radius of the fountain's exit hole is 4.00×10^{-3} m. Calculate the volume rate of flow of the water.

(c) The fountain is fed by a pipe that at one point has a radius of 7.00×10^{-3} m and is 3.00 m below the fountain's opening. The density of water is 1.0×10^3 kg/m^3. Calculate the gauge pressure in the feeder pipe at this point.

B2008B4.



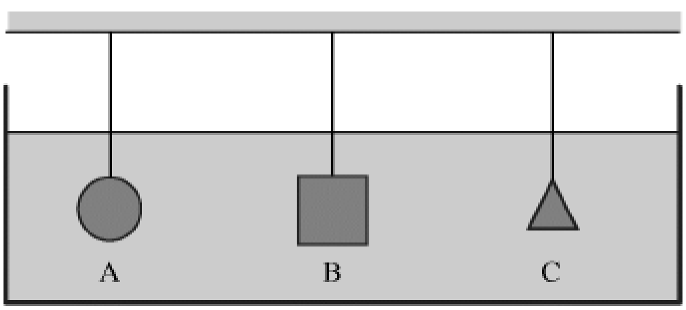
A fountain with an opening of radius 0.015 m shoots a stream of water vertically from ground level at 6.0 m/s. The density of water is 1000 kg/m^3.

(a) Calculate the volume rate of flow of water.

(b) The fountain is fed by a pipe that at one point has a radius of 0.025 m and is 2.5 m below the fountain's opening. Calculate the absolute pressure in the pipe at this point.

(c) The fountain owner wants to launch the water 4.0 m into the air with the same volume flow rate. A nozzle can be attached to change the size of the opening. Calculate the radius needed on this new nozzle.

2009B5.



Three objects of identical mass attached to strings are suspended in a large tank of liquid, as shown above.

(a) Must all three strings have the same tension?

____ Yes ____ No

Justify your answer.

Object A has a volume of 1.0 x 10^{-5} m^3 and a density of 1300 kg m^3. The tension in the string to which object A is attached is 0.0098 N.

(b) Calculate the buoyant force on object A.

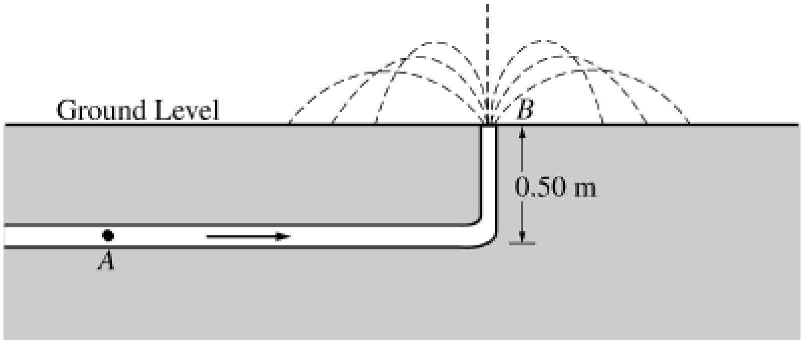
(c) Calculate the density of the liquid.

(d) Some of the liquid is now drained from the tank until only half of the volume of object A is submerged. Would the tension in the string to which object A is attached increase, decrease, or remain the same?

____ Increase ____ Decrease ____ Remain the same

Justify your answer.

B2009B3.



An underground pipe carries water of density 1000 kg/m^3 to a fountain at ground level, as shown above. At point *A*, 0.50 m below ground level, the pipe has a cross-sectional area of 1.0 x 10^{-4} m^2. At ground level, the pipe has a cross-sectional area of 0.50 x 10^{-4} m^2. The water leaves the pipe at point *B* at a speed of 8.2 m/s.
(a) Calculate the speed of the water in the pipe at point *A*.
(b) Calculate the absolute water pressure in the pipe at point *A*.
(c) Calculate the maximum height above the ground that the water reaches upon leaving the pipe vertically at ground level, assuming air resistance is negligible.
(d) Calculate the horizontal distance from the pipe that is reached by water exiting the pipe at 60° from the level ground, assuming air resistance is negligible.

Supplemental Problems

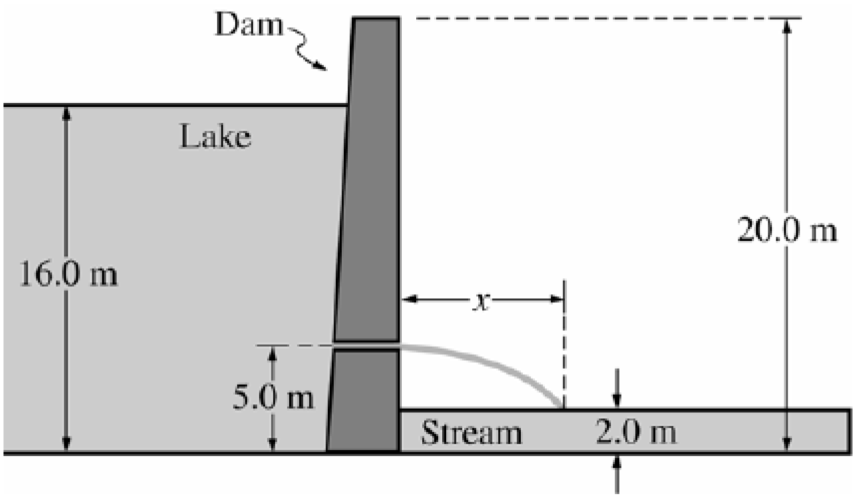
SUP1. A block of wood has a mass of 12 kg and dimensions 0.5 m by 0.2 m by 0.2 m.
(a) Find the density ρ_o of the wooden block.
(b) If the block is placed in water (ρ = 1000 kg/m^3) with the square sides parallel to the water surface, how far beneath the surface of the water is the bottom of the block?
(c) A weight is placed on the top of the block. The block sinks to a point that the top of the block is exactly even with the water surface. Find the mass of the added weight.

SUP2. A tapered horizontal pipe carries water from one building to another on the same level. The wider end has a cross-sectional area of 4 m^2. The narrower end has a cross-sectional area of 2 m^2. Water enters the wider end at a velocity of 10 m/sec.
(a) What is the speed of the water at the narrow end of the pipe?
(b) The gauge pressure of the water at the wide end of the pipe is 2 x 10^5 pascals. Using Bernoulli's equation, find the gauge pressure at the narrow end of the pipe.

SUP3. A small airplane has wings with surface area 9 m^2 each. The speed of the air across the top of the wing is 50 m/sec, and across the bottom of the wing, 40 m/sec. Take the density of air to be 1.2 kg/m^3.
(a) Find the difference in the pressure between the top and the bottom of the wing.
(b) i) Find the net lift upward on the plane.
ii) If there is no other lift on the plane, what would be the mass of the plane? Assume the plane is not accelerating up or down.

SUP4. A block of wood floats in water, with 2/3 of it submerged. The wood is then placed in oil, and 9/10 of it is submerged. Find the density of the wood, and of the oil.

SUP5.



A 20 m high dam is used to create a large lake. The lake is filled to a depth of 16 m as shown above. The density of water is 1000 kg/m^3 .

(a) Calculate the absolute pressure at the bottom of the lake next to the dam.

A release valve is opened 5.0 m above the base of the dam, and water exits horizontally from the valve.
(b) Use Bernoulli's equation to calculate the initial speed of the water as it exits the valve.

(c) The stream below the surface of the dam is 2.0 m deep. Assuming that air resistance is negligible, calculate the horizontal distance *x* from the dam at which the water exiting the valve strikes the surface of the stream.

(d) Suppose that the atmospheric pressure in the vicinity of the dam increased. How would this affect the initial speed of the water as it exits the valve?

___It would increase. ____It would decrease. ____It would remain the same.
Justify your answer.

ANSWERS - AP Physics Multiple Choice Practice – Fluids

	Solution	Answer
1.	FBD has F_t pointing down F_b pointing up and weight (mg) down. $F_{net} = 0$ $F_b - F_t - mg = 0$ The buoyant force is given by the weight of the displaced water. Since the waters displaced volume is equal to the corks displaced volume and the water weight for the same volume would be 4 times heavier (based on the given cork weight = 25% water weight) compared to the cork, the buoyant force is equal to 4 x the cork weight = 4mg. Using the force equation created initially. $F_t = F_b - mg = 4mg - mg = 3mg$	D
2.	Definition of Pascal's principle	A
3.	A 1 m^3 volume cube under water displaces $1m^3$ of water. This weight of water = pVg = 1000(1)(10) = 10000 N which is equivalent to the buoyant force. The apparent weight in water is $m_{app}g$ = 18300(10) = 183000 N. This apparent weight is lessened by the buoyant force pulling up with 10000 N of force. So outside of the water, this upwards force would not exist and the actual weight would be 193000 N which equal 19300 kg of mass.	D
4.	Using fluid continuity. $A_1v_1 = A_2v_2$ $\pi R^2 v_1 = \pi(2R)^2 v_2$ $v_1 = 4\ v_2$	E
5.	This is based on two principles. 1 – Bernoulli's principle says that when speed increases pressure drops. Second, continuity says more area means less speed based on $A_1v_1 = A_2v_2$ So the smallest area would have the largest speed and therefore most pressure drop.	B
6.	Since A and B have the same mass and density, they have the same volume. C has the same volume as A and B since it's the same shape as B. So all three objects have the same volume. When submerged, they will all displace the same amount of water and therefore all have the same buoyant force acting on them. *Note: if the objects were floating instead of submerged than the heavier ones would have larger buoyant forces.*	E
7.	Pascals principle of equal pressure transfer in a fluid allows for hydraulic lifts to function.	A
8.	Pascals principle says $P_1 = P_2$ $F_1/A_1 = F_2/A_2$ $F_2 = F_1A_2 / A_1$ = 500(40)/(2)	C
9.	Buoyant force is equal to weight of displaced fluid. Since the density is constant and the volume displaced is always the same, the buoyant force stays constant	B
10.	The wood is floating and is only partially submerged. It does not displace a weight of water related to its entire volume. The iron however is totally submerged and does displace a weight of water equal to its entire volume. Since the iron displaces more water, it has a larger buoyant force acting on it.	B
11.	For floating objects, the weight of the displaced fluid equals the weight of the object. For a more dense fluid, less of that fluid needs to be displaced to create a fluid weight equal to the weight of the object. Since the salt water is more dense, it will not need as much displaced.	B
12.	Definition of specific gravity. s.g = ρ_x / ρ_{H20}	A
13.	Same as question #5, but moving to more area → less speed → more pressure	B
14.	Flow continuity. $A_1v_1 = A_2v_2$ $\pi(0.02)^2(1) = \pi(0.01)^2 v_2$	D
15.	Buoyant force is based on how much weight of water is displaced. Since all three are completely submerged they all displace the same amount of water so have equal buoyant forces.	C

16. For floating objects, the buoyant force equals the weight of the objects. Since each object has the same weight, they must have the same buoyant force to counteract that weight and make them float. *IF the equal mass objects sunk, then the one with the smaller density would have a larger volume and displace more water so have a larger buoyant force. But that is not the case here.* D

17. P = F / A $1 \times 10^5 = F / (22*5)$ E

18. P = F / A = ma / A $= kg\ (m/s^2) / m^2$ $= kg / (m \cdot s^2)$ D

19. Three forces act on the block, F_t down, mg down and F_b up. $F_{net} = 0$ $F_b - F_t - mg = 0$ D
$F_b - 3 - 5 = 0$ $F_b = 8$ N – weight of displaced water $= \rho_{h20} V_{disp}\, g$
$8 = (1000)\, V\, (10) \rightarrow V = 0.0008\ m^3$

20. For floating objects $mg = F_b$ $\rho_{obj} V_{obj}\, g = \rho_{h20} V_{disp}\, g$ E
$\rho_{obj}\, (V)g = 1000\, (0.6V)\, g$ $\rho_{obj} = 600$

In oil the same is true $\rho_{obj} V_{obj}\, g = \rho_{oil} V_{disp}\, g$ $(600)Vg = (800)\, x\%V\, g$ $x\% = 0.75$

21. Same as question 4 A

22. Based on continuity, less area means more speed and based on Bernoulli, more speed means less pressure. B

23. The weight of the mass is 4N. The scale reading apparent weight is 3N so there must be a 1N buoyant force acting to produce this result. B

24. Since the pressure in a fluid is only dependent on the depth, they all have the same fluid pressure at the base. Since all of the bases have the same area and the same liquid pressure there, the force of the liquid given by P=F/A would be the same for all containers. *Note: IF instead this question asked for the pressure of the container on the floor below it, the container with more total mass in it would create a greater pressure, but that is not the case here.* E

25. As the fluid flows into the smaller area constriction, its speed increases and therefore the pressure drops. Since the pressure in the constriction is less than that outside at the water surface, fluid is forced up into the lower tube. A

26. The buoyant force would be the difference between the two scale readings … $(.09kg)(10\ m/s^2) =$ 0.9 N of buoyant force. This equals the weight of displaced water. $F_b = \rho_{h20} V_{disp}\, g$ E
0.9 = 1000 (V) (10) … gives the volume of the displaced water = 0.00009 which is the same as the volume of the object since its fully submerged.

Now using ρ = m/V for the object we have … $\dfrac{0.45}{0.00009} = \dfrac{45}{100} * \dfrac{10000}{9} = 5000$

27. Use flow continuity. $A_1 v_1 = A_2 v_2$ C
since the area is the same at both locations the speed would also have to be the same.

28. Apply Bernoulli's equation. $P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$ D
$(9.5)(100000) + 0 + \frac{1}{2}(1000)(10)^2 = P_2 + (1000)(10)(15) + \frac{1}{2}(1000)(10)^2$

$P_2 = 800000\ N/m^2$

29. Both object are more dense than water and will sink in the pool. Since both have the same volume, they will displace the same amount of water and will have the same buoyant forces. C

30. Again both samples sink. Also, both samples have the same mass but different densities. For the same mass, a smaller density must have a larger volume, and the larger volume displaces more water making a larger buoyant force. So the smaller density with the larger volume has a larger buoyant force. B

31. V of this ball is $4/3\ \pi\ r^3 = 4/3\ \pi\ (0.4)^3 = 0.2681\ m^3$. For the ball to just sink, it is on the verge of floating, meaning the weight of the ball equals the buoyant force of the fully submerged ball. B
$mg = \rho_{fl}\ V_{disp}\ g$ $\quad$ $m\ (10) = 1400\ (0.2681)\ (10)$ $\quad$ $m = 375\ kg$

32. This object will float, so $m_{obj} g = F_b$ $\quad$ $\rho_{obj} V_{obj}\ g = \rho_{ocean}\ V_{disp}\ g$ A
$(0.95\text{x}10^3)(V)(10) = (1.1\text{x}10^3)(x\%\ V)\ (10)$
Gives x% = 0.86 but that is the amount submerged, so the visible about would be 1 – 0.86

33. Statement associated with Bernoulli's principle E

34. $s.g = \rho_{obj} / \rho_{h20}$ $\quad$ $0.82 = \rho_{obj} / 1000$ $\quad$ $\rho_{obj} = 820$ … then $\rho = m/V$ $\quad$ $820 = m / 1.3$ D

35. The apparent weight is the air weight – the upwards buoyant force. The buoyant force is given by $F_b = \rho_{fl}\ V_{disp}\ g = 1.25\text{x}10^3\ (0.375)\ (10) = 4687.5$ N. D
The apparent weight is then (600)(10) – 4687.5 = 1312.5 N

36. Using fluid continuity. $A_1 v_1 = A_2 v_2$ $\quad$ $\pi(7R)^2 v_1 = \pi(R)^2 V$ $\quad$ $v_1 = V / 49$ A

37. The fluid flow is occurring in a situation similar to the diagram for question #27. B
Apply Bernoulli's equation. $P_1 + \rho g y_1 + ½\ \rho v_1^2 = P_2 + \rho g y_2 + ½\ \rho v_2^2$
$P + 0 + ½\ \rho v^2 = P_2 + \rho g y + ½\ \rho\ (2v)^2$
$P_2 = P + ½\ \rho v^2 - ½\ 4\rho v^2 - \rho g y$ $\quad$ $= P - 3/2\ \rho v^2 - \rho g y$

38. s.g is density / density and has no units E

39. Definition of Archimedes principle D

40. $P_{abs} = P_g + P_o = 2.026\text{x}10^5 + 1.01\text{x}10^5 = 3.03\text{x}10^5$ Pa D

41. Definition of buoyant force C

42. Using fluid continuity with W as river width. $A_1 v_1 = A_2 v_2$ $\quad$ $4(W)(12) = (8)(W)v_2$ $\quad$ $v_2 = 6$ m/s C

43. The relevant formula here is $P = P_o + \rho g h$ B
Answer (a) is wrong, because at y1 on both arms, the pressure is just the atmospheric pressure. The pressure in the right arm at y3 is still just atmospheric, but on the left, it is atmospheric plus $\rho g(y1 - y3)$. That rules out (a). The pressure at the bottom of the tube is everywhere the same (Pascal's principle), which rules out (c), and at the same time, tells us (b) is right. At y2, we can say $P = P_{bottom} - \rho_{Hg} g y2$ on both sides, so the pressure is equal. Answer (d) is wrong because at y3, the right arm is supporting only the atmosphere, while the left arm is supporting the atmosphere plus ρ_{H20} gh. Finally, (e) is silly because both arms at height y1 are at atmospheric pressure.

44. Apply Bernoulli's equation. $P_1 + \rho g y_1$ ~~$+ ½\ \rho v_1^2$~~ $= P_2 + \rho g y_2 +$ ~~$½\ \rho v_2^2$~~ A
$P_1 = P_2 + \rho g(y_2 - y_1)$

45. $P = F / A = 30 / \pi r^2$ … use 3 for π since its an estimate … $30 / (3*(.01)^2) = 100000$ Pa E

46. From a force standpoint, for the object to be completely submerged there would be three forces acting. F_b up, mg down and F_{push} down. $F_b = F_{push} + mg$ $F_{push} = F_b - mg$ D
$F_{push} = p_{h20} V_{disp} g - mg$
$= (1000)(2.5\text{x}10^{-2})(10) - (5)(10) = 200$ N

47. Using fluid continuity. $A_1 v_1 = A_2 v_2$ $\pi(D/2)^2 v_1 = \pi(d/2)^2 v_2$ solve for v_2 E

2002B6.

a) Example 1: Measure the unstretched length of the spring. Hang it with the object at rest and measure the stretched length. Call the difference in these lengths Δx. Equating the weight of the object and the force exerted by the extended spring gives $mg = k\Delta x$ from which k can be determined.
Example 2: Set the hanging mass into oscillation. Determine the period T by timing n oscillations and dividing that time by n. The equation $T = 2\pi\sqrt{m / k}$ can then be used to find k.

b) The spring is stretched less when the object is at rest in the fluid. The fluid exerts an upward buoyant force on the object. Since the net force on the object is still zero, the spring does not need to exert as much force as before and thus stretches less.

c&d)

1) Measure the length of the spring when the object is immersed in the liquid, and subtract the unstretched length to determine the amount the spring is stretched. This will allow calculation of the force exerted by the spring on the object.
2) The volume of fluid displaced is equal to the volume of the object, which can be determined from the given mass and density of the object.
3) The buoyant force on the object is equal to the difference of the object's weight and the force exerted by the spring.
4) The buoyant force also equals the weight of the displaced fluid, which equals the product of the fluid density, displaced volume, and g.

Symbol	Quantity
ρ	fluid density
V	object volume = displaced water volume
g	acceleration of gravity
m	mass of object
x	spring stretch in air
x_w	spring stretch in water

First solving for k in air. $mg = kx$

Then in the fluid. $F_{sp} = kx_w$

$F_{net} = 0$ $\quad F_b = mg - F_{sp}$ $\quad \rho Vg = mg - kx_w$ $\quad \rho Vg = mg - (mg / x)\, x_w$ solve for ρ

B2003B6.

a) i) The total mass of water moved can be found with the density and volume $m = \rho V = (1000)(0.35) = 350$ kg of water. This water is moved a distance 85 m so the work done to move it is $W=Fd = (350)(9.8)(85) = 291{,}500$ J.

ii) The force needed to move the water = the weight of the water (mg).
Using. $P = Fd /t = (350)(9.8)(85) / (2\text{hrs} * 3600 \text{ s/hr}) = 40.5$ W

b) i) Using fluid continuity. $A_1 v_1 = A_2 v_2$ $\pi(.03/2)^2(0.5) = \pi(0.0125/2)^2 v_2$ $v_2 = 2.88$ m/s

ii) Apply Bernoulli's equation. $P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$
$y_1 = 0$ P_2 = atmospheric (open faucet)

$P_1 + 0 + \frac{1}{2}(1000)(0.5)^2 = 1.01\text{x}10^5 + (1000)(9.8)(85) + \frac{1}{2}(1000)(2.88)^2$ $P_1 = 938000$ Pa

---

2003B6.

a) $P = \rho gh = (1025)(9.8)(35) = 351{,}600$ Pa

b) $P_{abs} = P_o + \rho gh = 1.01\text{x}10^5 + 343{,}000 = 452{,}600$ Pa

c) The FBD has three forces acting on it. The upwards lifting force, the upwards buoyant force and the downwards weight, mg. Constant velocity → $F_{net} = 0$
$F_t + F_b - mg = 0$ $F_t = mg - F_b$ $F_t = (\rho_{obj} V_{obj})g - (\rho_{h20} V_{disp})g$ $V_{disp} = V_{obj}$ call it V

→ $F_t = Vg(\rho_{al} - \rho_{h20}) = (1\text{x}2\text{x}0.03)(9.8)(2700 - 1025) = 985$ N

d) $F_t + F_b - mg = ma$ $F_t = mg - F_b + ma$. Comparing this tension equation to the one in part c you see that the tension will <u>increase</u> since the quantity "ma" is being added here

---

2004B2.

a) $P_{abs} = P_o + P_{gauge}$ 413 atm = 1 atm – P_{gauge} P_{gauge} = 412 atm

b) $P_{gauge} = \rho gh$ $412(1.01\text{x}10^5) = 1024\ (9.8)(h)$ h = 4140 m

c) The fluid pressures acting are the outside water pressure (which includes the atmosphere at the surface acting down on it) and, the inside air pressure which is atmospheric. Since the atmospheric pressure acts both inside and is also included in the water pressure, the net force due to fluid pressure can be found by using the water's gauge pressure since the air pressures effectively cancel each other out.
P_{gauge} = F /A $412(1.01\text{x}10^5)$ = F / 0.01 F = 416,000 N

d) The force from c is not the true net force. The actual net force is zero as the window is at rest. This force from c is due to fluid pressures and is resisted by normal forces acting on the edges of the window where it is connected to the submarine.

e) $v_f = v_i + at$ 10 = 0 + a(30) a = 0.33 m/s^2

f) $d = v_i t + ½ at^2$ $d = 0 + ½\ (0.33)(30)^2$ d = 150 m

g) The total depth is 4140 m. There are two parts to the trip, the first 150 m covered while accelerating and the second (3990m) covered while moving at constant speed. The parts must be calculated separately. Part one, during acceleration, was already given as taking 30 second. The second part at a constant speed can simply be found using $\bar{v}$ = d/t, 10 = 3990 / t, t_2 =399 seconds. So the total time of travel was 429 seconds.

B2004B2.

a) The descent occurs at two different accelerations and must be analyzed in the two sections.
Section 1 starts from rest and accelerates, find the time in that part
$v_{1f} = v_{1i} + a_1 t_1$ 2 = 0 + 0.10 t t_1 = 20 seconds.

$d_1 = v_{1i} t + ½\ a_1 t_1^2$ $d_1 = ½\ (0.10)(20)^2$ d_1 = 20 m

Section 2 occurs at a constant speed equal to the final speed in section 1 and will occur over the remaining distance d_2 = 60m.
$\bar{v}_2 = d_2 / t_2$ $2 = 60 / t_2$ t_2 = 30 seconds $t_{total} = t_1 + t_2$ = 50 seconds

b) Weight of water above the bell is a cylindrical column with a height of h=80 m and area of A=9 m^2. This gives us the volume of the water above the bell given by V = Ah = 720 m^3.
The weight of this column = m_{h20} g = (ρ_{h20}V) g = (1025)(720)(9.8) = $7.2\text{x}10^5$ N

c) $P_{abs} = P_o + \rho gh = 1.01\text{x}10^5 + (1025)(9.8)(80) = 9\text{x}10^5$ Pa

d) Since there is air pressure inside the bell, and the absolute pressure on the outside also includes the air pressure, these two pressures essentially cancel each other out and we only need to push against the water pressure alone so we should use the gauge pressure to find the needed force.
$P_{abs} = P_o + P_{gauge}$ $9\text{x}10^5 = 1.01\text{x}10^5 + P_{gauge}$ $P_{gauge} = 8\text{x}10^5$ Pa.
$F = PA = (8\text{x}10^5)(\pi(0.25)^2) = 1.58\text{x}10^5$ N

e) To reduce the pushing force needed, you could increase the pressure inside the bell to create a smaller pressure difference between inside and outside. Or, by making the area of the hatch smaller the pushing force would be less. Or, you could use a lever inside that uses torque to provide mechanical advantage to amplify an applied force to one side of the lever. This would makes the force pushing the hatch open the same but the required pushing force of a person less.

2005B5.

a) We are given the volume of the raft and the surface area as well. Use this to first find the total height of the raft h_t

$V = Ah_t$ $1.8 = 8.2\ h_t$ $h_t = 0.22$ m

Since the raft is floating, the weight of the raft must equal the weight of the displaced fluid. We will define "h_s" as being the portion of the height of the raft below the water so that the displaced volume is given by $V=Ah_s$

$m_{raft}g = \rho_{h20}\ V_{disp}\ g$

$\rho_{raft}V_{raft}\ g = \rho_{h20}\ V_{disp}\ g$ $\rho_{raft}V_{raft} = \rho_{h20}\ (Ah_s)$ $(650)(1.8) = (1000)(8.2)h_s$ $h_s = 0.143$ m

$h = h_t - h_s = 0.22 - 0.143 = 0.077$ m (the visible portion of the raft)

b) F_B equals weight of displaced water $= \rho_{h20}\ V_{disp}\ g = \rho_{h20}\ (Ah_s)\ g = (1000)(8.2)(0.143)(9.8) = 11500$ N directed ↑

c) Determine the extra buoyant force that will come from submerging the exposed raft volume $V_{exp} = Ah$

$F_{b(extra)} = \rho_{h20}\ V_{disp}\ g = \rho_{h20}\ (Ah)\ g = (1000)(8.2)(0.077)(9.8) = 6187.7$ N

1 persons weight = mg = 735 N. Total weight allowed / person weight = 6187.7 / 735 = 8.41

So, an extra 8 people could come on without submerging the raft. You could also chop some arms or legs off and throw them on there also until you get up to the extra 0.41 of a person limit.

B2005B5.

a) The force on the plug from the water inherently includes the atmosphere above it, so we use the absolute pressure.

$P_{abs} = P_o + \rho gh = 1.01x10^5 + (1025)(9.8)(20m) = 3x10^5$ Pa

The force is then found with P = F/A $3x10^5 = F / (4x10^{-5}) \rightarrow F = 12$ N

Note: This calculation of pressure (ρgh) only works since the fluid is at rest (static). For moving fluids, only Bernoulli's equation (or F/A in rare cases) can be applied for determining pressures.

b) Though many of you may know the Torricelli theorem shortcut to this problem, when the AP exam graded this question, simply stating that equation and plugging in lost points. To be safe you should always start with Bernoulli's equation in its full form, cancel out terms that don't exist or are assumed zero, and solve from there.

$P_1 + \rho gy_1 + ½\ \rho v_1^2 = P_2 + \rho gy_2 + ½\ \rho v_2^2$

P_1 and P_2 are both open to atmosphere so are at P_o and cancel. Tank is large so v_1 is assumed small enough to be 0, y_2 is set as zero height.

$\rho gy_1 = ½\ \rho v_2^2$ $v_2 = \sqrt{2gy_1}$ (as expected from Torricelli) … $v_2 = \sqrt{2(9.8)(20)} = 19.8$ m/s

c) Volume flow rate = $Q = Av = (4x10^{-5})(19.8) = 7.92x10^{-4}\ m^3/s$

2007B4.

a) Volume flow rate = $Q = V/t = 7.2x10^{-4}$ / (2min * 60 sec/min) = $6x10^{-6}\ m^3/s$

b) Your first thought is probably Bernoulli, but there are too many unknowns so this does not work. We can use the volume flow rate above the find the velocity.

$Q = Av$ $6x10^{-6} = (2.5x10^{-6})\ v$ $v = 2.4$ m/s

c) Use Bernoulli, same derivation as in the problem above (B2005B5) … $v_2 = \sqrt{2gh}$ $(2.4) = \sqrt{2(9.8)h}$ h=0.29m

d) Left of beaker. Based on the formula derived above, the exit velocity is dependent on the height and with less horizontal exit velocity the range will be less ($d_x = v_xt$). This makes sense because less height would result in less pressure and decrease the speed the fluid is ejected at, thus lessening the range.

B2007B4.

a) Use Bernoulli, same derivation as problem B2005B5 … $v_2 = \sqrt{2gh}$ … $v_2 = \sqrt{2(9.8)(0.7)}$… $v_2 = 3.7$ m/s

b) Volume flow rate $Q = Av = \pi(0.001)^2(3.7) = 1.16\text{x}10^{-5}\ m^3/s$

c) $Q = V / t$ $\quad 1.16\text{x}10^{-5} = V / (2\text{min} * 60\ \text{s/min})$ $\quad V = 0.0014\ m^3$

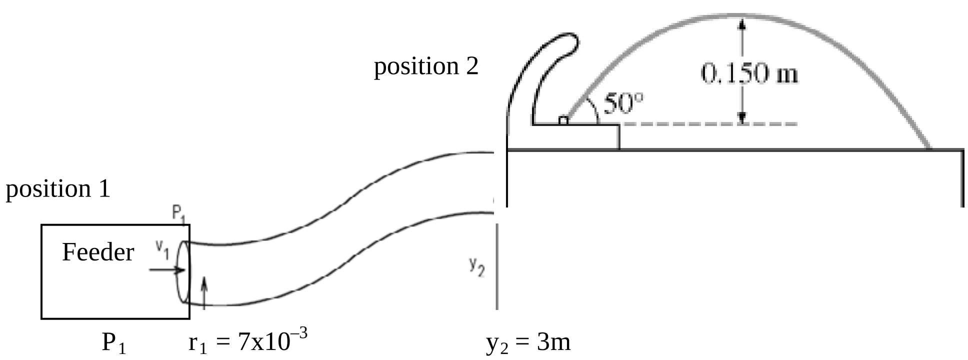
d) Free fall. $\quad d = v_it + ½\ gt^2 \quad -0.25 = (-3.7\ t) + ½(-9.8)\ t^2$ $\quad$ solve quadratic $\quad t = 0.062$ s
Alternatively, first determine v_f at the 0.25 m location then use $v_f = v_i + at$ to solve for t.

2008B4.

a) Using projectile methods. $\quad v_{fy}^2 = v_{iy}^2 + 2ad_y \quad 0 = (v_i \sin 50)^2 + 2(-9.8)(0.15) \quad v_i =$ 2.24 m/s

b) Volume flow rate = $Q = Av = \pi(4\text{x}10^{-3})^2\ (2.24) = 1.13\text{x}10^{-4}\ m^3/s$

c) If you don't understand the wording, here is what the problem is saying



First we need to find the velocity of the water at the feeder using continuity
$A_1v_1 = Q_2 \quad \pi(7\text{x}10^{-3})^2(v_1) = 1.13\text{x}10^{-4} \quad v_1 = 0.73$ m/s

Bernoulli. Position 2 is the fountain spigot which is open so at atmospheric pressure. $y_1=0$ no height.

$P_1 + \rho gy_1 + ½\ \rho v_1^2 = P_2 + \rho gy_2 + ½\ \rho v_2^2$

$P_1 + 0 + ½\ (1000)(0.73)^2 = (1.01\text{x}10^5) + (1000)(9.8)(3m) + ½\ (1000)(2.24)^2$

$P_1 = 1.32\text{x}10^5$ Pa which is the absolute pressure of the feeder.

To find the gauge pressure of the feeder. $P_{abs} = P_{gauge} + P_o \quad 1.32\text{x}10^5 = P_{gauge} + 1.01\text{x}10^5$

$P_{gauge} = 31600$ Pa.

Note: This gauge pressure could be determined directly in Bernoulli's equation by realizing that P_1 includes atmospheric pressure as part of its total value and that P_2 was equal to atmospheric pressure, so by elimination of the term P_2, P_1 becomes the gauge pressure. This should be stated in the solution if it is the chosen solution method.

B2008B4

a) Volume flow rate = Q = Av = $\pi(0.015)^2$ (6) = 0.0042 m^3/s

b) First we need to find the velocity of the water in the pipe below using continuity
$A_1v_1 = Q_2$ $\pi(0.025)^2(v_1) = 0.0042$ $v_1 = 2.16$ m/s

Bernoulli. Position 2 is the fountain spigot which is open so at atmospheric pressure. y_1=0 no height.
$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$
$P_1 + 0 + \frac{1}{2}(1000)(2.16)^2 = (1.01\text{x}10^5) + (1000)(9.8)(2.5\text{m}) + \frac{1}{2}(1000)(6)^2$ $P_1 = 141000$ Pa

c) Determine the launch speed needed to reach 4m.
Free fall of a water droplet. $v_f^2 = v_i^2 + 2gd$ $(0) = v_i^2 + 2(-9.8)(4)$ $v_i = 8.85$ m/s

Use flow rate to find new area needed. Q = Av (0.0042) = A (8.85) $A_{new} = 4.75\text{x}10^{-4}$ m^3

Find new radius $A_{new} = \pi r_{new}^2$ $4.75\text{x}10^{-4}$ $m^3 = \pi r_{new}^2$ $r_{new} = 0.0122$ m

---

2009B5.

a) There are three forces acting on the masses in each case. Tension up, buoyant force up, weight down. Since they are at rest we have. $F_{net} = 0$ $F_t + F_b = mg$ $F_t = mg - F_b$ so the largest F_b makes the largest F_t

We are to assume the diagram is to scale and that clearly the volumes of the three containers are different. The one with the largest volume displaces the largest amount and weight of water and will have the largest buoyant force acting on it. So since they all displace different volumes (and weights) of water they all have different buoyant forces, and based on the equation shown above will have different tensions.

b) The mass of the object is given by m = $\rho_{obj}V_{obj.}$
Using the equation from part a,
$F_t + F_b = mg$, $F_t + F_b = (\rho_{obj}V_{obj.})$ g $(0.0098) + F_b = (1300)(1\text{x}10^{-5})(9.8)$ F_b = 0.1176 N

c) The buoyant force is by definition equal to the weight of the displaced fluid.
$F_b = (\rho_{fluid}V_{disp.})$ g $0.1176 = \rho_{fluid}$ $(1\text{x}10^{-5})(9.8)$ $\rho_{fluid} = 1200$ kg/m^3

d) With only half of the volume submerged, ½ as much water will be displaced and the buoyant force will be half the size. Based on the formula from part A, less buoyant from will make a larger tension. This also makes sense conceptually. Objects have large apparent weights in air than water so having some of it in the air will increase its apparent weight.

B2009B3.

a) Using fluid continuity. $A_1v_1 = A_2v_2$ $(1x10^{-4})(v_1) = (0.5x10^{-4})(8.2)$ $v_a = 4.1$ m/s

b) Bernoulli. Position B is the fountain spigot which is open so at atmospheric pressure. $y_1=0$ no height.

$P_1 + \rho gy_1 + ½ \rho v_1^2 = P_2 + \rho gy_2 + ½ \rho v_2^2$

$P_1 + 0 + ½ (1000)(4.1)^2 = (1.01x10^5) + (1000)(9.8)(0.5m) + ½ (1000)(8.2)^2$ $P_1 = 1.3x10^5$ Pa

c) Free fall of a water droplet. $v_f^2 = v_i^2 + 2gd$ $(0)^2 = (8.2)^2 + 2(–9.8)(d)$ d = 3.43m

d) Projectile method, in y direction.

$d_y = v_{iy}t + ½ gt^2$ $d_y = (v_i \sin \theta) t + ½ gt^2$ $0 = (8.2 \sin 60)t + ½ (–9.8)t^2$ t = 1.45 sec

X direction. $d_x = v_x t$ $d_x = (v_i \cos \theta) t$ $d_x = (8.2 \cos 60)(1.45)$ $d_x =$ 5.95 m

---

SUP1.

a) $\rho = m/V$ = 12 / (0.5x0.2x0.2) = 600 kg / m^3

b) The block will float based on its density. For floating, block weight = buoyant force.

$m_{obj}g = p_{h20} V_{disp} g$ $m_{obj} = p_{h20} A_{sqaure}(h_{submerged})$ $12 = 1000(0.2x0.2)h_{sub}$ $h_{sub} = 0.3$ m

c) The extra weight added should equal the extra buoyant force created by submerging the remaining 0.2 m of height.

$F_{b(extra)} = p_{h20} V_{disp} g = (1000)(0.2x0.2x0.2)(9.8) = 78.4$ N 78.4 / 9.8 = 8kg of extra mass.

SUP2.

a) Using fluid continuity. $A_1v_1 = A_2v_2$ $(4)(10) = (2)(v_2)$ $v_2 = 20$ m/s

b) Bernoulli. ρgy_1 terms cancel out since the pipe stays on the same level.

$P_1 + \rho gy_1 + ½ \rho v_1^2 = P_2 + \rho gy_2 + ½ \rho v_2^2$

$2x10^5 + 0 + ½ (1000)(10)^2 = P_2 + 0 + ½ (1000)(20)^2$ $P_2 = 50000$ Pa.

Since P_1 was the gauge pressure and did not include P_o, P_2 will also come out as the gauge pressure.

SUP3.

a) Bernoulli. ρgy_1 terms cancel out since the height difference is negligible.

$P_1 + \rho gy_1 + ½ \rho v_1^2 = P_2 + \rho gy_2 + ½ \rho v_2^2$ … rearrange equation so we can find $P_2 – P_1$ which is the ΔP

$P_2 – P_1 = ½ \rho v_1^2 – ½ \rho v_2^2$ $\Delta P = ½ (1.2) (50^2 – 40^2) = 540$ Pa

b) i) $\Delta P = F_{lift} / A$ $540 = F_{lift} / (9x2 \text{ wings})$ $F_{lift} = 9720$ N

ii) $F_{net} = 0$ $F_{lift} = mg$ $9720 = m (9.8)$ m = 992 kg.

SUP4.

This problem involves floating objects, so weight of object = buoyant force $m_{obj}\, g = \rho_{fluid}\, V_{disp}\, g$

In general … $m_{obj} = \rho_{obj} V_{obj}$

Giving … $\rho_{obj} V_{obj}\, g = \rho_{fluid}\, V_{disp}\, g$ … $\rho_{obj} V_{obj} = \rho_{fluid}\, V_{disp}$

Water	Oil
$\rho_{obj} V_{obj} = \rho_{fluid}\, V_{disp}$	$\rho_{obj} V_{obj} = \rho_{fluid}\, V_{disp}$
$\rho_w V = (1000)(2/3\ V)$	$(666.67)V = \rho_{oil}\ (9/10\ V)$
$\rho_w = 666.67\ kg / m^3$	$\rho_{oil} = 740.74\ kg / m^3$

SUP5.

a) $P_{abs} = P_{gauge} + P_o$ $P_{abs} = \rho gh + 1.01x10^5$ $P_{abs} = (1000)(9.8)(16) + 1.01x10^5 =$ 260000 Pa

b) Use Bernoulli, same derivation as problem B2005B5 … $v_2 = \sqrt{2gh}$ … $v_2 = \sqrt{2(9.8)(11)}$… $v_2 = 14.7$ m/s

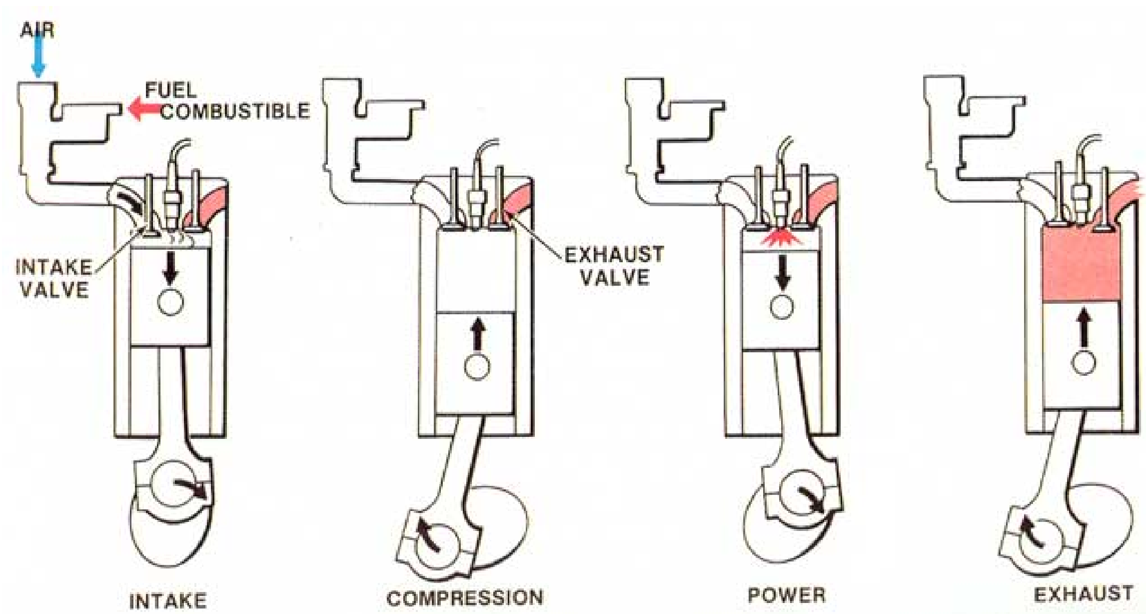
c) Using projectile methods. $d_y = v_{iy}t + ½ at^2$ $-3 = 0 + ½ (-9.8)\, t^2$ $t = 0.78$ sec

$d_x = v_x t = (14.7)\,(0.78) = 11.5$ m

d) An increase in atmospheric pressure around the damn increases both P_1 and P_2 equally so there is no net effect on these terms in Bernoulli's equation, which means the exit velocity would be the same.

Chapter 9

Thermodynamics

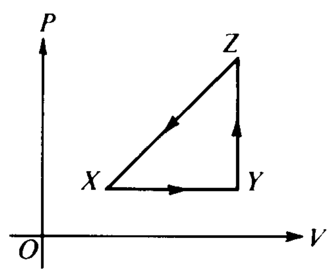


AP Physics Multiple Choice Practice – Thermodynamics

1. The maximum efficiency of a heat engine that operates between temperatures of 1500 K in the firing chamber and 600 K in the exhaust chamber is most nearly
   (A) 33% (B) 40% (C) 60% (D) 67% (E) 100%

2. An ideal gas is made up of N diatomic molecules, each of mass M. All of the following statements about this gas are true EXCEPT:
   (A) The temperature of the gas is proportional to the average translational kinetic energy of the molecules.
   (B) All of the molecules have the same speed.
   (C) The molecules make elastic collisions with the walls of the container.
   (D) The molecules make elastic collisions with each other.
   (E) The average number of collisions per unit time that the molecules make with the walls of the container depends on the temperature of the gas.

Questions 3 – 4



A thermodynamic system is taken from an initial state X along the path XYZX as shown in the PV-diagram.

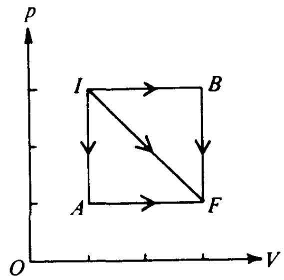
3. For the process X→Y, ΔU is greater than zero and
   (A) Q < 0 and W = 0 (B) Q < 0 and W > 0 (C) Q > 0 and W < 0 (D) Q > 0 and W = 0 (E) Q > 0 and W > 0

4. For the process Y→Z, Q is greater than zero and
   (A) W < 0 & ΔU = 0 (B) W = 0 & ΔU < 0 (C) W = 0 & ΔU > 0 (D) W > 0 & ΔU = 0 (E) W > 0 & ΔU > 0

5. An ideal gas confined in a box initially has pressure p. If the absolute temperature of the gas is doubled and the volume of the box is quadrupled, the pressure is
   (A) p/8 (B) p/4 (C) p/2 (D) p (E) 2p

6. James Joule did much to establish the value of the
   (A) universal gravitational constant (B) speed of light (C) mechanical equivalent of heat
   (D) charge of an electron (E) specific heat capacity of helium

7. An ideal gas in a closed container initially has volume V, pressure P. and Kelvin temperature T. If the temperature is changed to 3T, which of the following pairs of pressure and volume values is possible?
   (A) 3P and V (B) P and V (C) P and V/3 (D) P/3 and V (E) 3P and 3V



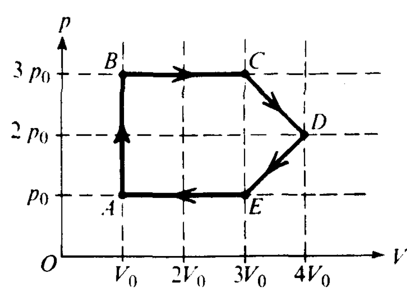
8. If three identical samples of an ideal gas are taken from initial state I to final state F along the paths IAF, IF, and IBF as shown in the pV-diagram above, which of the following must be true?
   (A) The work done by the gas is the same for all three paths.
   (B) The heat absorbed by the gas is the same for all three paths.
   (C) The change in internal energy of the gas is the same for all three paths.
   (D) The expansion along path IF is adiabatic.
   (E) The expansion along path IF is isothermal.

9. If the average kinetic energy of the molecules in an ideal gas at a temperature of 300 K is E, the average kinetic energy at a temperature of 600 K is
   (A) $\frac{E}{\sqrt{2}}$ (B) E (C) $\sqrt{2}E$ (D) $2E$ (E) $4E$

10. A metal rod of length L and cross-sectional area A connects two thermal reservoirs of temperatures T_1 and T_2. The amount of heat transferred through the rod per unit time is directly proportional to
    (A) A and L (B) A and 1/L (C) 1/A and L (D) 1/A and 1/L (E) $\sqrt{A}$ and L^2

11. Which of the following is always a characteristic of an adiabatic process?
    (A) The temperature does not change ($\Delta T = 0$).
    (B) The pressure does not change ($\Delta P = 0$).
    (C) The internal energy does not change ($\Delta U = 0$).
    (D) No heat flows into or out of the system ($Q = 0$)
    (E) No work is done on or by the system ($W = 0$)

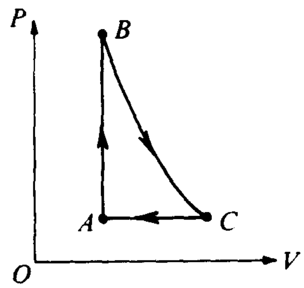
<u>Questions 12 – 13</u>



An ideal gas undergoes a cyclic process as shown on the graph above of pressure *p* versus volume V.

12. During which process is no work done on or by the gas?
    (A) AB (B) BC (C) CD (D) DE (E) EA

13. At which point is the gas at its highest temperature?
    (A) A (B) B (C) C (D) D (E) E

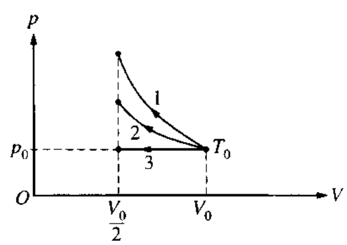


14. Gas in a chamber passes through the cycle ABCA as shown in the diagram above. In the process AB, 12 joules of heat is added to the gas. In the process BC, no heat is exchanged with the gas. For the complete cycle ABCA, the work done by the gas is 8 joules. How much heat is added to or removed from the gas during process CA?
(A) 20 J is removed. (B) 4 J is removed. (C) 4 J is added. (D) 20 J is added.
(E) No heat is added to or removed from the gas.

15. If the gas in a container absorbs 275 joules of heat, has 125 joules of work done on it, and then does 50 joules of work, what is the increase in the internal energy of the gas?
(A) l00 J (B) 200 J (C) 350 J (D) 400 J (E) 450 J

16. In each cycle of a certain Carnot engine, 100 joules of heat is absorbed from the high-temperature reservoir and 60 joules is exhausted to the low-temperature reservoir. What is the efficiency of the engine?
(A) 40% (B) 60% (C) 67% (D) 150% (E) 167%

<u>Questions 17 – 18</u>

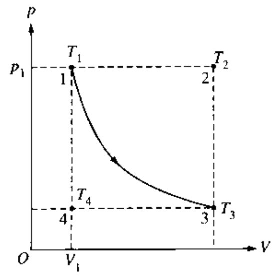


A certain quantity of an ideal gas initially at temperature T_0, pressure p_0, and volume V_0 is compressed to one-half its initial volume. As shown above, the process may be adiabatic (process 1), isothermal (process 2), or isobaric (process 3).

17. Which of the following is true of the mechanical work done on the gas?
(A) It is greatest for process 1.
(B) It is greatest for process 3.
(C) It is the same for processes I and 2 and less for process 3.
(D) It is the same for processes 2 and 3 and less for process 1.
(E) It is the same for all three processes.

18. Which of the following is true of the final temperature of this gas?
(A) It is greatest for process 1. (B) It is greatest for process 2.
(C) It is greatest for process 3. (D) It is the same for processes 1 and 2.
(E) It is the same for processes 1 and 3.

19. In a certain process, 400 J of heat is added to a system and the system simultaneously does 100 J of work. The change in internal energy of the system is
(A) 500 J (B) 400 J (C) 300 J (D) –100 J (E) –300 J



20. An ideal gas is initially in a state that corresponds to point 1 on the graph above, where it has pressure p_1, volume V_1, and temperature T_1. The gas undergoes an isothermal process represented by the curve shown, which takes it to a final state 3 at temperature T_3. If T_2 and T_4 are the temperatures the gas would have at points 2 and 4, respectively, which of the following relationships is true?
(A) $T_1 < T_3$ (B) $T_1 < T_2$ (C) $T_1 < T_4$ (D) $T_1 = T_2$ (E) $T_1 = T_4$

21. The absolute temperature of a sample of monatomic ideal gas is doubled at constant volume. What effect, if any, does this have on the pressure and density of the sample of gas?

Pressure	Density
(A) Remains the same	Remains the same
(B) Remains the same	Doubles
(C) Doubles	Remains the same
(D) Doubles	Is multiplied by a factor of 4
(E) Is multiplied by a factor of 4	Doubles

22. Which of the following statements is NOT a correct assumption of the classical model of an ideal gas?
(A) The molecules are in random motion.
(B) The volume of the molecules is negligible compared with the volume occupied by the gas.
(C) The molecules obey Newton's laws of motion.
(D) The collisions between molecules are inelastic.
(E) The only appreciable forces on the molecules are those that occur during collisions.

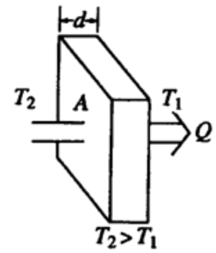
23. A sample of an ideal gas is in a tank of constant volume. The sample absorbs heat energy so that its temperature changes from 300 K to 600 K. If v_1 is the average speed of the gas molecules before the absorption of heat and v_2 is their average speed after the absorption of heat, what is the ratio v_2/v_1?
(A) 1/2 (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

24. Two blocks of steel, the first of mass 1 kg and the second of mass 2 kg, are in thermal equilibrium with a third block of aluminum of mass 2 kg that has a temperature of 400 K. What are the respective temperatures of the first and second steel blocks?
(A) 400 K and 200 K (B) 200 K and 400 K (C) 400 K and 400 K
(D) 800 K and 400 K (E) None of the above

25. An ideal gas may be taken from one state to another state with a different pressure, volume, and temperature along several different paths. Quantities that will always be the same for this process, regardless of which path is taken, include which of the following?
I. The change in internal energy of the gas
II. The heat exchanged between the gas and its surroundings
III. The work done by the gas
(A) I only (B) II only (C) I and III only (D) II and III only (E) I, II, and III

26. A square steel plate with sides of length 1.00 m has a hole in its center 0.100 m in diameter. If the entire plate is heated to such a temperature that its sides become 1.01 m long, the diameter of the hole will be
(A) 0.090 m (B) 0.099 m (C) 0.100 m (D) 0.101 m (E) 0.110 m

27. Which of the following will occur if the average speed of the gas molecules in a closed rigid container is increased?
   (A) The density of the gas will decrease.
   (B) The density of the gas will increase.
   (C) The pressure of the gas will increase.
   (D) The pressure of the gas will decrease.
   (E) The temperature of the gas will decrease.



28. In time *t*, an amount of heat *Q* flows through the solid door of area *A* and thickness *d* represented above. The temperatures on each side of the door are T_2 and T_1, respectively. Which of the following changes would be certain to decrease *Q*?
   (A) Increasing *A* only (B) Decreasing *d* only (C) Increasing *d* and $T_2 - T_1$ only
   (D) Decreasing *A* and $T_2 - T_1$ only (E) Increasing *d*, *A*, and $T_2 - T_1$

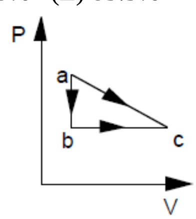
29. A gas with a fixed number of molecules does 32 J of work on its surroundings, and 16 J of heat are transferred from the gas to the surroundings. What happens to the internal energy of the gas?
   (A) It decreases by 48 J. (B) It decreases by 16 J. (C) It remains the same. (D) It increases by 16 J.
   (E) It increases by 48 J.

30. Which of the following could *NOT* be used to indicate a temperature change? A change in:
   (A) color of a metal rod. (B) length of a liquid column. (C) pressure of a gas at constant volume.
   (D) electrical resistance. (E) mass of one mole of gas at constant pressure.

31. A mass *m* of helium gas is in a container of constant volume *V*. It is initially at pressure *p* and absolute (Kelvin) temperature *T*. Additional helium is added, bringing the total mass of helium gas to *3m*. After this addition, the temperature is found to be *2T*. What is the gas pressure?
   (A) 2/3 *p* (B) 3/2 *p* (C) 2 *p* (D) 3 *p* (E) 6 *p*

32. Which would be the most comfortable temperature for your bath water?
   (A) 0ºC (B) 40 K (C) 110ºC (D) 310 K (E) 560 K

33. The theoretical (Carnot) efficiency of a heat engine operating between 600ºC and 100ºC is:
   (A) 16.7% (B) 20.0% (C) 42.7% (D) 57.3% (E) 83.3%



34. A gas can be taken from state *a* to *c* by two different reversible processes, *a*⇨*c* or *a*⇨*b*⇨*c*. During the direct process *a*⇨*c*, 20.0 J of work are done by the system and 30.0 J of heat are added to the system. During the process *a*⇨*b*⇨*c*, 25.0 J of heat are added to the system. How much work is done by the system during *a*⇨*b*⇨*c*?
   (A) 5.0 J (B) 10.0 J (C) 15.0 J (D) 20.0 J (E) 25.0 J

35. What temperature change on the Kelvin scale is equivalent to a 10 degree change on the Celsius scale?
   (A) 283 K (B) 273 K (C) 18 K (D) 10 K (E) 0

36. When an ideal gas is isothermally compressed:
    (A) thermal energy flows from the gas to the surroundings.
    (B) the temperature of the gas decreases.
    (C) no thermal energy enters or leaves the gas.
    (D) the temperature of the gas increases.
    (E) thermal energy flows from the surroundings to the gas.

37. Which of the following is always true for an isothermal process of an ideal gas?
    (A) The internal energy does not change.
    (B) No heat flows into or out of the system.
    (C) The pressure does not change.
    (D) The volume does not change.
    (E) No work is done by or on the system.

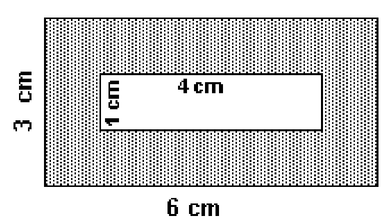
38. The average speed of the atoms of a gas at 100 K is 200 m/s. What would most nearly be the average speed of the atoms at 300 K?
    (A) 67 m/s (B) 140 m/s (C) 200 m/s (D) 350 m/s (E) 600 m/s

39. Which of the following temperatures would be most appropriate to keep milk at inside a refrigerator?
    (A) –20ºC (B) 5 K (C) 40ºC (D) 278 K (E) 350 K

40. A heat engine takes in 200 J of thermal energy and performs 50 J of work in each cycle. What is its efficiency?
    (A) 50 % (B) 40 % (C) 25 % (D) 20 % (E) 12 %

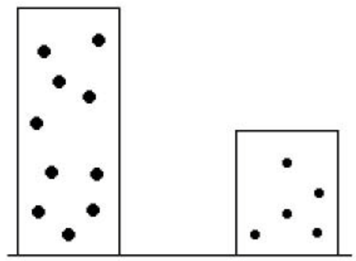
41. The 1st Law of Thermodynamics is a simple statement of the Law of the Conservation of Energy. It was first announced about the time of
    (A) the First World War
    (B) the U.S. Civil War
    (C) the French Revolution
    (D) Christopher Columbus' discovery of the New World
    (E) the fall of the Roman Empire

42. A 200 gram sample of copper is submerged in 100 grams of water until both the copper and water are at the same temperature. Which of the following statements would be true?
    (A) the molecules of the water and copper would have equal average speeds
    (B) the molecules of the water and copper would have equal average momenta
    (C) the molecules of the water and copper would have equal average kinetic energies
    (D) the water molecules would have twice the average momentum of the copper molecules
    (E) the copper molecules would have twice the average speed of the water molecules



43. A rectangular piece of metal 3 cm high by 6 cm wide has a hole cut in its center 1cm high by 4 cm wide as shown in the diagram at right. As the metal is warmed from 0^oC to 100^0C, what will happen to the dimensions of the hole?
    (A) both height and width will increase
    (B) both height and width will decrease
    (C) both height and width will remain unchanged
    (D) height will decrease while width will increase
    (E) height will increase while width will decrease

44. A gas is enclosed in a cylindrical piston. When the gas is heated from 0ºC to 100ºC, the piston is allowed to move to maintain a constant pressure. According to the Kinetic-Molecular Theory of Matter
(A) the mass of the gas will increase
(B) the number of molecules of gas must increase
(C) the size of the individual molecules has increased
(D) the average speed of the molecules has increased
(E) the molecules continue to strike the sides of the container with the same energy



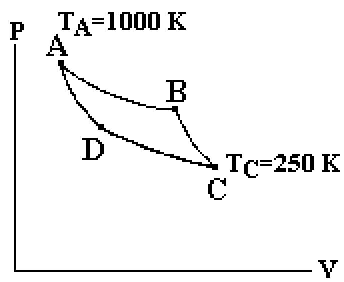
45. Two containers are filled with gases at the same temperature. In the container on the left is a gas of molar mass *2M*, volume *2V,* and number of moles *2n.* In the container on the right is a gas of molar mass *M*, volume *V*, and moles n. Which is most nearly the ratio of the pressure of the gas on the left to the pressure of the gas on the right?
(A) 1:1 (B) 8:1 (C) 2:1 (D) 16:1 (E) 4:1

46. A thermally insulating container has a membrane separating the container into two equal parts. In one part is a vacuum. In the other part is an ideal gas of temperature *T* and internal energy *U.* The membrane is punctured and the gas rushes into the region which was a vacuum. After the system has returned to equilibrium, which of the following is NOT true for the gas?
(A) The temperature of the gas is unchanged.
(B) No work is done by the gas on the surroundings.
(C) There is no heat exchanged by the gas with the surroundings.
(D) There is no entropy change of the system.
(E) The internal energy of the gas is unchanged.

47. A fan blows the air and gives it kinetic energy. An hour after the fan has been turned off, what has happened to the kinetic energy of the air?
(A) it disappears (B) it turns into sound energy (C) it turns into potential energy
(D) it turns into electrical energy (E) it turns into thermal energy

48. According to the kinetic theory of gases, when the absolute temperature of an ideal gas doubles, the average kinetic energy of the molecules of the gas
(A) quadruples (B) doubles (C) stays the same (D) is cut in half (E) is quartered

49. When gas escapes from a pressurized cylinder, the stream of gas feels cool. This is because
(A) work is being done at the expense of thermal energy
(B) of the convection inside the cylinder
(C) pressurized cylinders are good thermal insulators
(D) the gas inside the cylinder is actually frozen
(E) the moisture in the air condenses and cools

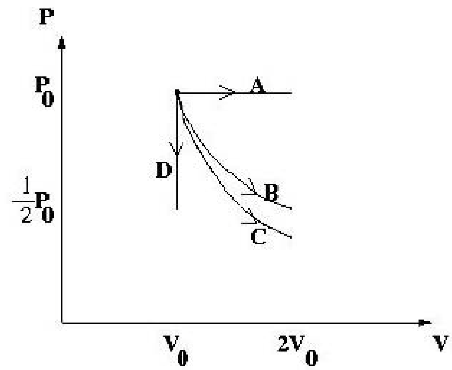


50. A monatomic ideal gas is used as the working substance for the Carnot cycle shown in the figure. Processes A → B and C → D are isothermal, while processes B → C and D → A are adiabatic. During process A → B, there are 400 J of work done by the gas on the surroundings. How much heat is expelled by the gas during process C → D?
(A) 1600 J (B) 800 J (C) 400 J (D) 200 J (E) 100 J

51. Two completely identical samples of the same ideal gas are in equal volume containers with the same pressure and temperature in containers labeled A and B. The gas in container A performs non-zero work W on the surroundings during an isobaric (constant pressure) process before the pressure is reduced isochorically (constant volume) to ½ its initial amount. The gas in container B has its pressure reduced isochorically (constant volume) to ½ its initial value and then the gas performs non-zero work W on the surroundings during an isobaric (constant pressure) process.
After the processes are performed on the gases in containers A and B, which is at the higher temperature?
(A) The gas in container A
(B) The gas in container B
(C) The gases have equal temperature
(D) The value of the work W is necessary to answer this question.
(E) The value of the work W is necessary, along with both the initial pressure and volume, in order to answer the question.

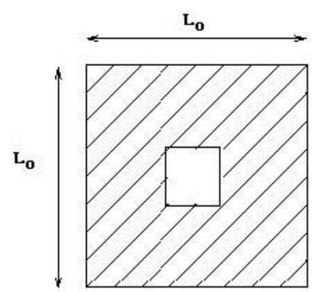
52. The volume of an ideal gas changes as the gas undergoes an isobaric (constant pressure) process starting from temperature 273^{O}C and ending at 546^{O}C What is the ratio of the new volume of the gas to the old volume (V_{new}/V_{old})?
(A) 2 (B) $\frac{3}{2}$ (C) 1 (D) $\frac{2}{3}$ (E) $\frac{1}{2}$

53. A frozen hamburger in plastic needs to be thawed quickly. Which of the methods described provides the most rapid thawing of the burger?
(A) Place the burger itself in a metal pan at room temperature.
(B) Place the burger in its package on the kitchen counter at room temperature.
(C) Place the burger in its package in a pot of non-boiling warm water.
(D) Place the burger itself on a plastic plate in the refrigerator.
(E) Place the burger itself on the ceramic kitchen counter at room temperature.



54. The PV diagram shows four different possible reversible processes performed on a monatomic ideal gas. Process A is isobaric (constant pressure). Process B is isothermal (constant temperature). Process C is adiabatic. Process D is isochoric (constant volume). For which process(es) does the temperature of the gas decrease?
(A) Process A only (B) Process C only (C) Only Processes C and D (D) Only Processes B, C and D
(E) All 4 Processes.

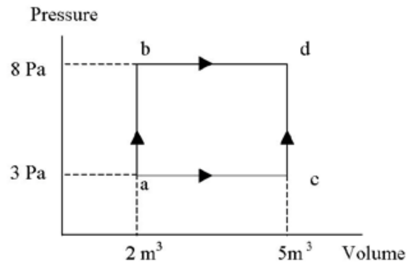
55. A new monatomic ideal gas is discovered and named "Wellsium". A pure 4-mole sample is sitting in a container at equilibrium in a 20.0ºC environment. According to the kinetic theory of gases, what is the average kinetic energy per molecule for this gas?
(A) 4.14×10^{-22} J (B) 3652 J (C) 6.07×10^{-21} J (D) 2.02×10^{-21}J
(E) The molar mass of the gas is needed to answer this question.



56. A uniform square piece of metal has initial side length L_0 . A square piece is cut out of the center of the metal. The temperature of the metal is now raised so that the side lengths are increased by 4%. What has happened to the area of the square piece cut out of the center of the metal?
(A) It is increased by 16 % (B) It is increased by 8 % (C) It is increased by 4 % (D) It is decreased by 4 %
(E) It is decreased by 8 %

57. A monatomic ideal gas at pressure $P = 10^5$ Pa is in a container of volume $V = 12$ m^3 while at temperature $T =$ 50ºC. How many molecules of gas are in the container?
(A) 447 (B) 2888 (C) 6.02×10^{23} (D) 2.69×10^{26} (E) 1.74×10^{27}

58. Absolute zero is best described as that temperature at which
(A) water freezes at standard pressure.
(B) water is at its triple point.
(C) the molecules of a substance have a maximum kinetic energy.
(D) the molecules of a substance have a maximum potential energy.
(E) the molecules of a substance have minimum kinetic energy.



59. In the Pressure versus Volume graph shown, in the process of going from a to b 60 J of heat are added, and in the process of going from b to d 20 J of heat are added. In the process of going a to c to d, what is the total heat added?
(A) 80 J (B) 65 J (C) 60 J (D) 56 J (E) 47 J

60. Which is not true of an isochoric process on an enclosed ideal gas in which the pressure decreases?
(A) The work done is zero. (B) The internal energy of the gas decreases. (C) The heat is zero.
(D) The rms speed of the gas molecules decreases. (E) The gas temperature decreases.

61. One mole of an ideal gas has a temperature of 100°C. If this gas fills the $10.0 m^3$ volume of a closed container, what is the pressure of the gas?
(A) 0.821 *Pa* (B) 3.06 *Pa* (C) 83.1 *Pa* (D) 310 *Pa* (E) 1.84×10^{24} *Pa*

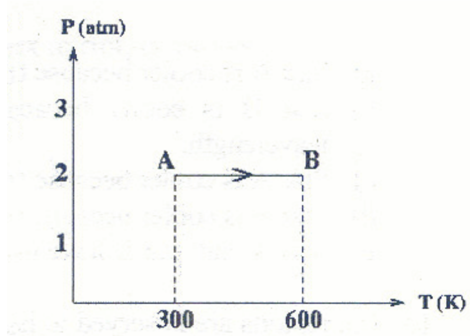
62. An ideal gas is enclosed in a container. The volume of the container is reduced to half the original volume at constant temperature. According to kinetic theory, what is the best explanation for the increase in pressure created by the gas?
(A) The average speed of the gas particles decreases, but they hit the container walls more frequently.
(B) The average speed of the gas particles is unchanged, but they hit the container walls more frequently.
(C) The average speed of the gas particles increases as does the frequency with which they hit the container walls.
(D) The average speed of the gas particles increases, overcoming the decreased frequency that they hit the container walls.
(E) The internal energy of the gas increases.

63. A mole of a monatomic ideal gas has pressure P, volume V, and temperature T. Which of the following processes would result in the greatest amount of energy added to the gas from heat?
(A) A process doubling the temperature at constant pressure.
(B) An adiabatic free expansion doubling the volume.
(C) A process doubling the pressure at constant volume.
(D) An adiabatic expansion doubling the volume.
(E) A process doubling the volume at constant temperature.

64. An ideal gas in a closed container of volume 6.0 *L* is at a temperature of 100ºC. If the pressure of the gas is 2.5 atm, how many moles of gas are in the container?
(A) 0.0048 (B) 0.018 (C) 0.49 (D) 1.83 (E) 490

65. A scientist claims to be investigating "The transfer of energy that results from the bulk motion of a fluid." Which of the following terms best describes the energy transfer method that this scientist is studying?
(A) radiation (B) convection (C) conduction (D) latent heat (E) specific heat

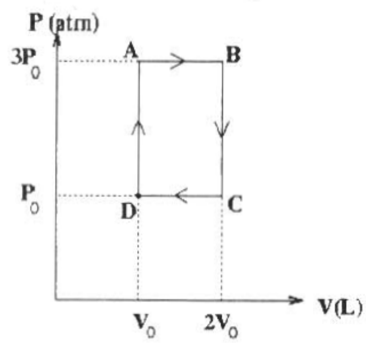
66. An ideal gas undergoes an isobaric expansion followed by an isochoric cooling. Which of the following statements *must* be true after the completion of these processes?
(A) The final pressure is less than the original pressure.
(B) The final volume is less than the original volume.
(C) The final temperature is less than the original temperature.
(D) The total quantity of heat, Q, associated with these processes is positive.
(E) The internal energy of the gas is unchanged.



67. Two moles of a monatomic ideal gas undergoes the process from A to B, shown in the diagram above by the solid line. Using the sign convention that work is positive when surroundings do work on the system, how much work is done in the process AB?
(A) 5000 J (B) 1200 J (C) –1200 J (D) –5000 J
(E) More information is required to determine the amount of work done.

Questions 68 – 69

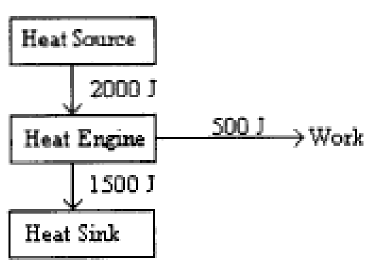
An engine operates on the cycle shown in the PV diagram below. The working substance of the engine is an ideal monatomic gas. The processes $A \rightarrow B$ and $C \rightarrow D$ are isobaric, while processes $B \rightarrow C$ and $D \rightarrow A$ are isochoric.



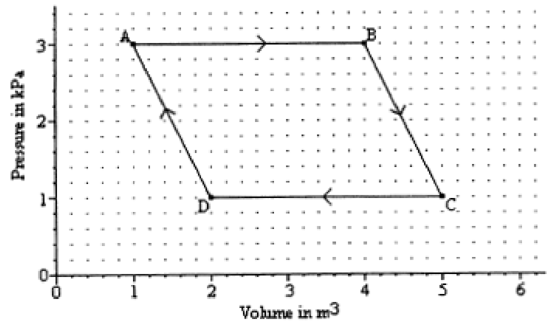
68. What is the efficiency of a Carnot engine operating between the same maximum and minimum temperatures as this engine?
(A) 1/6 (B) 1/3 (C) 1/2 (D) 3/5 (E) 5/6

69. What is the actual efficiency of this engine?
(A) 1/6 (B) 4/21 (C) 5/17 (D) 7/19 (E) 8/15

70. What would be the efficiency of the heat engine diagramed as shown below?



(A) 300 % (B) 133 % (C) 75 % (D) 33 % (E) 25 %



71. A sample of gas is caused to go through the cycle shown in the pV diagram shown above. What is the net work done by the gas during the cycle?
(A) 12,000 J (B) 8000 J (C) 6000 J (D) 4000 J (E) 2000 J

72. A sample of an ideal monatomic gas is confined in a rigid 0.008 m^3 container. If 40 joules of heat energy were added to the sample, how much would the pressure increase?
(A) 5 Pa (B) 320 Pa (C) 1,600 Pa (D) 3,333 Pa (E) 5000 Pa

73. Hydrogen gas (H_2) and oxygen gas (O_2) are in thermal equilibrium. How does the average speed of the hydrogen molecules compare to the average speed of oxygen molecules?
(A) equal (B) 4 times greater (C) 8 times greater (D) 16 times greater (E) 32 times greater

74. A reversible heat engine works between a high temperature reservoir at 227°C and low temperature reservoir of 27°C. If the engine absorbs an amount of heat *Q* at the high temperature reservoir, how much heat will it exhaust at the low temperature reservoir?
(A) 227Q/27 (B) 27Q/227 (C) 5Q/3 (D) 3Q/5 (E) 0

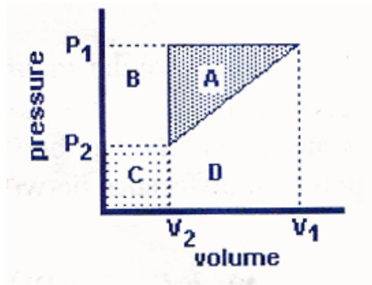
75. If an ideal Carnot engine takes in 500 kJ of heat at 1500 K and expels 200 kJ of heat to the low temperature reservoir during each cycle, which of the following would be closest to the temperature of the low temperature reservoir?
(A) 900 K (B) 750 K (C) 600 K (D) 450 K (E) 300 K

76. Hydrogen gas is contained in a rigid container. A second rigid container of equal volume contains oxygen gas. If the average rms velocities of the molecules in each container is the same, which of the following *must* be true?
(A) The oxygen gas would apply the greater pressure.
(B) The hydrogen gas would apply the greater pressure.
(C) There would be an equal pressure in each container.
(D) The oxygen gas would have the higher temperature.
(E) The temperature of both gasses would be identical.

77. If an idea Carnot heat engine with an efficiency of 30% absorbs heat from a reservoir at 727° C, what must be the exhaust temperature of the engine?
(A) 509°C (B) 427°C (C) 273°C (D) 218°C (E) 0°C

78. A mole of ideal gas at STP is heated in an insulated constant volume container until the average velocity of its molecules doubled. Its pressure would therefore increase by what factor?
(A) 0.5 (B) 1 (C) 2 (D) 4 (E) 8

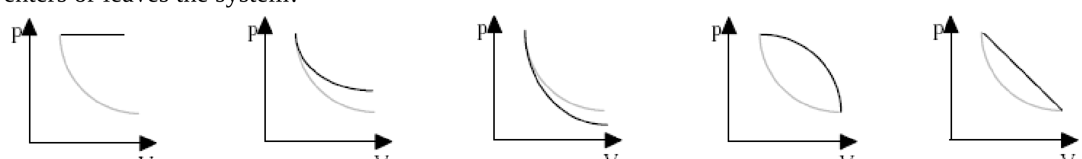
79. An ideal heat engine takes in heat energy at a high temperature and exhausts energy at a lower temperature. If the amount of energy exhausted at the low temperature is 3 times the amount of work done by the heat engine, what is its efficiency?
(A) 0.25 (B) 0.33 (C) 0.67 (D) 0.9 (E) 1.33



80. A sample of gas was first compressed from V_1 to V_2 at a constant pressure of P_1. The sample was then cooled so that the pressure went from P_1 to P_2 while the volume remained constant at V_2. Finally the sample was allowed to expand from V_2 back to V_1 while the pressure increased from P_2 back to P_1 as shown in the diagram. Which of the following statements is correct?
(A) The area A represents the energy that is lost by the gas in this cycle.
(B) The area A + B represents the energy gained by the gas in this cycle.
(C) The area A + C represents the energy gained by the gas in this cycle.
(D) The area B + D represents the energy gained by the gas in this cycle.
(E) There was no energy lost or gained by the gas in this cycle.

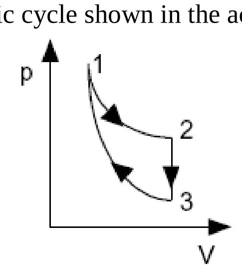
81. One end of a metal rod of length L and cross-sectional area A is held at a constant temperature T_1. The other end is held at a constant T_2. Which of the statements about the amount of heat transferred through the rod per unit time are true?
I. The rate of heat transfer is proportional to $1/(T_1 - T_2)$.
II. The rate of heat transfer is proportional to A
III. The rate of heat transfer is proportional to L.
(A) II only (B) III only (C) I and II only (D) I and III only (E) II and III only

82. On all of the pV diagrams shown below the lighter curve represents isothermal process, a process for which the temperature remains constant. Which dark curve best represents an adiabatic process, a process for which no heat enters or leaves the system?



(A) (B) (C) (D) (E)

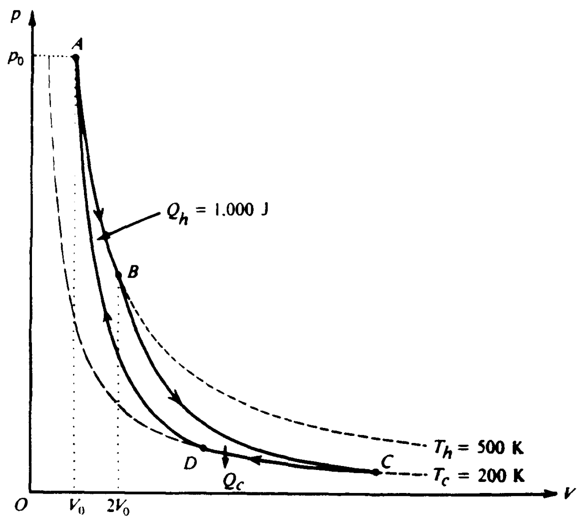
83. Three processes compose a thermodynamic cycle shown in the accompanying pV diagram of an ideal gas.



Process 1→2 takes place at constant temperature (300 K). During this process 60 J of heat enters the system.
Process 2→3 takes place at constant volume. During this process 40 J of heat leaves the system.
Process 3→1 is adiabatic. T_3 is 275 K.

What is the change in internal energy of the system during process 3→1?
(A) –40 J (B) –20 J (C) 0 J (D) +20 J (E) +40 J

AP Physics Free Response Practice – Thermodynamics



1983B4. The pV-diagram above represents the states of an ideal gas during one cycle of operation of a reversible heat engine. The cycle consists of the following four processes.

Process	**Nature of Process**
AB	Constant temperature (T_h = 500 K)
BC	Adiabatic
CD	Constant temperature (T_c = 200 K)
DA	Adiabatic

During process A B, the volume of the gas increases from V_o to $2V_o$ and the gas absorbs 1,000 joules of heat.

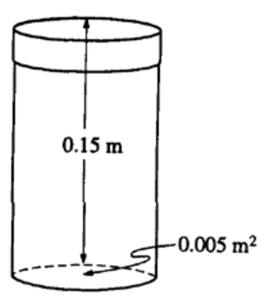
a. The pressure at A is $p_{o.}$ Determine the pressure at B.

b. Using the first law of thermodynamics, determine the work performed on the gas during the process AB.

c. During the process AB, does the entropy of the gas increase, decrease, or remain unchanged? Justify your answer.

d. Calculate the heat Q_c given off by the gas in the process CD.

e. During the full cycle ABCDA is the total work the performed on the gas by its surroundings positive, negative, or zero? Justify your answer.



1996B7 The inside of the cylindrical can shown above has cross-sectional area 0.005 m^2 and length 0.15 m. The can is filled with an ideal gas and covered with a loose cap. The gas is heated to 363 K and some is allowed to escape from the can so that the remaining gas reaches atmospheric pressure (1.0×10^5 Pa). The cap is now tightened, and the gas is cooled to 298 K.

a. What is the pressure of the cooled gas?

b. Determine the upward force exerted on the cap by the cooled gas inside the can.

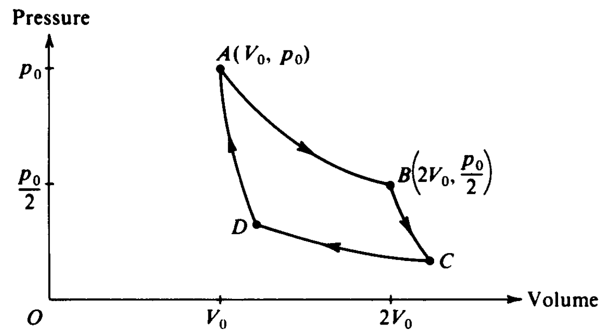
c. If the cap develops a leak, how many moles of air would enter the can as it reaches a final equilibrium at 298 K and atmospheric pressure? (Assume that air is an ideal gas.)

1986B5 (modified) A proposed ocean power plant will utilize the temperature difference between surface seawater and seawater at a depth of 100 meters. Assume the surface temperature is 25° Celsius and the temperature at the 100-meter depth is 3° Celsius.

a. What is the ideal (Carnot) efficiency of the plant?

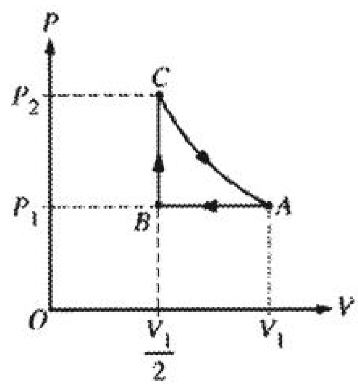
b. If the plant generates useful energy at the rate of 100 megawatts while operating with the efficiency found in part (a), at what rate is heat given off to the surroundings?

The diagram below represents the Carnot cycle for a simple reversible (Carnot) engine in which a fixed amount of gas, originally at pressure p_o and volume V_o follows the path ABCDA.



c. In the chart below, for each part of the cycle indicate with +, -, or 0 whether the heat transferred Q and temperature change ΔT are positive, negative, or zero, respectively. (Q is positive when heat is added to the gas, and ΔT is positive when the temperature of the gas increases.)

	Q	Δ*T*
AB		
BC		
CD		
DA		



2004Bb5 One mole of an ideal gas is initially at pressure P_1, volume V_1, and temperature T_1, represented by point *A* on the *PV* diagram above. The gas is taken around cycle *ABCA* shown. Process *AB* is isobaric, process *BC* is isochoric, and process *CA* is isothermal.

a. Calculate the temperature T_2 at the end of process *AB* in terms of temperature T_1.

b. Calculate the pressure P_2 at the end of process *BC* in terms of pressure P_1.

c. Calculate the net work done on the gas when it is taken from *A* to *B* to *C*. Express your answer in terms of P_1 and V_1.

d. Indicate below all of the processes that result in heat being added to the gas.

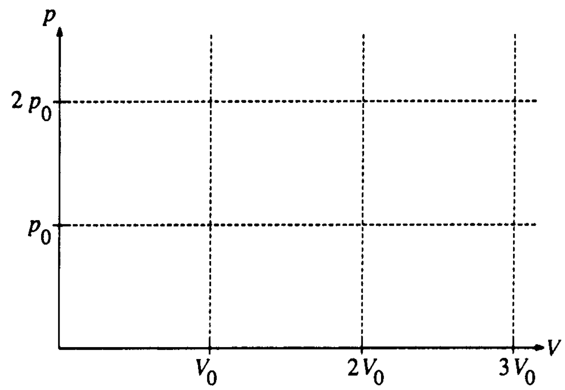
____ *AB* ____ *BC* ____*CA*

Justify your answer.

1989B4 (modified) An ideal gas initially has pressure p_o, volume V_o, and absolute temperature T_o. It then undergoes the following series of processes:

I. It is heated, at constant volume, until it reaches a pressure $2p_o$.
II. It is heated, at constant pressure, until it reaches a volume $3\ V_o$.
III. It is cooled, at constant volume, until it reaches a pressure p_o.
IV. It is cooled, at constant pressure, until it reaches a volume V_o.

a. On the axes below
i. draw the p-V diagram representing the series of processes;
ii. label each end point with the appropriate value of absolute temperature in terms of T_o.

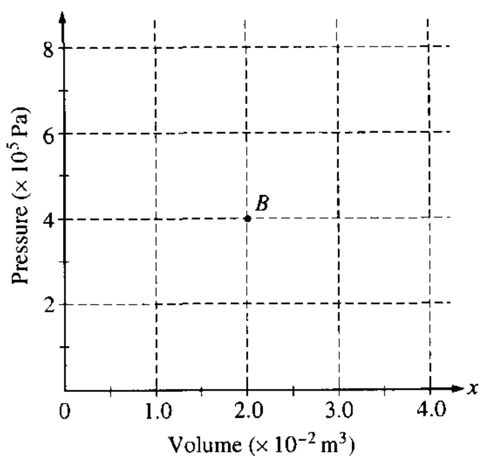


b. For this series of processes, determine the following in terms of p_o and V_o.
i. The net work done on the gas
ii. The net change in internal energy
iii. The net heat absorbed

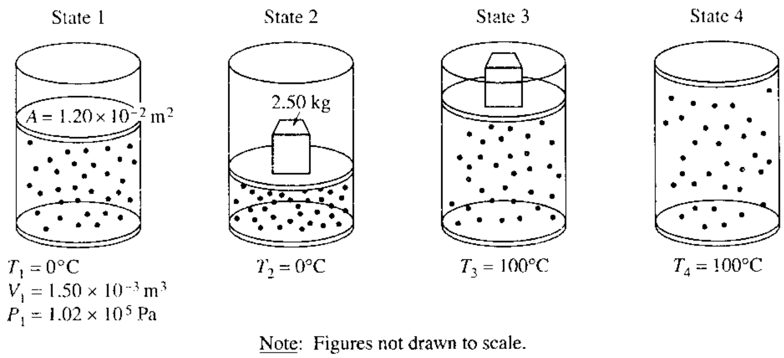
c. Determine the heat transferred during process 2 in terms of p_o and V_o.

1999B7. A cylinder contains 2 moles of an ideal monatomic gas that is initially at state A with a volume of $1.0 \times 10^{-2}\ m^3$ and a pressure of 4.0×10^5 Pa. The gas is brought isobarically to state B. where the volume is $2.0 \times 10^{-2}\ m^3$. The gas is then brought at constant volume to state C, where its temperature is the same as at state A. The gas is then brought isothermally back to state A.

a. Determine the pressure of the gas at state C.
b. On the axes below, state B is represented by the point B. Sketch a graph of the complete cycle. Label points A and C to represent states A and C, respectively.



c. State whether the net work done on the gas during the complete cycle is positive, negative, or zero. Justify your answer.
d. State whether this device is a refrigerator or a heat engine. Justify your answer.



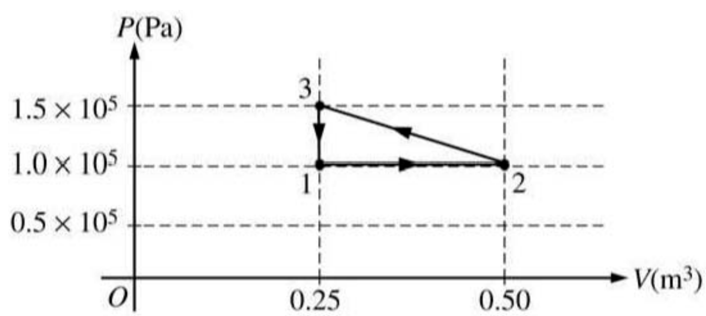
2001B6. A cylinder is fitted with a freely moveable piston of area 1.20×10^{-2} m^2 and negligible mass. The cylinder below the piston is filled with a gas. At state 1, the gas has volume 1.50×10^{-3} m^3, pressure 1.02×10^5 Pa, and the cylinder is in contact with a water bath at a temperature of 0°C. The gas is then taken through the following four-step process.

- A 2.50 kg metal block is placed on top of the piston, compressing the gas to state 2, with the gas still at 0°C.
- The cylinder is then brought in contact with a boiling water bath, raising the gas temperature to 100°C at state 3.
- The metal block is removed and the gas expands to state 4 still at 100°C.
- Finally, the cylinder is again placed in contact with the water bath at 0°C, returning the system to state 1.

a. Determine the pressure of the gas in state 2.
b. Determine the volume of the gas in state 2.
c. Indicate below whether the process from state 2 to state 3 is isothermal, isobaric, or adiabatic.

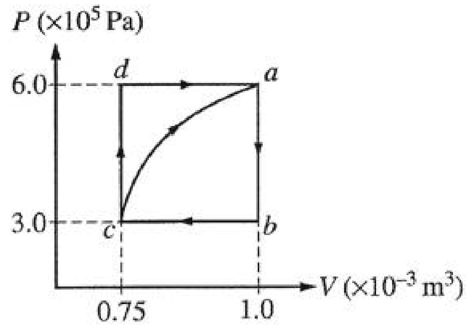
_____Isothermal _____Isobaric _____Adiabatic
Explain your reasoning.

d. Is the process from state 4 to state 1 isobaric? _____Yes _____No
Explain your reasoning.
e. Determine the volume of the gas in state 4.



2006B5 A cylinder with a movable frictionless piston contains an ideal gas that is initially in state 1 at 1×10^5 Pa, 373 K, and 0.25 m^3. The gas is taken through a reversible thermodynamic cycle as shown in the *PV* diagram above.

a. Calculate the temperature of the gas when it is in the following states.
i. State 2
ii. State 3
b. Calculate the net work done on the gas during the cycle.
c. Was heat added to or removed from the gas during the cycle?
Added _____ Removed _____ Neither added nor removed _____
Justify your answer.



2003B5. A cylinder with a movable piston contains 0.1 mole of a monatomic ideal gas. The gas, initially at state *a*, can be taken through either of two cycles, *abca* or *abcda*, as shown on the PV diagram above. The following information is known about this system.

$Q_{c \to a}$ = 685 J along the curved path
$W_{c \to a}$ = –120 J along the curved path
$U_a - U_b$ = 450 J
$W_{a \to b \to c}$ = 75 J

a. Determine the change in internal energy, $U_a - U_c$, between states *a* and *c*.
b. i. Is heat added to or removed from the gas when the gas is taken along the path *abc ?*

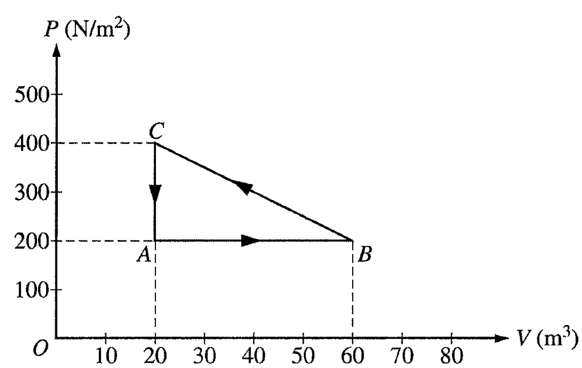
_____added to the gas _____removed from the gas

ii. Calculate the amount added or removed.

c. How much work is done on the gas in the process *cda?*
d. Is heat added to or removed from the gas when the gas is taken along the path *cda?*

_____added to the gas _____removed from the gas

Explain your reasoning.



2003Bb5. One mole of an ideal gas is taken around the cycle $A \to B \to C \to A$ as shown on the PV diagram above.

a. Calculate the temperature of the gas at point *A*.
b. Calculate the net work done on the gas during one complete cycle.
c. i. Is heat added to or removed from the gas during one complete cycle?

_____added to the gas _____removed from the gas

ii. Calculate the heat added to or removed from the gas during one complete cycle.

d. After one complete cycle, is the internal energy of the gas greater, less, or the same as before?

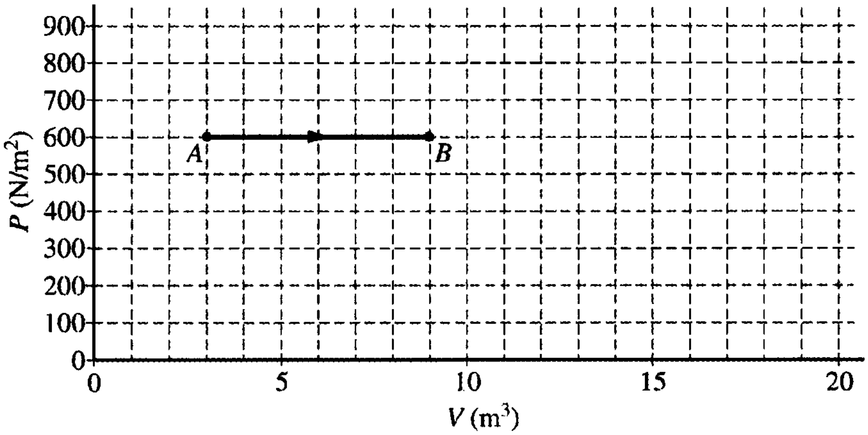
_____greater _____less _____the same

Justify your answer.

e. After one complete cycle, is the entropy of the gas greater, less, or the same as before?

_____greater _____less _____the same

Justify your answer.

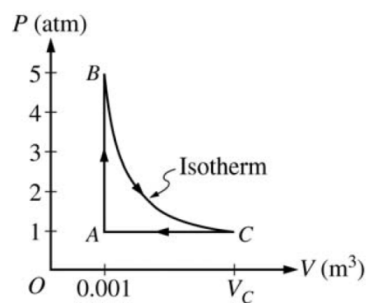


2004B5. The diagram above of pressure P versus volume V shows the expansion of 2.0 moles of a monatomic ideal gas from state A to state B. *As* shown in the diagram, $P_A = P_B = 600$ N/m^2, V_A = 3.0 m^3, and V_B = 9.0 m^3.

a. i. Calculate the work done <u>by the gas</u> as it expands.
ii. Calculate the change in internal energy of the gas as it expands.
iii. Calculate the heat added to or removed from the gas during this expansion.

b. The pressure is then reduced to 200 N/m^2 without changing the volume as the gas is taken from state B to state C. Label state C on the diagram and draw a line or curve to represent the process from state B to state C.

c. The gas is then compressed isothermally back to state A.
i. Draw a line or curve on the diagram to represent this process.
ii. Is heat added to or removed from the gas during this isothermal compression?
_______added to   _______removed from
Justify your answer.



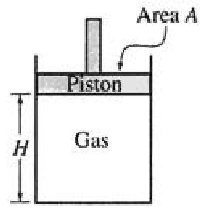
2008B5. A 0.03 mol sample of helium is taken through the cycle shown in the diagram above. The temperature of state A is 400 K.

a. For each process in this cycle, indicate in the table below whether the quantities W, Q, and ΔU are positive (+), negative (–), or zero (0). W is the work done on the helium sample.

Process	W	Q	ΔU
$A \rightarrow B$			
$B \rightarrow C$			
$C \rightarrow A$			

b. Explain your response for the signs of the quantities for process A⇨B.

c. Calculate V_c.

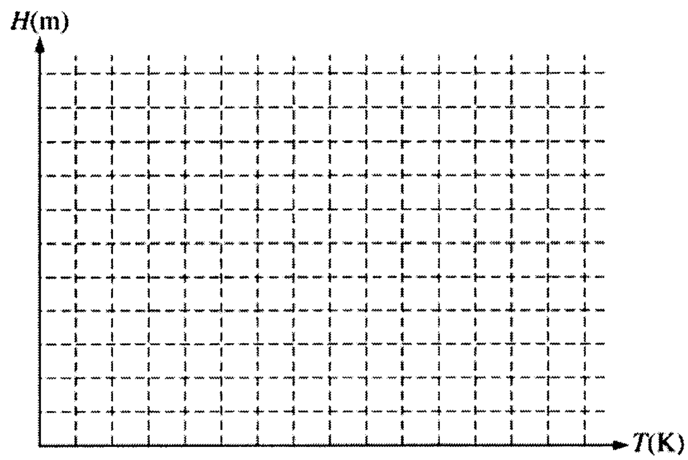


2005B6. An experiment is performed to determine the number *n* of moles of an ideal gas in the cylinder shown above. The cylinder is fitted with a movable, frictionless piston of area *A*. The piston is in equilibrium and is supported by the pressure of the gas. The gas is heated while its pressure P remains constant. Measurements are made of the temperature *T* of the gas and the height *H* of the bottom of the piston above the base of the cylinder and are recorded in the table below. Assume that the thermal expansion of the apparatus can be ignored.

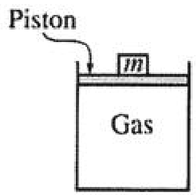
T (K)	*H* (m)
300	1.11
325	1.19
355	1.29
375	1.37
405	1.47

a. Write a relationship between the quantities *T* and *H,* in terms of the given quantities and fundamental constants, that will allow you to determine *n.*

b. Plot the data on the axes below so that you will be able to determine *n* from the relationship in part (a). Label the axes with appropriate numbers to show the scale.



c. Using your graph and the values A = 0.027 m^2 and P = 1.0 atmosphere, determine the experimental value of *n.*



Note: Figure not drawn to scale.

2005Bb6. You are given a cylinder of cross-sectional area A containing n moles of an ideal gas. A piston fitting closely in the cylinder is lightweight and frictionless, and objects of different mass m can be placed on top of it, as shown in the figure above. In order to determine n, you perform an experiment that consists of adding 1 kg masses one at a time on top of the piston, compressing the gas, and allowing the gas to return to room temperature T before measuring the new volume V. The data collected are given in the table below.

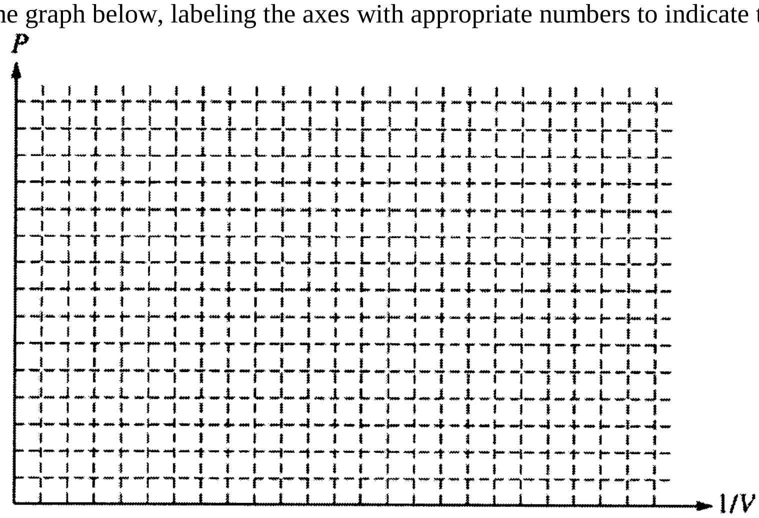
m (kg)	V (m^3)	$1/V$ (m^{-3})	P (Pa)
0	6.0×10^{-5}	1.7×10^{4}	
1	4.5×10^{-5}	2.2×10^{4}	
2	3.6×10^{-5}	2.8×10^{4}	
3	3.0×10^{-5}	3.3×10^{4}	
4	2.6×10^{-5}	3.8×10^{4}	

a. Write a relationship between total pressure P and volume V in terms of the given quantities and fundamental constants that will allow you to determine n.

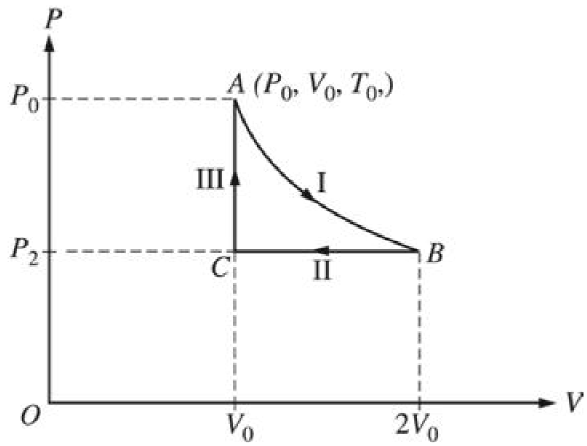
You also determine that $A = 3.0 \times 10^{-4}\ m^2$ and $T = 300$ K.

b. Calculate the value of P for each value of m and record your values in the data table above.

c. Plot the data on the graph below, labeling the axes with appropriate numbers to indicate the scale.



d. Using your graph in part (c), calculate the experimental value of n.



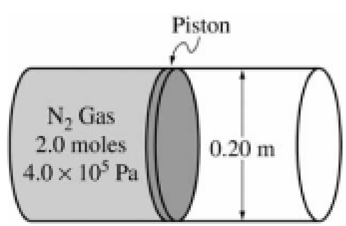
2006Bb5. A sample of ideal gas is taken through steps I, II, and III in a closed cycle, as shown on the pressure P versus volume V diagram above, so that the gas returns to its original state. The steps in the cycle are as follows.

I. An isothermal expansion occurs from point A to point B, and the volume of the gas doubles.
II. An isobaric compression occurs from point B to point C, and the gas returns to its original volume.
III. A constant volume addition of heat occurs from point C to point A and the gas returns to its original pressure.

a. Determine numerical values for the following ratios, justifying your answers in the spaces next to each ratio.

i. $\frac{P_B}{P_A} =$ ii. $\frac{P_C}{P_A} =$ iii. $\frac{T_B}{T_A} =$ iv. $\frac{T_C}{T_A} =$

b. During step I, the change in internal energy is zero. Explain why.
c. During step III, the work done on the gas is zero. Explain why.



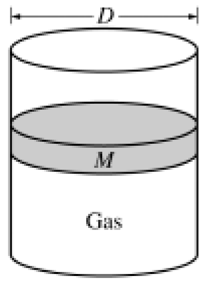
2007B5. The figure above shows a 0.20 m diameter cylinder fitted with a frictionless piston, initially fixed in place. The cylinder contains 2.0 moles of nitrogen gas at an absolute pressure of 4.0×10^5 Pa. Nitrogen gas has a molar mass of 28 g/mole and it behaves as an ideal gas.
a. Calculate the force that the nitrogen gas exerts on the piston.
b. Calculate the volume of the gas if the temperature of the gas is 300 K.
c. In a certain process, the piston is allowed to move, and the gas expands at constant pressure and pushes the piston out 0.15 m. Calculate how much work is done by the gas.
d. Which of the following is true of the heat energy transferred to or from the gas, if any, in the process in part (c)?

_______Heat is transferred to the gas.

_______Heat is transferred from the gas.

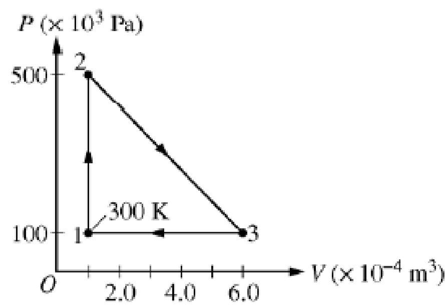
_______No heat is transferred in the process.

Justify your answer.

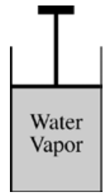


B2007b5. The cylinder above contains an ideal gas and has a movable, frictionless piston of diameter D and mass M. The cylinder is in a laboratory with atmospheric pressure P_{atm} . Express all algebraic answers in terms of the given quantities and fundamental constants.

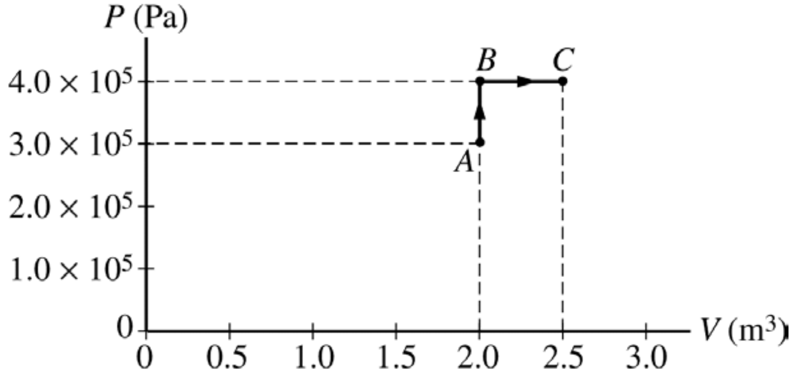
a. Initially, the piston is free to move but remains in equilibrium. Determine each of the following.
   i. The force that the confined gas exerts on the piston
   ii. The absolute pressure of the confined gas
b. If a net amount of heat is transferred to the confined gas when the piston is fixed, what happens to the pressure of the gas?
   _______Pressure goes up. _______Pressure goes down. _______Pressure stays the same.
   Explain your reasoning.
c. In a certain process the absolute pressure of the confined gas remains constant as the piston moves up a distance x_o. Calculate the work done by the confined gas during the process.



2008Bb6. A 0.0040 mol sample of a monatomic gas is taken through the cycle shown above. The temperature T_1 of state 1 is 300 K.
a. Calculate T_2 and T_3.
b. Calculate the amount of work done on the gas in one cycle.
c. Is the net work done on the gas in one complete cycle positive, negative, or zero?
   _____Positive _____Negative _____Zero
d. Calculate the heat added to the gas during process $1 \rightarrow 2$.



2009B4. The cylinder represented above contains 2.2 kg of water vapor initially at a volume of 2.0 m^3 and an absolute pressure of 3.0×10^5 Pa. This state is represented by point *A* in the *PV* diagram below. The molar mass of water is 18 g, and the water vapor can be treated as an ideal gas.



a. Calculate the temperature of the water vapor at point *A*.

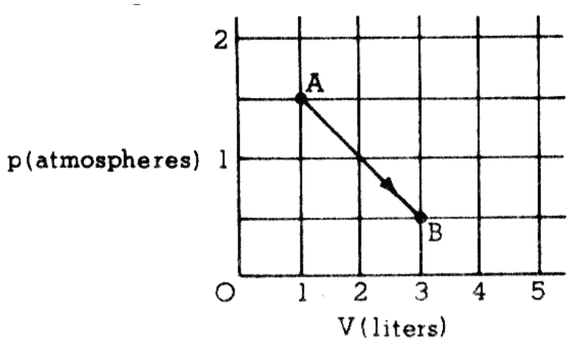
The absolute pressure of the water vapor is increased at constant volume to 4.0×10^5 Pa at point *B*, and then the volume of the water vapor is increased at constant pressure to 2.5 m^3 at point *C*, as shown in the *PV* diagram.

b. Calculate the temperature of the water vapor at point *C*.

c. Does the internal energy of the water vapor for the process *A*→*B*→*C* increase, decrease, or remain the same?
____Increase ____Decrease ____Remain the same
Justify your answer.

d. Calculate the work done on the water vapor for the process *A*→*B*→*C*.

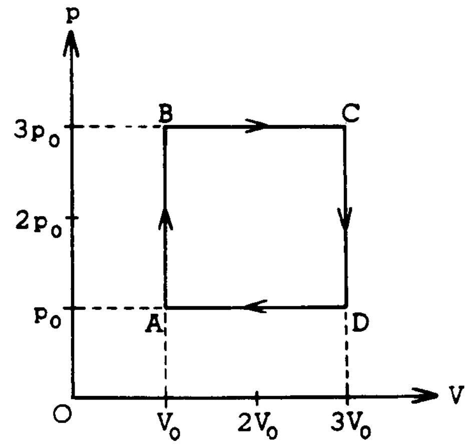
1974B6. One-tenth of a mole of an ideal monatomic gas undergoes a process described by the straight-line path AB shown in the p-V diagram below.



a. Show that the temperature of the gas is the same at points A and B.

b. How much heat must be added to the gas during the process described by A $\rightarrow$ B?

c. What is the highest temperature of the gas during the process described by A $\rightarrow$ B?



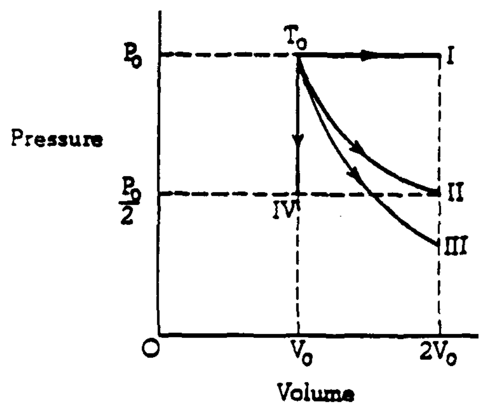
1975B3. One mole of a monatomic ideal gas enclosed in a cylinder with a movable piston undergoes the process ABCDA shown on the p-V diagram above.

a. In terms of p_o and V_o calculate the work done on the gas in the process.

b. In terms of p_o and V_o calculate the net heat absorbed by the gas in the process.

c. At what two lettered points in the process are the temperatures equal? Explain your reasoning.

d. Consider the segments AB and BC. In which segment is the amount of heat added greater? Explain your reasoning.



1979B5. Four samples of ideal gas are each initially at a pressure P_o and volume V_o, and a temperature T_o as shown on the diagram above. The samples are taken in separate experiment from this initial state to the final states I, II, III, and IV along the processes shown on the diagram.

a. One of the processes is isothermal. Identify which one and explain.

b. One of the processes is adiabatic. Identify this one and explain.

c. In which process or processes does the gas do work? Explain.

d. In which process or processes is heat removed from the gas? Explain.

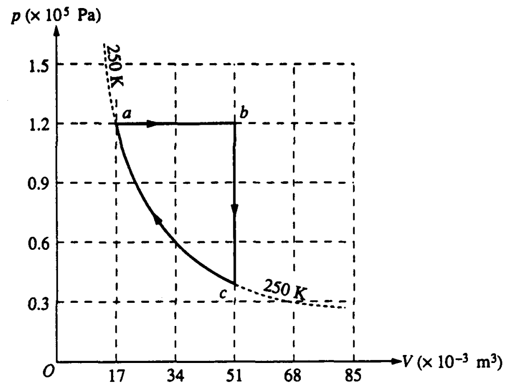
e. In which process or processes does the root-mean-square speed of the gas molecules increase? Explain.

1991B3 (modified) A heat engine consists of an oil-fired steam turbine driving an electric power generator with a power output of 120 megawatts. The thermal efficiency of the heat engine is 40 percent.

a. Determine the time rate at which heat is supplied to the engine.

b. If the heat of combustion of oil is 4.4×10^7 joules per kilogram, determine the rate in kilograms per second at which oil is burned.

c. Determine the time rate at which heat is discarded by the engine.



1993B5. One mole of an ideal monatomic gas is taken through the cycle abca shown on the diagram above. State a has volume $V_a = 17 \times 10^{-3}$ cubic meter and pressure $P_a = 1.2 \times 10^5$ pascals, and state c has volume $V_c = 51 \times 10^{-3}$ cubic meter. Process ca lies along the 250 K isotherm. Determine each of the following.

a. The temperature T_b of state b

b. The heat Q_{ab} added to the gas during process ab

c. The change in internal energy $U_b - U_a$

d. The work W_{bc} done by the gas on its surroundings during process bc

The net heat added to the gas for the entire cycle 1,800 joules. Determine each of the following.

e. The net work done on the gas by its surroundings for the entire cycle

f. The efficiency of a Carnot engine that operates between the maximum and minimum temperatures in this cycle

---

1995B5. A heat engine operating between temperatures of 500 K and 300 K is used to lift a 10-kilogram mass vertically at a constant speed of 4 meters per second.

a. Determine the power that the engine must supply to lift the mass.

b. Determine the maximum possible efficiency at which the engine can operate.

c. If the engine were to operate at the maximum possible efficiency, determine the following.

i. The rate at which the hot reservoir supplies heat to the engine

ii. The rate at which heat is exhausted to the cold reservoir

ANSWERS - AP Physics Multiple Choice Practice – Thermodynamics

	Solution	Answer
1.	$e_c = \frac{T_H - T_C}{T_H}$	C
2.	While *all* collisions are elastic and $K_{avg} \propto T$, the molecules move with a wide range of speeds represented by the Maxwellian distribution.	B
3.	For X⇨Y, the process is isobaric. Since the gas is expanding, W < 0 and since the temperature is increasing, $\Delta U > 0$ and $\Delta U = Q + W$ so Q > 0 (it is also true because process XY lies above an adiabatic expansion from point X)	C
4.	For Y⇨Z, the process is isochoric, which means no work is done (W = 0) and since the temperature is increasing, $\Delta U > 0$	C
5.	$PV \propto T$, or $P \propto T/V$ and if T × 2 then P × 2 and if V × 4 then P ÷ 4 so the net effect is P×2÷4	C
6.	James Joule did experiments on changing the temperature of water through various means, including by doing work on it.	C
7.	$PV \propto T$ so to triple the temperature, the product of P and V must be tripled	A
8.	Changes in internal energy are path independent on a pV diagram as it depends on the change in temperature, which is based on the beginning and end points of the path and not the path taken	C
9.	$K_{avg} \propto T$	D
10.	$H = \frac{kA\Delta T}{L}$	B
11.	by definition	D
12.	No work is done in an isochoric process, or a process where $\Delta V = 0$ (a vertical line on the pv graph)	A
13.	The temperature at any point is proportional to the product of P and V. Point A at temperature T_0 is at pressure × volume p_0V_0. Point C is at $3p_0 \times 3V_0 = 9T_0$ and point D is at $2p_0 \times 4V_0 = 8T_0$	C
14.	For the entire cycle, $\Delta U = 0$ and W = –8 J so $Q = \Delta U - W$ = +8 J (8 J added). This means $Q_{AB} + Q_{BC} + Q_{CA}$ = +8 J = +12 J + 0 J + Q_{CA} = +8 J	B
15.	Q = +275 J; W = + 125 J + (– 50 J) = +75 J; $\Delta U = Q + W$	C
16.	Q_H = 100 J and Q_C = 60 J; $e = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$	A
17.	Work is the area under the curve, the line bounding the greatest area indicates the most work done	A
18.	Temperature rises as you travel up and to the right on a pV diagram. Since processes 1, 2 and 3 are at the same volume, the highest point is at the highest temperature	A
19.	Q = +400 J; W = – 100 J; $\Delta U = Q + W$	C
20.	Isothermal means the temperature is constant. Points to the right or above are at higher temperatures.	B

21. $P \propto T$ at constant volume. If $T \times 2$, then $P \times 2$. Since the mass and volume are unchanged, the density is unchanged as well — C

22. If the collisions were inelastic, the gas would change its temperature by virtue of the collisions with no change in pressure or volume. — D

23. related to average speed, $v_{rms} = \sqrt{\frac{3RT}{M}}$ — C

24. Being in thermal equilibrium means the objects are at the same temperature. Mass is irrelevant. The question describes the zeroth law of thermodynamics. — C

25. Changes in internal energy are path independent on a pV diagram as it depends on the change in temperature, which is based on the beginning and end points of the path and not the path taken. Different paths, with different areas under them will do different amounts of work and hence, different amounts of heat exchanged. — A

26. In linear expansion, every linear dimension of an object changes by the same fraction when heated or cooled. — D

27. "rigid container" = constant volume. If the speed increases, the temperature will increase, and if the temperature increases at constant volume, the pressure will increase. — C

28. $H = \frac{kA\Delta T}{L}$ — D

29. $Q = -16$ J; $W = -32$ J; $\Delta U = Q + W$ — A

30. Mass is independent of the state of a gas. ("color" will be addressed in a later topic, think about a yellow vs. blue flame or a "red hot" piece of metal) — E

31. $P \propto nT/V$; if $n \times 3$ then $P \times 3$ and if $T \times 2$ then $P \times 2$, the net effect is $P \times 3 \times 2$ — E

32. Comfortable bath water should be slightly above room temperature. Room temperature is about 20ºC, or 293 K — D

33. $e_c = \frac{T_H - T_C}{T_H}$ (use absolute temperatures) — D

34. ΔU for each process is equal so $Q_{AC} + W_{AC} = Q_{ABC} + W_{ABC}$, or +30 J + (–20 J) = +25 J + W_{ABC} — C

35. While temperatures are different on the Celsius and Kelvin scale, the temperature *intervals* are identical. 1ºC = 274 K, but 1 Cº = 1 K — D

36. In any compression, work is done on the gas (W is +). Since the compression is isothermal, $\Delta U = 0$ so $Q = -W$ and heat leaves the gas. — A

37. $\Delta U \propto \Delta T$ — A

38. $v_{rms} = \sqrt{\frac{3RT}{M}}$, if T is tripled, v is multiplied by $\sqrt{3}$ — D

39. A refrigerator should be less than room temperature, but above the freezing point of water (between 0ºC and 20ºC, or 273 K to 293 K) — D

40. $e = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$ — C

41. 1^{st} law was first described by Clausius in 1850. (this will not be tested, but it's always good to have a reference for important laws) B

42. $K_{avg} \propto T$ C

43. In linear expansion, every linear dimension of an object changes by the same fraction when heated or cooled. A

44. $K_{avg} \propto T$ D

45. $P \propto n/V$ at constant temperature A

46. Since the container is insulated, no heat is exchanged (C is true), since there is no work done (no force required to expand), choice B is true. Since Q = 0 and W = 0, ΔU and $\Delta T = 0$ (A and E are true). While entropy change does have a heat component (and if Q = 0, the change in entropy may be incorrectly regarded as zero) it also has a volume component (how "spread out" the gas is) D

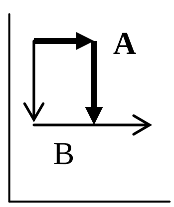
47. This question is a bit of a paradox as the energy form the fan giving the air kinetic energy is theoretically adding to the thermal energy of the air, But as the air lowers in temperature, this energy will dissipate into the walls and other outside areas of the room as thermal energy as well. E

48. $K_{avg} \propto T$ B

49. Gas escaping form a pressurized cylinder is an example of an adiabatic process. While the gas rapidly does work (W < 0), ΔU is negative since heat does not have time to flow into the gas in a rapid expansion. A

50. In a Carnot cycle $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$ and in process AB, $\Delta U = 0$ and since $W_{AB} = -400$ J, $Q_{AB} = +400$ J and this is Q_H E

51. Since process A and B perform the same amount of work, they must have the same area under their respective lines. Since A does the work at a higher pressure, it does not have to move as far to the right as process B, which performs the work at a lower temperature. Since the end of process B lies farther to the right, it is at the higher temperature. B



52. At constant pressure $V \propto T$ (use absolute temperature) B

53. Metals are the best heat conductors and will conduct heat out of the hamburger quickly A

54. Consider the isothermal line as the "dividing line" between process that increase the temperature of the gas (above the isotherm) and process that lower the temperature of the gas (below the isotherm). A similar analysis can be done to identify heat added or removed from a gas by comparing a process to an adiabat drawn from the same point. C

55. $K_{avg} = 3/2\ k_BT$ (use absolute temperature) C

56. In linear expansion, every linear dimension of an object changes by the same fraction when heated or cooled. Since each side increases by 4%,the area increases by $(1.04)^2 = 1.08$ B

57. $pV = nRT$ and $n = N/N_A$ D

58. $K_{avg} \propto T$ (absolute) E

59. $Q_{abd} = +60$ J + 20 J = + 80 J. W_{abd} = area, negative due to expansion = –24 J so $\Delta U = Q + W$ = +56 J and $\Delta U_{abd} = \Delta U_{acd}$ and W_{acd} = area = –9 J so $Q_{acd} = \Delta U - W_{acd}$ = +56 J – (–9 J) B

60. Since there is no area under the line (and no change in volume) W = 0. The temperature (and internal energy) decrease so Q cannot be zero ($Q = \Delta U - W$) C

61. pV = nRT D

62. pressure is the collisions of the molecules of the gas against the container walls. Even though the speed of the molecules is unchanged (constant temperature), the smaller container will cause the molecules to strike the walls more frequently. B

63. Q = 0 in adiabatic processes (choices B and D). $Q = \Delta U - W$. Choices A and C have the same ΔT and hence, same ΔU and since doubling the volume at constant pressure involves *negative* work, while doubling the pressure at constant volume does *no* work, $\Delta U - W$ is greater for the constant pressure process. (The constant temperature process has $\Delta U = 0$ and less work than the constant pressure process) A

64. pV = nRT (watch those units!) C

65. by definition B

66. Isochoric cooling is a path straight down on a pV diagram (to lower pressures) A

67. Work = area under the curve on a *pV diagram*. In the convention stated, work is negative for any expansion. Be careful with the graph since it is a graph of pressure vs. *temperature*. We can find the work by using $|W| = p\Delta V = nR\Delta T$ D

68. $e_c = \dfrac{T_H - T_C}{T_H}$ where $T_H \propto p_B V_B$ (the highest temperature) and $T_C \propto p_D V_D$ (the lowest temperature) gives $e_C = (6p_0V_0 - p_0V_0)/p_0V_0$ E

69. The heat input for this engine occurs during process D⇨A⇨B and the heat exhaust is B⇨C⇨D B
If P_0V_0 corresponds to temperature T_0, the temperatures at points A, B, C and D respectively are $3T_0$, $6T_0$, $2T_0$ and T_0. The change in temperature for each process is then AB = $+3T_0$, BC = $-4T_0$, CD = $-T_0$ and DA = $+2T_0$.
We also have $P_0V_0 = nRT_0$
For the isochoric process, where W = 0, $Q = \Delta U = 3/2\ nR\Delta T$

DA: $Q = 3/2\ nR(2T_0) = 3nRT_0$

BC: $Q = 3/2\ nR(-4T_0) = -6nRT_0$

For the isobaric processes, where $W = -p\Delta V = -nR\Delta T$,

$Q = \Delta U - W = 3/2\ nR\Delta T + nR\Delta T = 5/2\ nR\Delta T$

AB: $Q = 5/2\ nR(3T_0) = 7.5nRT_0$

CD: $Q = 5/2\ nR(-T_0) = -2.5nRT_0$

Putting it all together gives us $Q_{input} = Q_{DA} + Q_{AB} = 10.5nRT_0$ and $Q_{exhaust} = -8.5nRT_0$

$$e = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{10.5nRT_0 - 8.5nRT_0}{10.5nRT_0} = \frac{2}{10.5} = \frac{4}{21}$$

70. $e = \dfrac{W}{Q_H} = \dfrac{Q_H - Q_C}{Q_H}$ E

71. Work = area enclosed by the parallelogram. Since the work done *on* the gas is negative for a clockwise cycle and they are asking for the work done *by* the gas, the answer will be positive. C

72. At constant volume $\Delta U = Q = 3/2\ nR\Delta T$ where in an isochoric process $nR\Delta T = \Delta pV$ so $Q = 3/2\ \Delta pV$, or $\Delta p = 2 \times (+40\text{ J})/(3 \times 0.008\text{ m}^3)$ D

73. $v_{rms} = \sqrt{\frac{3RT}{M}}$ Since hydrogen is 16 times lighter and $v_{rms} \propto \frac{1}{\sqrt{M}}$, $v_H = 4 \times v_O$ B

74. In a reversible (Carnot) engine $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$ (use absolute temperature) D

75. In a reversible (Carnot) engine $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$ C

76. $v_{rms} = \sqrt{\frac{3RT}{M}}$ since $M_O > M_H$ for them to have the same v_{rms} $T_O > T_H$ D

77. $e_c = \frac{T_H - T_C}{T_H}$ (use absolute temperature) B

78. $v_{rms} = \sqrt{\frac{3RT}{M}}$ if v_{rms} is doubled, then T is quadrupled. If T × 4 at constant volume, then p × 4 D

79. $Q_C = 3W$ and $Q_H = Q_C + W = 4W$. $e = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H}$ E

80. The "energy" lost or gained would be the sum of the work done on the gas and the net heat added to the gas, which is the change in internal energy of the gas. Since the gas returns to its original state, $\Delta U = 0$. A

81. $H = \frac{kA\Delta T}{L}$ A

82. An adiabatic expansion is shaped like an isotherm, but brings the gas to a lower temperature. C

83 $Q_{cycle} = Q_{12} + Q_{23} + Q_{31} = +60\text{ J} - 40\text{ J} + 0\text{ J} = +20\text{ J}$ E
$W_{cycle} = \Delta U_{cycle} - Q_{cycle} = 0\text{ J} - (+20\text{ J}) = -20\text{ J} = W_{12} + W_{23} + W_{31}$
where $W_{12} = -Q_{12}$ since $\Delta U_{12} = 0$ and $W_{23} = 0$
so we have $-20\text{ J} = -60\text{ J} + 0\text{ J} + W_{31}$ which gives $W_{31} = +40\text{ J}$
Process 3⇨1 is adiabatic so $\Delta U_{31} = W_{31}$

1983B4

a. Since T is constant, $p_BV_B = p_0V_0$ and $V_B = 2V_0$ gives $p_B = ½\ p_0$

b. $\Delta U = Q + W$, since AB is isothermal, $\Delta U = 0$ and $W = –Q = –1000$ J

c. The entropy of the gas increases because $\Delta S = Q/T$ and Q is positive (heat was added)

d. In a reversible (Carnot) engine $\frac{Q_H}{Q_C} = \frac{T_H}{T_C}$ giving $Q_c = 400$ J

e. Negative. In a clockwise cycle, the work done on the gas is negative. Or for the cycle $Q_{net} = +600$ J and $\Delta U = 0$ so $W = –Q = – 600$ J

1996B7

a. $p_1/T_1 = p_2/T_2$ gives $p_2 = 0.82$ atm $= 8.2 \times 10^4$ Pa

b. F = p × Area = 410 N

c. Since volume and temperature are constant, we can use $p_1V = n_1RT$ and $p_2V = n_2RT$. SUbtractnig the two equations gives $\Delta pV = \Delta nRT$, or $\Delta n = \Delta pV/RT = 5.45 \times 10^{-3}$ mol

1986B5

a. $e_c = \frac{T_H - T_C}{T_H}$ (use absolute temperature) gives $e_c = 0.074$

b. $e = W/Q_H$, or $Q_H = W/e = (100\text{ MW})/(0.074) = 1350$ MW and $Q_C = Q_H – W = 1250$ MW (note Q may represent heat in Joules or rate in Watts)

c. AB is isothermal so $\Delta T = 0$. It is an expansion so W is – and Q = –W
BC is adiabatic so Q = 0. Temperature drops so ΔT is negative.
CD is isothermal so $\Delta T = 0$. It is a compression so W is + and Q = –W
BC is adiabatic so Q = 0. Temperature rises so ΔT is positive.

	Q	ΔT
AB	+	0
BC	0	–
CD	–	0
DA	0	+

2004B5B

a. Since $P_A = P_B$ and $V_A/T_A = V_B/T_B$ giving $T_B = T_2 = T_1/2$

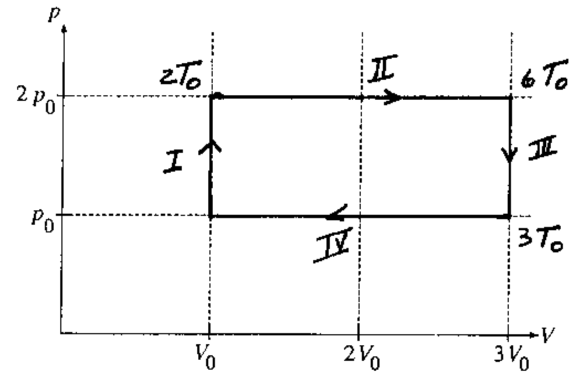
b. CA is an isotherm so $T_A = T_C$ so $P_AV_A = P_CV_C$; $P_1V_1 = P_2(V_1/2)$ giving $P_2 = 2P_1$

c. Work is the area under the line. No work is done from B to C so we just need the area under line AB.
Specifically, $W = –P\Delta V = –P_1(V_1/2 – V_1) = +½P_1V_1$

d. Heat was added in processes BC and CA, but not in AB.
BC: W = 0 so $\Delta U = Q$ and temperature rises so ΔU is positive
CA: $\Delta U = 0$ (isotherm) so Q = –W and it is an expansion so W is negative and therefore Q is positive
AB: Compression so W is + and temperature drops so ΔU is negative and $Q = \Delta U – W$ which must be negative

1989B4

a.



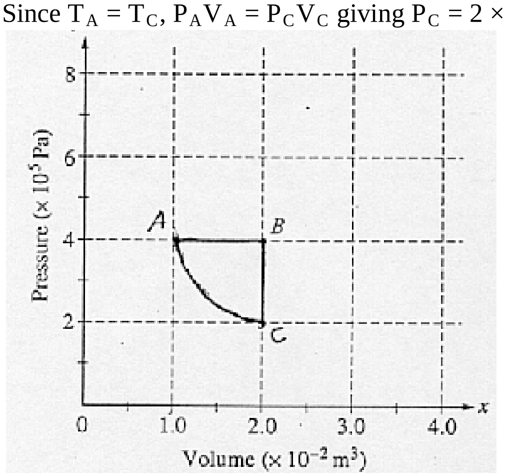
b. i. The work done on the gas is the area enclosed. Area = width × height = $2V_0 \times P_0 = –2P_0V_0$ (negative since it is a clockwise cycle)
   ii. $\Delta U = 0$ for any cycle
   iii. since $\Delta U = 0$, $Q = –W = +2P_0V_0$

c. For process 2, $W = –P\Delta V = –2P_0 \times (3V_0 – V_0) = –4P_0V_0$
   and $\Delta U = 3/2\ nR\Delta T = 3/2\ nR(6T_0 – 2T_0) = +6\ nRT_0 = +6P_0V_0$
   $Q = \Delta U – W = +6\ P_0V_0 – (–4P_0V_0) = +10P_0V_0$

1999B7

a. Since $T_A = T_C$, $P_AV_A = P_CV_C$ giving $P_C = 2 \times 10^5$ Pa

b.



c. This is a clockwise cycle so the work done on the gas is negative.

d. This is a clockwise cycle so this is a heat engine.

2001B6

a. The additional pressure comes from the weight of the added block. $\Delta P = F/A = mg/A = 2.04 \times 10^3$ Pa and $P_2 = P_1 + \Delta P = 1.04 \times 10^5$ Pa

b. A constant temperature, $P_1V_1 = P_2V_2$, or $V_2 = P_1V_1/P_2 = 1.47 \times 10^{-3}\ m^3$

c. Since the external pressure and the added weight do not change, the pressure remains constant, therefore the process from state 2 to state 3 is isobaric

d. For similar reasons as stated above, the process from state 4 to state 1 is also isobaric.

e. Comparing state 1 and state 4, which have equal pressures: $V_1/T_1 = V_4/T_4$, giving $V_4 = V_1T_4/T_1 = 2.05 \times 10^{-3}\ m^3$

2006B5

a. i. $P_1 = P_2$ so $V_1/T_1 = V_2/T_2$ giving $T_2 = 746$ K

ii. $V_1 = V_3$ so $P_1/T_1 = P_3/T_3$ giving $T_3 = 560$ K

b. The net work done is the area enclosed by the triangle = ½ base × height = +6250 J (positive since the cycle is counterclockwise)

c. Since the cycle is counterclockwise, the work done on the gas is positive (more area under the process 2⇨3 in which positive work is done than in process 1⇨2 where negative work is done). In any cycle $\Delta U = 0$ so we have $Q = -W$, therefore Q is negative meaning heat is removed.

2003B5

a. $U_a - U_c = \Delta U_{ca} = Q_{ca} + W_{ca} = 685 \text{ J} + (-120 \text{ J}) = 565 \text{ J}$

b. i/ii. Heat is removed. $\Delta U_{abc} = -\Delta U_{ca} = -565$ J since it is the opposite beginning and end points, the path doesn't matter. $Q = \Delta U - W = -565 \text{ J} - 75 \text{ J} = -640 \text{ J}$

c. $W_{cda} = W_{cd} + W_{da} = 0 + -P\Delta V_{da} = -150 \text{ J}$

d. Heat is added. $\Delta U = +565$ J and $W = -150$ J and $Q = \Delta U - W$

2003B5B

a. $pV = nRT$ so $T = pV/nR = (200 \text{ Pa})(20 \text{ m}^3)/(1 \text{ mol})(8.32 \text{ J/(mol-K)}) = 481 \text{ K}$

b. The net work done is the area enclosed by the triangle = ½ base × height = +4000 J (positive since the cycle is counterclockwise)

c. i/ii. Heat is removed. In one cycle $\Delta U = 0$ so $Q = -W = -4000$ J

d. In a cyclic process $\Delta U = 0$ (the temperature returns to the same value)

e. The entropy is a function of the state of the gas, and after one complete cycle the gas has returned to its original state so the entropy is the same.

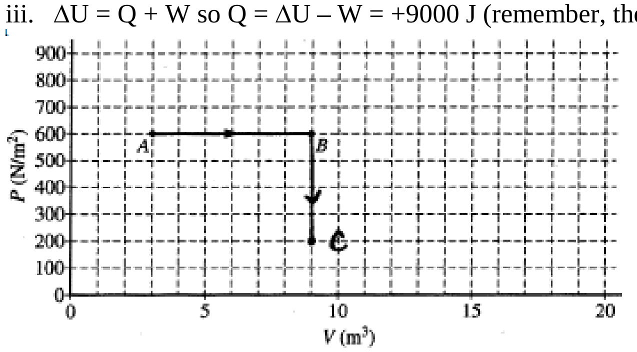
2004B5

a. i. $W = -P\Delta V = -3600$ J. The work done *by* the gas is the negative of the work done *on* the gas, +3600 J

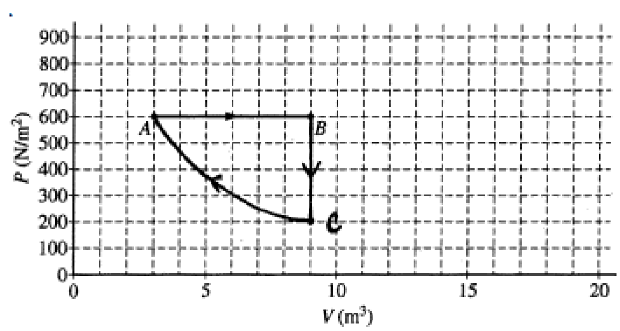
ii. $\Delta U = 3/2\ nR\Delta T$ and the temperatures can be found from $PV = nRT$ giving $T_A = 108$ K and $T_B = 325$ K so $\Delta U = 5400$ J

iii. $\Delta U = Q + W$ so $Q = \Delta U - W = +9000$ J (remember, the W in this equation is the work done *on* the gas)

b.



c.



ii. Heat is removed. In an isothermal process, $\Delta U = 0$ so $Q = -W$ and in a compression W is positive.

2008B5

a.

Process	W	Q	ΔU
$A \rightarrow B$	0	+	+
$B \rightarrow C$	–	+	0
$C \rightarrow A$	+	–	–

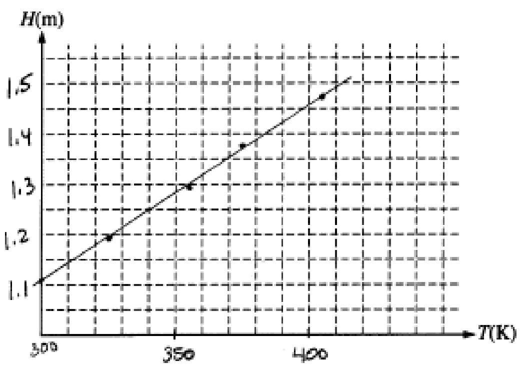
b. ⇨Since process AB is isochoric, $\Delta V = 0$ therefore $W = -P\Delta V = 0$ (also, there is no area under the line)
⇨At constant volume for a fixed number of moles, pressure is directly related to temperature and since the pressure increases, so does the temperature. ΔU is directly related to ΔT so it is positive.
⇨$Q = \Delta U - W$ and $W = 0$

c. Since $T_B = T_C$, $P_BV_B = P_CV_C$ so $V_C = P_BV_B/P_C = 0.005\ m^3$

2005B6

a. The volume of the cylinder = Area × height = AH. PV = nRT then becomes PAH = nRT so H = nRT/PA

b.



c. Calculating the slope of the line above and setting it equal to the slope from the equation of part a: nR/PA gives n = 1.11 moles

---

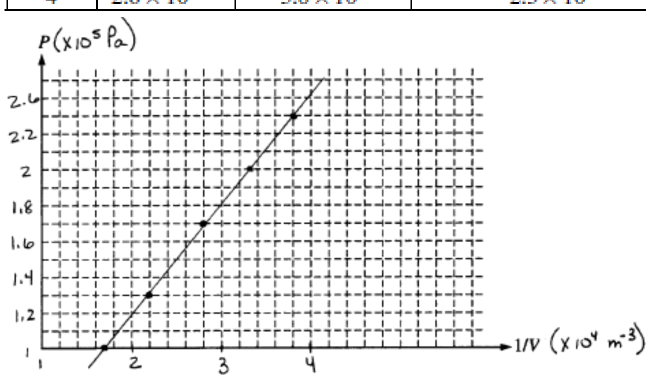
2005B6B

a. PV = nRT or P = (1/V)nRT

b. The total pressure is the atmospheric pressure plus the pressure due to the added mass $P = P_{atm} + mg/A$

m (kg)	V (m^3)	$1/V$ (m^{-3})	P (Pa)
0	6.0×10^{-5}	1.7×10^4	1.0×10^5
1	4.5×10^{-5}	2.2×10^4	1.3×10^5
2	3.6×10^{-5}	2.8×10^4	1.7×10^5
3	3.0×10^{-5}	3.3×10^4	2.0×10^5
4	2.6×10^{-5}	3.8×10^4	2.3×10^5

c.



d. From P = (1/V)nRT, the slope of the above line = nRT. Slope = 6.19 Pa-m^3 so n = .0025 moles

2006B5B

a. i. $T_A = T_B$ so $P_AV_A = P_BV_B$: $P_B/P_A = ½$
   ii. $P_B = P_C$ so $P_C/P_A = P_B/P_A = ½$
   iii. A and B are on the same isotherm so $T_B/T_A = 1$
   iv. $V_C = V_A$ so $P_C/P_A = T_C/T_A = ½$
b. Internal energy depends only on the temperature. Since step I is isothermal there is no change in temperature and thus no change in internal energy
c. $W = –P\ \Delta V$. In step III there is no change in volume, and thus no work done.

2007B5

a. $P = F/A$ so $F = PA = P(\pi R^2) = (4.0 \times 10^5\ \text{Pa})\pi(½\ 0.20\ \text{m})^2 = 1.3 \times 10^4\ \text{N}$
b. $PV = nRT$ gives $V = 1.2 \times 10^{-2}\ \text{m}^3$
c. $W_{\text{on the gas}} = -P\Delta V$ so $W_{\text{by the gas}} = +P\Delta V$ where $\Delta V = Ax = \pi R^2 x$ and x = extra distance pushed by the piston giving $W_{by} = 1.9 \times 10^3\ \text{J}$
d. Heat is transferred to the gas. This is an expansion so W_{on} is negative. For the gas to expand at constant pressure, the temperature must also increase so ΔU is positive. $Q = \Delta U - W$.

2007B5B

a. i. For the piston to be in equilibrium, the gas must hold it up against its own weight and the external force due to the outside pressure: $F = P_{atm}A + Mg$ where $A = \pi R^2 = \pi(D/2)^2 = \pi D^2/4$ so we have $F = ¼P_{atm}\pi D^2 + Mg$
   ii. $P = F/A = F$ from above $\div ¼\pi D^2$ giving $P_{abs} = P_{atm} + 4Mg/\pi D^2$
b. Pressure goes up. If heat is added at constant volume, the temperature goes up and so must the pressure since $P \propto T$ at constant volume.
c. $W = Fx$ (from mechanics) $= (¼P_{atm}\pi D^2 + Mg)x_0$

2008B6B

a. $V_1 = V_2$ so $P_1/T_1 = P_2/T_2$ giving $T_2 = 1500\ \text{K}$; $P_1 = P_3$ so $V_1/T_1 = V_3/T_3$ giving $T_3 = 1800\ \text{K}$
b/c. The net work done is the area enclosed by the triangle = ½ base × height = –100 J (negative since clockwise)
d. For process 1⇨2 W = 0 so $Q = \Delta U = 3/2\ nR\Delta T = (1.5)(0.004\ \text{mol})(8.31\ \text{J/mol-K})(1500\ \text{K} - 300\ \text{K}) = 60\ \text{J}$

2009B4

a. $PV = nRT$ so $T = PV/nR$ and the number of moles $= (2.2 \times 10^3\ \text{g of}\ H_2O)/(18\ \text{g/mole}) = 122.2$ moles. This gives $T = (3 \times 10^5\ \text{Pa})(2\ \text{m}^3)/(122.2\ \text{moles})(8.31\ \text{J/mol-K}) = 591\ \text{K}$
b. The temperature is proportional to the product of P and V. $(PV)_A = 6 \times 10^5\ \text{J}$ and $(PV)_C = 10 \times 10^5\ \text{J}$ so $T_C/T_A = 10/6$ giving $T_C = 985\ \text{K}$
c. Since the temperature increases for process A⇨B⇨C and U is dependent on the temperature, U increases.
d. $W_{ABC} = W_{AB} + W_{BC} = 0 + -P\Delta V = -(4 \times 10^5\ \text{Pa})(2.5\ \text{m}^3 - 2\ \text{m}^3) = -2 \times 10^5\ \text{J}$

1974B6

a. $P_AV_A/T_A = P_BV_B/T_B$; $(1.5\ \text{atm})(1\ \text{L})/T_A = (0.5\ \text{atm})(3\ \text{L})/T_B$ giving $T_A = T_B$
b. Since $T_A = T_B$, $\Delta U_{AB} = 0$. W is the area under the line = –2 L-atm (negative for an expansion) and we have $Q = \Delta U - W = +2$ L-atm = +202.6 J
c. PV/T is constant so highest temperature is at the highest value of PV where P = 1 atm and V = 2 L. PV = nRT gives T = 243 K

1975B3

a. The work done on the gas is the area enclosed by the cycle = length×width $= -4p_0V_0$ (negative since clockwise)
b. In the cycle $\Delta U = 0$ so $Q = -W = +4p_0V_0$
c. Temperature is the same where the product p × V is the same: $A = p_0V_0$; $B = 3p_0V_0$; $C = 9p_0V_0$; $D = 3p_0V_0$; $T_B = T_D$
d. AB: $Q = \Delta U - W = 3/2\ nR\Delta T - 0$ and $\Delta T = 2T_0$; so $Q = +3nRT_0 = +3p_0V_0$
   BC: $Q = \Delta U - W = 3/2\ nR\Delta T - (-p\Delta V)$ and $\Delta T = 6T_0$; so $Q = 9p_0V_0 - (-6p_0V_0) = +15p_0V_0$
   $Q_{BC} > Q_{AB}$

1979B5

a. Process II is isothermal. An isothermal process is one in which the temperature is constant. Thus, from the ideal gas law, the product of pressure and volume is a constant. This condition is satisfied by process II.

b. Process III is adiabatic. In an adiabatic process, both the pressure and the volume must change. Thus, processes I and IV are eliminated. Since process II is isothermal, process III is the only possible adiabatic one.

c. The gas does work in processes I, II and III. Work is done by the gas whenever the volume increases. (negative work is done by the gas when the volume decreases as well)

d. In process IV, no work is done. Since the pressure decreases at constant volume, the temperature also decreases, giving ΔU is negative and with $W = 0$, $\Delta U = Q$ and therefore Q is negative. One could also use the adiabatic process as the dividing line between process in which heat is added and those for which heat is removed. On the adiabatic line, $Q = 0$. For any process from the same initial point that lies above the adiabat, heat is added and for any process that lies below the adiabat, heat is removed.

e. RMS speed is proportional to the kinetic energy which, in turn, is proportional to the temperature. Only in process I does the temperature increase.

1991B3

a. Power is the rate of useful work form an engine so W (which here represents the rate in MW) = 120 MW and $e = W/Q_H = 0.40 = 120\ \text{MW}/Q_H$ giving $Q_H = 300$ MW

b. The rate of heat input from the combustion of oil is 300 Joules per second. Since oil provides 4.4×10^7 joules per kilogram burned we can divide to find the number of kg per second that must be combusted:
$\Delta m/\Delta t = (300 \times 10^6\ \text{J/s}) \div (4.4 \times 10^7\ \text{J/kg}) = 6.82$ kg/s

c. $Q_C = Q_H - W = 180$ MW

1993B5

a. Since $P_a = P_b$, $V_a/T_a = V_b/T_b$ giving $T_b = 750$ K

b/c. $\Delta U_{ab} = 3/2\ nR\Delta T = (1.5)(1\ \text{mole})(8.32\ \text{J/mol-K})(750\ \text{K} - 250\ \text{K}) = 6240$ J
$W_{ab} = -P\Delta V = -(1.2 \times 10^5\ \text{Pa})(51 \times 10^{-3}\ \text{m}^3 - 17 \times 10^{-3}\ \text{m}^3) = -4080$ J
$Q = \Delta U - W = 10{,}320$ J

d. $W = -P\Delta V = 0$ (no area under the line)

e. In a cycle $\Delta U = 0$ so $W = -Q = -1800$ J

f. $e_c = \dfrac{T_H - T_C}{T_H} = 0.66$

1995B5

a. $P = Fv$ (from mechanics) $= mgv = (10\ \text{kg})(10\ \text{m/s}^2)(4\ \text{m/s}) = 400$ W

b. $e_c = \dfrac{T_H - T_C}{T_H} = 0.4$ or 40%

c. i. With an efficiency of 0.4 and useful work done at the rate of 400 W we have $e = (W/t)/(Q_H/t)$ or $(Q_H/t) = 1000$ W
ii. $(Q_C/t) = Q_H/t) - (W/t) = 600$ W
