

# TRIANGLE TRIGONOMETRY

## Definitions

**polygon** - A closed plane figure formed by three or more line segments joined at their endpoints.  
**triangle** - A three sided polygon.     **right angle** - A 90° angle.     **right triangle** - A triangle with a right angle.  
**hypotenuse** - The side opposite the right angle of a right triangle; also the longest side of a right triangle.  
**altitude** - A perpendicular (90°) line segment from one side of a triangle to the opposite vertex.  
**median** - A line segment from one vertex of a triangle to the midpoint of the opposite side.

## Triangle Centers

<b>circumcenter</b> intersection of perpendicular bisectors	<b>incenter</b> intersection of angle bisectors	<b>centroid</b> intersection of medians; also the center of gravity	<b>orthocenter</b> intersection of altitudes
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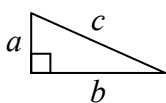
## Postulates, theorems, and corollaries

**Angle sum theorem** - The sum of the angles in a triangle is 180°. **corollaries:** The acute angles of a right triangle are complimentary; There can be at most one right or obtuse angle in a triangle.  
**3<sup>rd</sup> angle theorem** - If two angles of a triangle are congruent ( $\cong$ ) to those of another, then the 3<sup>rd</sup> angles are  $\cong$ .  
**Exterior angle theorem** - An exterior angle of a triangle is equal to the sum of the two remote interior angles.  
**SSS postulate** - If the sides of one triangle are  $\cong$  to those of another, then the triangles are  $\cong$ .  
**SAS postulate** - If two sides and the included angle of a triangle are  $\cong$  to those of another, then the triangles are  $\cong$ .  
**ASA postulate** - If two angles and the included side of a triangle are  $\cong$  to those of another, then the triangles are  $\cong$ .  
**AAS theorem** - If two angles and a nonincluded side of a triangle are  $\cong$  to those of another, then the triangles are  $\cong$ .  
**Isosceles triangle theorem** - If two sides of a triangle are  $\cong$ , then the angles opposite those sides are  $\cong$ .

## Right Triangles

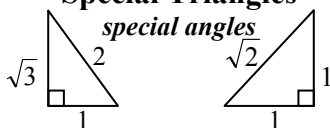
### Pythagorean Theorem

$$a^2 + b^2 = c^2$$



### Special Triangles

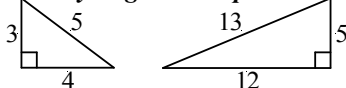
*special angles*



**30-60-90**

**45-45-90**

### Pythagorean triplets



### Trigonometric Functions

**SOH-CAH-TOA**

$$\sin = \frac{\text{Opp}}{\text{Hyp}} \quad \cos = \frac{\text{Adj}}{\text{Hyp}} \quad \tan = \frac{\text{Opp}}{\text{Adj}}$$

### Reciprocal functions

$$\csc = \frac{\text{hyp}}{\text{opp}} \quad \sec = \frac{\text{hyp}}{\text{adj}} \quad \cot = \frac{\text{adj}}{\text{opp}}$$

### Inverse functions

$$\sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) = \theta \quad \cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right) = \theta$$

$$\tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right) = \theta$$

## Oblique Triangles

### acute

*all angles less than 90°*



**equilateral = equiangular**



**isosceles**     **scalene**

*2 sides equal     no sides equal*

### obtuse

*one angle greater than 90°*



### Law of cosines

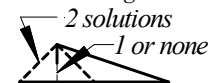
$$a^2 = b^2 + c^2 - 2bc \cos A$$

*used if SAS or SSS known*

### Law of sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

*used if AAS or ASA known or for SSA the "ambiguous case"*



\*AAA has no solution

## Area

### Right triangles

$$A = \frac{1}{2}bh$$

### Oblique triangles

SAS known

$$A = \frac{1}{2}ab \sin C$$

SSS known

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{1}{2}(a+b+c)$$

**Heron's formula**