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LIMITS

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DEFINITION

Limits describe a function's behavior as the independent variable approaches a certain value; a limit does not depend on the value of the function at that point. Oscillation is a common case where no limit exists; unbounded increase/decrease has limits of +/− infinity.

Notation

Two-sided limits

"The limit of $f(x)$ as x approaches a"

$$\lim_{x\to a} f(x)$$

A two-sided limit only exists if both one-sided limits exist and are equal.

One-sided limits

"The limit of $f(x)$ as x approaches a from the left"

$$\lim_{x\to a^-} f(x)$$

"The limit of $f(x)$ as x approaches a from the right"

$$\lim_{x\to a^+} f(x)$$

Limits at infinity

"The limit of $f(x)$ as x approaches positive infinity"

$$\lim_{x\to +\infty} f(x)$$

"The limit of $f(x)$ as x approaches negative infinity"

$$\lim_{x\to -\infty} f(x)$$

* $\lim f(x)$ is used when all preceding cases apply.

Properties of Limits

The limit of a constant is that constant:

$$\lim c = c$$

The limit of a sum is the sum of the limits:

$$\lim[f(x)+g(x)] = \lim f(x) + \lim g(x)$$

The limit of a difference is the difference of the limits:

$$\lim[f(x)-g(x)] = \lim f(x) - \lim g(x)$$

The limit of a product is the product of the limits:

$$\lim[f(x)\cdot g(x)] = \lim f(x)\cdot \lim g(x)$$

The limit of a quotient is the quotient of the limits:

$$\lim \frac{f(x)}{g(x)} = \frac{\lim f(x)}{\lim g(x)}, \lim g(x) \neq 0$$

The limit of a power is the power of the limit:

$$\lim[f(x)]^n = [\lim f(x)]^n$$

The limit of a root is the root of the limit:

$$\lim \sqrt[n]{f(x)} = \sqrt[n]{\lim f(x)}$$

Evaluating Limits

Limits are evaluated using many different techniques, among which are: examination (pattern recognition), substitution, and algebraic simplification. The following theorems apply for n > 0.

Polynomials

$$\lim cx = c\lim x \qquad \lim_{x\to a} x = a \qquad \lim_{x\to a} f(x) = f(a)$$

$$\lim_{x\to a}(c_n x^n + c_{n0-1}x^{n-1} + \cdots) = c_n a^n + c_{n-1}a^{n-1} + \cdots$$

$$\lim_{x\to\pm\infty}(c_n x^n + c_{n-1}x^{n-1} + \cdots) = c_n \lim_{x\to\pm\infty} x^n$$

Rationals

$$\lim_{x\to a^-}\frac{1}{x-a} = -\infty \qquad \lim_{x\to\pm\infty}\frac{1}{x-a} = 0$$

$$\lim_{x\to a^+}\frac{1}{x-a} = +\infty \qquad \lim_{x\to 0^+}\frac{1}{x^n} = +\infty \qquad \lim_{x\to\pm\infty}\frac{1}{x^n} = 0$$

$$\lim_{x\to 0^-}\frac{1}{x^n} = -\infty, n\,\text{odd} \qquad \lim_{x\to 0^-}\frac{1}{x^n} = +\infty, n\,\text{even}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f(a)}{g(a)}, g(a)\neq 0$$

$$\lim_{x\to\pm\infty}\frac{ax^n + bx^{n-1} + \cdots}{cx^m + dx^{m-1} + \cdots} = \frac{a}{c}\lim_{x\to\pm\infty} x^{n-m}$$

Powers

$$\lim_{x\to a} x^n = a^n$$

$$\lim_{x\to+\infty} x^n = +\infty \qquad \lim_{x\to\pm\infty} x^{-n} = 0$$

$$\lim_{x\to-\infty} x^n = -\infty, n\,\text{odd}$$

$$\lim_{x\to-\infty} x^n = +\infty, n\,\text{even}$$

Trigonometric

$$\lim_{x\to a}\sin x = \sin a \qquad \lim_{x\to a}\cos x = \cos a$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1 \qquad \lim_{x\to 0}\frac{1-\cos x}{x} = 0$$

Composition of Functions

$$\lim f(g(x)) = f(\lim g(x))$$