AP® Physics C: Electricity and Magnetism
2012 Free-Response Questions

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TABLE OF INFORMATION DEVELOPED FOR 2012

CONSTANTS AND CONVERSION FACTORS	
Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant, $G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8$ m/s^2
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit,	$1 \text{ u} = 1.66 \times 10^{-27}$ kg $= 931$ MeV/c^2
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^{3}$ eV•nm
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}$ C^2/N•m^2
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}$ N•m^2/C^2
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	$1 \text{ atm} = 1.0 \times 10^{5}$ N/m^2 $= 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS DEVELOPED FOR 2012

MECHANICS

$\upsilon = \upsilon_0 + at$

$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2$

$\upsilon^2 = \upsilon_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \bullet d\mathbf{r}$

$K = \frac{1}{2}m\upsilon^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \bullet \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{\upsilon^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$\upsilon = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
υ = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$x = x_{\max}\cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \bullet d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$\mathbf{E} = \rho\mathbf{J}$

$I = Nev_d A$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
υ = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$\oint \mathbf{B} \bullet d\boldsymbol{\ell} = \mu_0 I$

$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \bullet d\mathbf{A}$

$\mathcal{E} = \oint \mathbf{E} \bullet d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

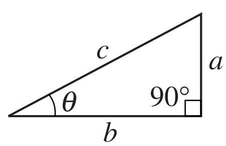
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

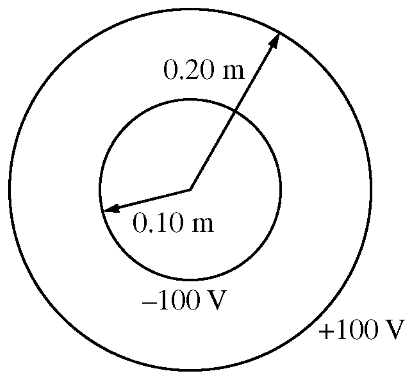
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of -100 V , and the outer shell is set at an electric potential of +100 V, with each potential defined relative to the conventional reference point. Let Q_i and Q_o represent the net charge on the inner and outer shells, respectively, and let *r* be the radial distance from the center of the shells. Express all algebraic answers in terms of Q_i, Q_o, *r*, and fundamental constants, as appropriate.

(a) Using Gauss's Law, derive an algebraic expression for the electric field $E(r)$ for $0.10 \text{ m} < r < 0.20 \text{ m}$.

(b) Determine an algebraic expression for the electric field $E(r)$ for $r > 0.20 \text{ m}$.

(c) Determine an algebraic expression for the electric potential $V(r)$ for $r > 0.20 \text{ m}$.

(d) Using the numerical information given, calculate the value of the total charge Q_T on the two spherical shells $(Q_T = Q_i + Q_o)$.

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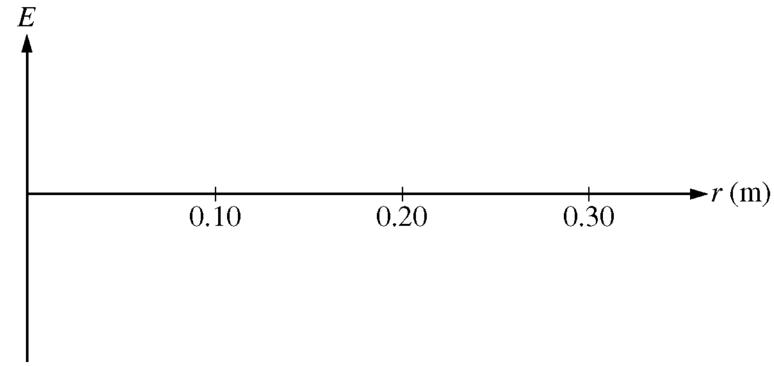
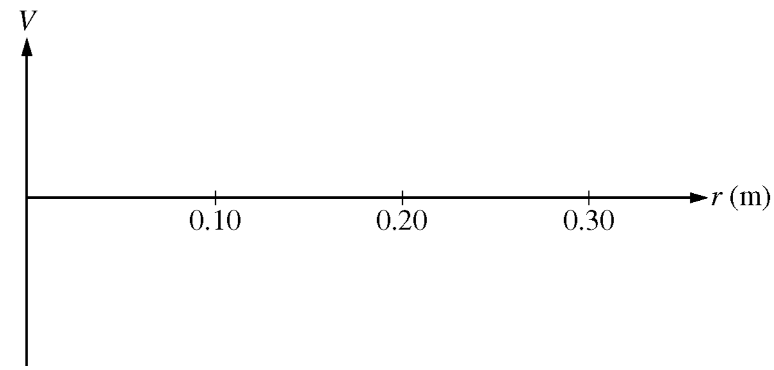
(e) On the axes below, sketch the electric field E as a function of r . Let the positive direction be radially outward.



(f) On the axes below, sketch the electric potential V as a function of r .



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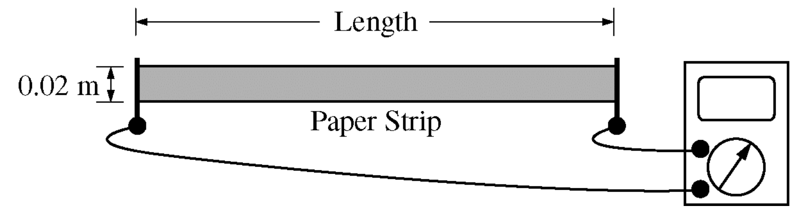
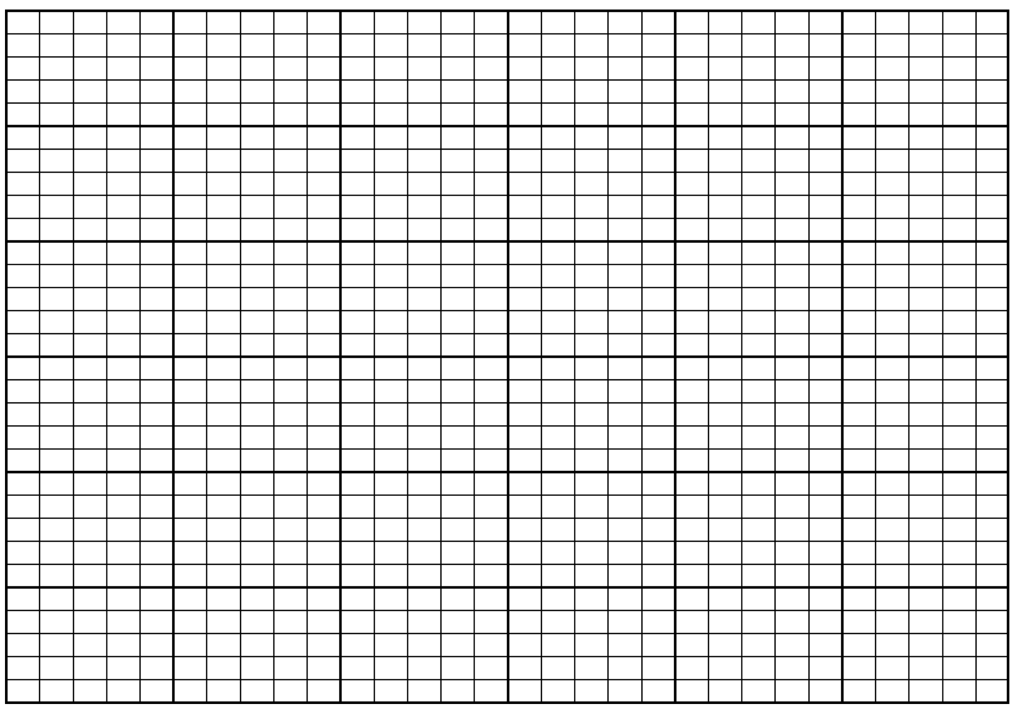


E&M. 2.

A physics student wishes to measure the resistivity of slightly conductive paper that has a thickness of 1.0×10^{-4} m . The student cuts a sheet of the conductive paper into strips of width 0.02 m and varying lengths, making five resistors labeled R1 to R5. Using an ohmmeter, the student measures the resistance of each strip, as shown above. The data are recorded below.

Resistor	R1	R2	R3	R4	R5
Length (m)	0.020	0.040	0.060	0.080	0.100
Resistance (Ω)	80,000	180,000	260,000	370,000	440,000

(a) Use the grid below to plot a linear graph of the data points from which the resistivity of the paper can be determined. Include labels and scales for both axes. Draw the straight line that best represents the data.



(b) Using the graph, calculate the resistivity of the paper.

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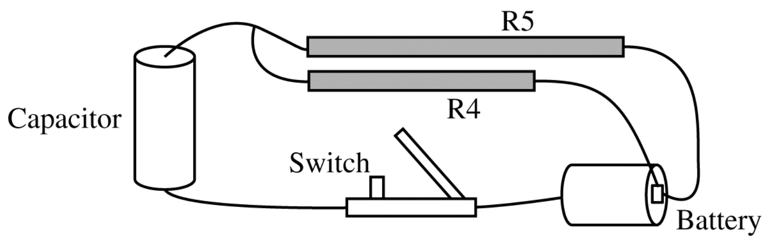
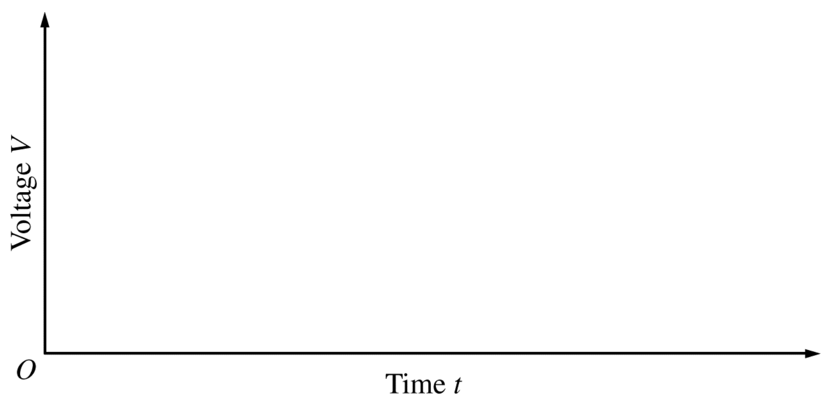


The student uses resistors R4 and R5 to build a circuit using wire, a 1.5 V battery, an uncharged 10 μF capacitor, and an open switch, as shown above.

(c) Calculate the time constant of the circuit.

(d) At time t = 0, the student closes the switch. On the axes below, sketch the magnitude of the voltage V_c across the capacitor and the magnitudes of the voltages V_{R4} and V_{R5} across each resistor as functions of time *t*. Clearly label each curve according to the circuit element it represents. On the axes, explicitly label any intercepts, asymptotes, maxima, or minima with values or expressions, as appropriate.



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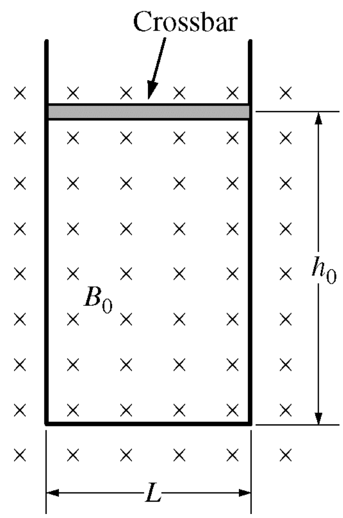

E&M. 3.

A closed loop is made of a U-shaped metal wire of negligible resistance and a movable metal crossbar of resistance R. The crossbar has mass m and length L. It is initially located a distance h_0 from the other end of the loop. The loop is placed vertically in a uniform horizontal magnetic field of magnitude B_0 in the direction shown in the figure above. Express all algebraic answers to the questions below in terms of B_0, L, m, h_0, R, and fundamental constants, as appropriate.

(a) Determine the magnitude of the magnetic flux through the loop when the crossbar is in the position shown.

The crossbar is released from rest and slides with negligible friction down the U-shaped wire without losing electrical contact.

(b) On the figure below, indicate the direction of the current in the crossbar as it falls.

Justify your answer.

(c) Calculate the magnitude of the current in the crossbar as it falls as a function of the crossbar's speed v.

(d) Derive, but do NOT solve, the differential equation that could be used to determine the speed v of the crossbar as a function of time t.

(e) Determine the terminal speed v_T of the crossbar.

(f) If the resistance R of the crossbar is increased, does the terminal speed increase, decrease, or remain the same?

____ Increases   ____ Decreases   ____ Remains the same

Give a physical justification for your answer in terms of the forces on the crossbar.

STOP

END OF EXAM