AP® Physics C: Mechanics 2014 Scoring Guidelines

Question 1

15 points total | **Distribution of points**

(a) 2 points

For using a correct expression for the potential energy of a spring expressed as the integral of the force — 1 point

$$U_S = \int F(x)dx$$

$$U_S = \int_0^s \left(Ax^2 + Bx\right)dx$$

Evaluate the definite integral to get an answer with the correct magnitude.

$$U_S = \left[\frac{1}{3}Ax^3 + \frac{1}{2}Bx^2\right]_0^s$$

For any correct answer with a local minimum at $x = 0$ — 1 point

$$U_S = \frac{1}{3}As^3 + \frac{1}{2}Bs^2$$

Note: Full credit is given for any consistent use of sign, since the guidelines are concerned with the magnitude. Full credit is also given if there is a constant term added to the correct expression.

(b)

i. 3 points

For any statement of conservation of mechanical energy — 1 point

$$U_S = K$$

For using the expression for potential energy from part (a) and a correct expression for kinetic energy — 1 point

$$\frac{1}{3}As^3 + \frac{1}{2}Bs^2 = \frac{1}{2}mv^2$$

Solve for v^2

$$v^2 = 2\left(\frac{1}{3}As^3 + \frac{1}{2}Bs^2\right)\Big/m$$

For correct substitution — 1 point

$$v^2 = 2\left(\left(\frac{1}{3}\right)\left(200\,\mathrm{N/m^2}\right)(0.040\ \mathrm{m})^3 + \left(\frac{1}{2}\right)(150\,\mathrm{N/m})(0.040\ \mathrm{m})^2\right)\Big/(0.30\ \mathrm{kg})$$

$$v^2 = 0.828\ \mathrm{m^2/s^2}$$

$$v = 0.91\ \mathrm{m/s}$$

Question 1 (continued)

Distribution of points

(b) continued

ii. 3 points

For using a correct expression for impulse in terms of change in velocity — 1 point

$J = \Delta p = m(v_2 - v_1)$

For substitution of values consistent with the answer in part (b)i — 1 point

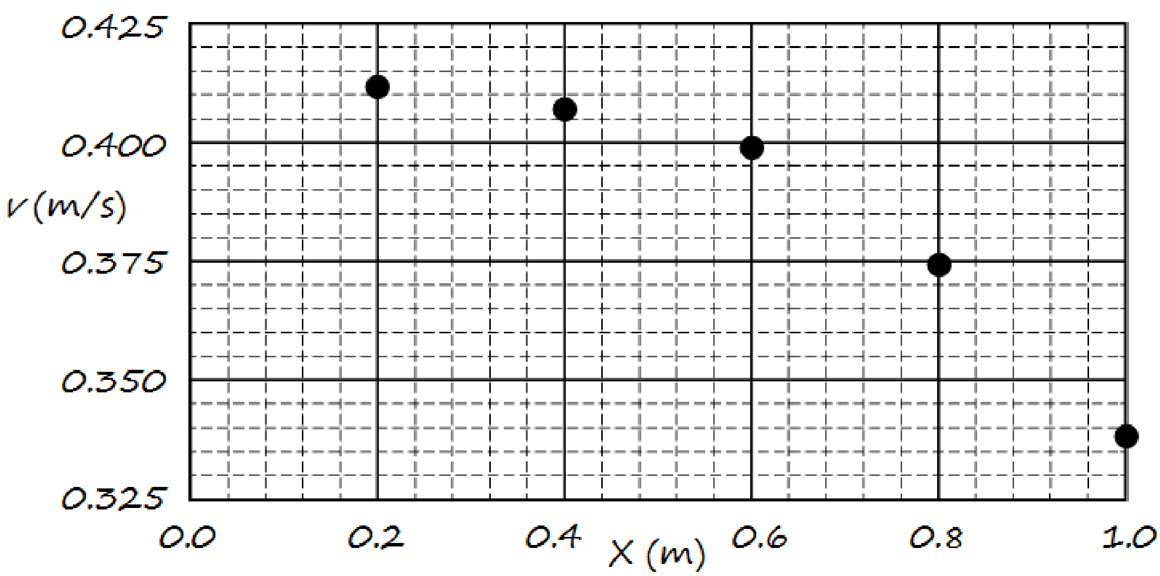
$J = (0.30\ \text{kg})(0.91\ \text{m/s} - 0)$

$J = 0.27\ \text{kg}\bullet\text{m/s} \quad \text{or} \quad \text{N}\bullet\text{s}$

Units 1 point

For correct units in both answers in part (b) — 1 point

(c) 3 points



For correctly labeling both axes with variables and units — 1 point

For correctly scaling both axes with scales that are linear and such that the curvature in the data is apparent — 1 point

For correctly plotting the data points — 1 point

Question 1 (continued)

	Distribution of points
(d) i. 2 points	
For stating that the measured initial speed of the cart is greater than the predicted value	1 point
For correctly identifying a source of error regarding the initial speed of the cart	1 point

Examples:

The student compressed the spring more than was determined. This would lead to more potential energy in the spring and greater kinetic energy for the cart. The cart would therefore move faster than predicted.

The table is not level, sloping downward would result in a greater measured speed.

The constants A and B for the spring are not accurate. The true values are larger than what is given. This would lead to smaller predicted potential energy of the spring and a smaller predicted value for the kinetic energy of the cart. Therefore, the cart would move faster than predicted

ii. 2 points	
For correctly identifying the trend	1 point
For a correct physical explanation for the cart slowing down	1 point

Examples:

Friction in the axles and air resistance against the cart are slowing it down.

The track is not perfectly level and the cart is going uphill. This is slowing down the cart.

Question 2

15 points total | **Distribution of points**

(a) 3 points

For some correct application of conservation of energy — 1 point

$U_i = K_f$

For substituting correct expressions for potential and kinetic energy — 1 point

$mgh_i = \frac{1}{2}mv_f^2$

$gh = \frac{1}{2}v_0^2$

For a correct answer — 1 point

$h = \frac{v_0^2}{2g}$

Alternate solution — *Alternate points*

For using correct kinematics and dynamics equations — *1 point*

$v_f^2 = v_i^2 + 2a(s_f - s_i)$

$F = ma$

For substituting correct variables — *1 point*

$mg\sin\theta = ma$, $\sin\theta = h/L$ *where L is the length of the ramp, so* $a = gh/L$

$v_0^2 = 0 + 2\left(\frac{gh}{L}\right)L$

For a correct answer — *1 point*

$h = \frac{v_0^2}{2g}$

(b)

i. 2 points

For selecting "Zero" — 1 point

For a correct explanation of why the vertical component of the net force is zero. The explanation must be linked to the fact that there is no acceleration. — 1 point

Example:

The block does not accelerate vertically, therefore the component of the net force in the vertical direction must be zero.

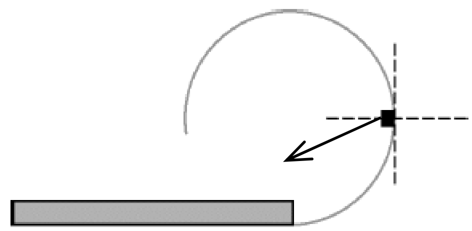
Note: No credit is earned if an incorrect choice is selected.

Question 2 (continued)

Distribution of points

(b) (continued)

ii. 3 points



For drawing a single arrow pointing into the third quadrant	2 points
For correct justification of both components	1 point

Example:

The horizontal component of the net force must provide a centripetal force and also slow down the block. So it must point inward and against the direction of motion.

Note: If components are drawn, partial credit can be earned for the following.

For drawing an arrow pointing left (toward the center of the circle) with correct justification	*1 point*
For drawing an arrow pointing toward the bottom of the page with correct justification	*1 point*

A point is deducted for each incorrect vector drawn, with a maximum 2 point deduction

(c) 1 point

The normal force exerted by the wall is the centripetal force.

$F_N = F_C$

For a correct answer — 1 point

$F_N = mv^2/R$

Note: Since the statement of part (c) says to "determine" an expression, credit is given for just stating the correct answer.

(d) 3 points

For correctly using the frictional force in an expression of Newton's second law — 1 point

$-F_f = ma_t$

$-\mu F_N = ma_t$

For substituting the normal force from part (c) into a correct expression — 1 point

$-\mu mv^2/R = ma_t$

For an answer consistent with part (c) — 1 point

$a_t = -\mu v^2/R$

Note: Since the question asks for the magnitude of the acceleration, the negative sign is not needed but students are not penalized for including it.

Question 2 (continued)

Distribution of points

(e) 3 points

For substituting dv/dt for a into the answer from part (d), <u>or</u> substituting dv/dt for a and the friction force for F_{net} into Newton's second law — 1 point

$$\frac{dv}{dt} = -\frac{\mu v^2}{R} \quad \text{or} \quad m\frac{dv}{dt} = -F_f$$

For including the negative sign — 1 point

Substituting for F_f produces the same relationship as the first equation above.

For separation of variables and using correct limits — 1 point

$$\frac{1}{v^2}dv = -\frac{\mu}{R}dt$$

$$\int_{v_0}^{v} \frac{1}{v^2}dv = \int_0^t -\frac{\mu}{R}dt$$

Integrate the equation to solve for v.

$$\left[-\frac{1}{v}\right]_{v_0}^{v} = \left[-\frac{\mu t}{R}\right]_0^t$$

$$\frac{1}{v} - \frac{1}{v_0} = \frac{\mu t}{R}$$

$$v = \frac{Rv_0}{R + \mu v_0 t} \quad \text{or} \quad \frac{v_0}{1 + \mu v_0 t/R}$$

Question 3

15 points total — **Distribution of points**

(a) 2 points

For using a correct kinematics expression for the vertical motion — 1 point

$$y - y_0 = v_1 t + \frac{1}{2}at^2$$

$$h = 0 + \frac{1}{2}gt^2$$

For a correct answer — 1 point

$$t = \sqrt{2h/g}$$

(b) 3 points

For a statement of conservation of momentum or Newton's third law — 1 point

$$p_i = p_f$$

For substituting the momentum of the stone into a correct expression for conservation of momentum — 1 point

For substituting the momentum of the person-disk system into a correct expression for conservation of momentum — 1 point

$$0 = m_1 v_1 + m_2 v_2$$

$$0 = \left(\frac{m}{20}\right)(v_0) + \left(m + \frac{m}{2}\right)v$$

$$\frac{3}{2}mv = -\frac{1}{20}mv_0$$

$$v = -\frac{1}{30}v_0$$

Note: Since the question asks for speed, the negative sign is not needed. There is no penalty for including it.

(c) 3 points

For using a correct expression of Newton's second law with friction as the net force — 1 point

$$f = ma$$

$$\mu mg = ma$$

$$a = \mu g$$

For correctly substituting the velocity from part (b) and the acceleration into an appropriate kinematics equation — 1 point

$$v_2 = v_1 + at$$

$$0 = -\frac{1}{30}v_0 + \mu g t$$

For an answer consistent with part (b) — 1 point

$$t = \frac{v_0}{30\mu g}$$

Question 3 (continued)

Distribution of points

(c) (continued)

Alternate Solution — *Alternate points*

For using a correct expression for impulse and change in momentum, with friction as the force — *1 point*

$Ft = \Delta p = m(\upsilon_2 - \upsilon_1)$

$-\mu mgt = m(\upsilon_2 - \upsilon_1)$

For correct substitutions — *1 point*

$-\mu mgt = m\left(0 - \dfrac{1}{30}\upsilon_0\right)$

For an answer consistent with part (b) — *1 point*

$t = \dfrac{\upsilon_0}{30\mu g}$

(d) 4 points

For a statement of conservation of total angular momentum — 1 point

$L_i = L_f$

$L = mr\upsilon$ for linear motion

$L = I\omega$ for rotation

For substituting the angular momentum of the stone into a correct expression of conservation of angular momentum — 1 point

For substituting the angular momentum of the person into a correct expression of conservation of angular momentum — 1 point

For substituting the angular momentum of the disk into a correct expression of conservation of angular momentum — 1 point

$0 = m_s r_s \upsilon_s + I_D\omega_D + I_P\omega_P$

$$0 = \left(\frac{m}{20}R\upsilon_0\right) - \left(\frac{mR^2}{2}\omega + \frac{m}{2}R^2\omega\right)$$

$$\left(\frac{m}{20}R\upsilon_0\right) = \left(\frac{mR^2}{2}\omega + \frac{m}{2}R^2\omega\right) = mR^2\omega$$

$$\omega = \frac{\frac{m}{20}R\upsilon_0}{mR^2}$$

$$\omega = \frac{\upsilon_0}{20R}$$

Question 3 (continued)

Distribution of points

(e) 3 points

For selecting "Less than"	1 point
For correctly identifying a decrease in the stone's angular momentum	1 point
For correctly identifying a decrease in the person's rotational inertia	1 point

Example: If the stone is thrown from a point closer to the center of the disk, its angular momentum around the center of the disk decreases, resulting in a decrease of the angular momentum gained by the disk/person system. In addition, the person's rotational inertia is decreased, which decreases the rotational inertia of the disk-person system. The effect of this decrease in the rotational inertia of the person is less than the effect of the decrease in the angular momentum. Therefore, the disk/person system must have a decrease in its angular speed.

Example: From angular momentum conservation, we have

$$L_{stone} = L_{disk+person}$$

$$L_{stone} = \left(I_{disk} + I_{person}\right)\omega$$

$$\boxed{m_{stone}v_0 r = \left(\frac{mR^2}{2} + \frac{m}{2}r^2\right)\omega}$$

where r is the distance of the person from the center of the disk. If r is decreased from R to $R/2$, then the stone's rotational inertia (left-hand-side of the boxed equation) is reduced to half of its previous value. The person's rotational inertia is reduced to one quarter of its previous value, but the combined disk+person rotational inertia (included in right-hand-side of boxed equation) is still greater than half of its previous value. Solving for ω therefore yields a value that is less than it was previously.

Note: If neither the stone nor the person are explicitly mentioned, one point may still be earned for the justification. If just one of either the stone or person are explicitly mentioned, both points for the justification may be earned.

Note: Indicating a decrease in the torque is equivalent to indicating a decrease in the change in angular momentum.